РЕДОВИ РЕАЛНИХ БРОЈЕВА

~друга контролна вежба~

I) КОЈЕМ ХИПЕРХАРМОНИЈСКОМ РЕДУ ЈЕ ЕКВИКОНВЕРГЕНТАН РЕД И ДА ЛИ ОН КОНВЕРГИРА

1. $\sum_{n=1}^{\infty}\frac{1}{n(n+2)}$ $\quad \frac{1}{n^2+2n}=\frac{1}{n^2(1+2/n)}=\frac{1}{n^2}$

$\sim\sum_{n=1}^{\infty}\frac{1}{n^2}$ КОНВЕРГИРА РЕД

2. $\sum_{n=2}^{\infty}\frac{n}{n^2-1}\sim\frac{n}{n^2(1-1/n^2)}=\frac{n}{n^2}=\frac{1}{n}$ $\quad \sum_{n=2}^{\infty}\frac{1}{n}$ РЕД ДИВЕРГИРА

3. $\sum_{n=1}^{\infty}\frac{n}{n^3+2}$ $\quad \frac{n}{n^3(1+2/n^3)}=\frac{n}{n^3}=\frac{1}{n^2}\sim\sum_{n=1}^{\infty}\frac{1}{n^2}$ РЕД КОНВЕРГИРА

4. $\sum_{n=1}^{\infty}\frac{n^2+2}{3n^2+n}$ $\quad \frac{n^2+2}{3n^2+n}=\frac{1}{3}$ РЕД ДИВЕРГИРА $\frac{1}{3}\sum_{n=1}^{\infty}\frac{1}{n^0}$

5. $\sum_{n=1}^{\infty}\frac{n+2}{n^2+2}$ $\quad \frac{n+2}{n^2+2}=\frac{n(1+2/n)}{n^2(1+2/n^2)}=\frac{1}{n}$ $\quad \sum_{n=1}^{\infty}\frac{1}{n}$ РЕД ДИВЕРГИРА

6. $\sum_{n=1}^{\infty}\frac{n^2+1}{n}\sim\sum_{n=1}^{\infty}\frac{n^2(1+1/n^2)}{n}\sim\frac{n^2}{n}=n$ РЕД ДИВЕРГИРА $\sum_{n=1}^{\infty}\frac{1}{n^{-1}}$

7. $\sum_{n=1}^{\infty}\frac{2n^2-5}{3n^3+2n-1}\sim\sum_{n=1}^{\infty}\frac{2n^2(1-5/2n^2)}{3n^3(1+2/3n^2-1/3n^3)}\sim\frac{2}{3}\sum_{n=1}^{\infty}\frac{1}{n}$ РЕД ДИВЕРГ

8. $\sum_{n=1}^{\infty}\frac{n^3+2n^2-1}{7n^7+n}\sim\sum_{n=1}^{\infty}\frac{n^3(1+2/n-1/n^3)}{7n^7(1+1/n^6)}\sim\frac{1}{7}\sum_{n=1}^{\infty}\frac{1}{n^4}$ РЕД КОНВЕРГ

9. $\sum_{n=1}^{\infty}\frac{n(n+1)}{2n^3-n-2}\sim\sum_{n=1}^{\infty}\frac{n^2(1+1/n)}{2n^3(1-1/2n^2-1/n^3)}\sim\frac{1}{2}\sum_{n=1}^{\infty}\frac{1}{n}$ РЕД ДИВЕРГИ

10. $\sum_{n=1}^{\infty}\frac{(n+1)^2}{n^3+n^2+n+1}\sim\sum_{n=1}^{\infty}\frac{n^2(1+1/n)^2}{n^3(1+1/n+1/n^2+1/n^3)}\sim\sum_{n=1}^{\infty}\frac{1}{n}$

11. $\sum_{n=1}^{\infty}\frac{1}{\sqrt{n(n+1)}}\sim\sum_{n=1}^{\infty}\frac{1}{n\sqrt{1+1/n}}\sim\sum_{n=1}^{\infty}\frac{1}{n}$ РЕД ДИВЕРГИРА

12. $\sum_{n=1}^{\infty} \frac{\dots}{n+2} \sim \sum_{n=1} \frac{\dots}{n(1+2/n)} \sim \sum_{n=1} \frac{1}{n^{1/2}}$ ДИВЕРГИРА

13. $\sum_{n=1}^{\infty} \frac{\sqrt{n+1}}{2n^2-1} \sim \sum_{n=1}^{\infty} \frac{\sqrt{n}\sqrt{1+1/n}}{2n^2(1-1/2n^2)} \sim \frac{1}{2}\sum_{n=1}^{\infty} \frac{1}{n^{3/2}}$ РЕД КОНВЕРГИРА

14. $\sum_{n=1}^{\infty} \frac{\sqrt{n^2+1}}{(2n)^2+3} \sim \sum_{n=1}^{\infty} \frac{n\sqrt{1+1/n^2}}{4n^2(1+3/4n^2)} \sim \frac{1}{4}\sum_{n=1}^{\infty} \frac{1}{n}$ РЕД ДИВЕРГИРА

15. $\sum_{n=1}^{\infty} \frac{\sqrt{4n^3+1}}{n^2-2n-5} \sim \sum_{n=1}^{\infty} \frac{2n^{3/2}\sqrt{1+1/4n^3}}{n^2(1-2/n-5/n^2)} \sim 2\sum_{n=1}^{\infty} \frac{1}{n^{1/2}}$ РЕД ДИВЕРГИРА

16. $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n^2-4}+2n} \sim \sum_{n=1}^{\infty} \frac{1}{n(\sqrt{1-4/n^2}+2)} \sim \sum_{n=1}^{\infty} \frac{1}{2n\cdot\frac{3}{2}} \sim$

$\sim \frac{1}{3}\sum_{n=1}^{\infty} \frac{1}{n}$ РЕД ДИВЕРГИРА

17. $\sum_{n=1}^{\infty} \frac{\sqrt[3]{n}+2}{5+n^2} \sim \sum_{n=1}^{\infty} \frac{n^{1/3}+2}{5+n^2} \sim \sum_{n=1}^{\infty} \frac{n^{1/3}(1+2/n^{1/3})}{n^2(5/n^2+1)} \sim \sum_{n=1}^{\infty} \frac{1}{n^{5/3}}$ РЕД КОНВЕРГ.

18. $\sum_{n=1}^{\infty} \frac{\sqrt[3]{n^2+2n}}{3n+\sqrt{n^3}} \sim \sum_{n=1}^{\infty} \frac{n^{2/3}\sqrt[3]{1+2/n}}{n^{3/2}(3/n^{1/2}+1)} \sim \sum_{n=1}^{\infty} \frac{1}{n^{5/6}}$ РЕД ДИВЕРГИРА

19. $\sum_{n=1}^{\infty} \frac{3n+2}{\sqrt[3]{n^3+2n+1}} \sim \sum_{n=1}^{\infty} \frac{3n(1+2/3n)}{n\sqrt[3]{1+2/n^2+1/n^3}} \sim 3\sum_{n=1}^{\infty} \frac{1}{n^0}$ РЕД ДИВЕРГИРА

20. $\sum_{n=1}^{\infty} \frac{1+\sqrt[5]{n^2}}{\sqrt[4]{n^7+3n^2}} \sim \sum_{n=1}^{\infty} \frac{n^{2/5}(1/n^{2/5}+1)}{n^{7/4}(\sqrt[4]{1+3/n^5})} \sim \sum_{n=1}^{\infty} \frac{1}{n^{27/20}}$ РЕД КОНВЕРГИРА

21. $\sum_{n=1}^{\infty} \frac{2n^3+4n^2+3}{n\sqrt[5]{n^{16}+n^{11}+1}} \sim \sum_{n=1}^{\infty} \frac{2n^3(1+2/n+3/2n^3)}{n^{21/5}\sqrt[5]{1+1/n^5+1/n^{16}}} \sim$

$\sim 2\sum_{n=1}^{\infty} \frac{1}{n^{16/5}}$ РЕД КОНВЕРГИРА

22. $\sum_{n=1}^{\infty} \frac{\sqrt[7]{n^5+3n^2-1}}{\sqrt[3]{(n^2+2)^2}+3n-5} \sim \sum_{n=1}^{\infty} \frac{n^{5/7}\sqrt[7]{1+3/n^3-1/n^5}}{n^{4/3}(\sqrt[3]{(1+\frac{2}{n^2})^2}+3/n^{1/3}-5/n^{4/3})} \sim$

$\sim \sum_{n=1}^{\infty} \frac{1}{n^{13/21}}$ РЕД КОНВЕРГИРА

23. $\sum_{n=1} \frac{\dots}{\sqrt[3]{n}+4,8} \sim \sum_{n=1} \frac{\dots}{n^{1/3}(1+\frac{4,8}{\sqrt[3]{n}})} \sim \sum_{n=1} n^{2/3}$ ДИВЕРГИРА

24. $\sum_{n=1}^{\infty} \frac{\sqrt[3]{n^3+2n-1}+\sqrt{n^3}}{n^2-1} \sim \sum_{n=1}^{\infty} \frac{n^{3/2}(\sqrt[3]{1/n^{3/2}+2/n^{3/2}-1/n^{3/2}}+1)}{n^2(1-1/n^2)}$

$\sim \sum_{n=1}^{\infty} \frac{1}{n^{1/2}}$ РЕД ДИВЕРГИРА

25. $\sum_{n=1}^{\infty} (\sqrt{n+2}-\sqrt{n+1})\cdot\frac{\sqrt{n+2}+\sqrt{n+1}}{\sqrt{n+2}+\sqrt{n+1}} \sim \sum_{n=1}^{\infty} \frac{n+2-n-1}{\sqrt{n}(\sqrt{1+2/n}+\sqrt{1+1/n})} \sim$

$\sim \frac{1}{2}\sum_{n=1}^{\infty} \frac{1}{n^{1/2}}$ РЕД КОНВЕРГИРА

26. $\sum_{n=1}^{\infty} (\sqrt{n^2+1}-\sqrt{n^2+3})\cdot\frac{\sqrt{n^2+1}+\sqrt{n^2+3}}{\sqrt{n^2+1}+\sqrt{n^2+3}} \sim$

$\sim \sum_{n=1}^{\infty} \frac{n^2+1-n^2-3}{n(\sqrt{1+1/n^2}+\sqrt{1+3/n^2})} \sim -1\sum_{n=1}^{\infty} \frac{1}{n}$ РЕД ДИВЕРГИРА

27. $\sum_{n=1}^{\infty} \frac{\sqrt{n+1}-\sqrt{n-1}}{\sqrt{n+2}}\cdot\frac{\sqrt{n+1}+\sqrt{n-1}}{\sqrt{n+1}+\sqrt{n-1}} \sim$

$\sim \sum_{n=1}^{\infty} \frac{n+1-n+1}{n\sqrt{1+2/n}(\sqrt{1+1/n}+\sqrt{1-1/n})} \sim \sum_{n=1}^{\infty} \frac{1}{n}$ РЕД ДИВЕРГИРА

28. $\sum_{n=1}^{\infty} \frac{\sqrt{n+1}-\sqrt{n}}{\sqrt[3]{n}}\cdot\frac{\sqrt{n+1}+\sqrt{n}}{\sqrt{n+1}+\sqrt{n}} \sim \sum_{n=1}^{\infty} \frac{n+1-n}{n^{1/3}\cdot n^{1/2}(\sqrt{1+1/n}+1)}$

$\sim \sum_{n=1}^{\infty} \frac{1}{n^{5/6}\cdot 2} \sim \frac{1}{2}\sum_{n=1}^{\infty} \frac{1}{n^{5/6}}$ РЕД ДИВЕРГИРА

29. $\sum_{n=1}^{\infty} \frac{\sqrt{n^2+3n}-\sqrt{n^2-2}}{\sqrt[3]{n}}\cdot\frac{\sqrt{n^2+3n}+\sqrt{n^2-2}}{\sqrt{n^2+3n}+\sqrt{n^2-2}} \sim \sum_{n=1}^{\infty} \frac{n^2+3n-n^2+2}{n^{1/3}\, n(1+1)} \sim$

$\sim \sum_{n=1}^{\infty} \frac{3n(1+2/3n)}{2n^{4/3}} \sim \frac{3}{2}\sum_{n=1}^{\infty} \frac{1}{n^{1/3}}$ РЕД ДИВЕРГИРА

30. $\sum_{n=1}^{\infty}\frac{\sqrt{n^2+2n}-\sqrt{n^2+1}}{\sqrt{n^3}}\cdot\frac{\sqrt{n^2+2n}+\sqrt{n^2+1}}{\sqrt{n^2+2n}+\sqrt{n^2+1}} \sim$

$\sim \sum_{n=1}^{\infty}\frac{n^2+2n-n^2-1}{n^{3/2}\cdot n\left(\sqrt{1+2/n}+\sqrt{1+1/n^2}\right)} \sim \sum_{n=1}^{\infty}\frac{2n-1}{2n^{5/2}} \sim$

$\sim \sum_{n=1}^{\infty}\frac{2n(1-1/2n)}{2n^{5/2}} \sim \sum_{n=1}^{\infty}\frac{1}{n^{3/2}}$ РЕД КОНВЕРГИРА

31. $\sum_{n=1}^{\infty}\frac{1}{n(n-\sqrt{n^2+2n})}\cdot\frac{n+\sqrt{n^2+2n}}{n+\sqrt{n^2+2n}} \sim$

$\sim \sum_{n=1}^{\infty}\frac{n(1+\sqrt{1+2/n})}{n(n^2-n^2-2n)} \sim \sum_{n=1}^{\infty}\frac{2n}{-2n^2} \sim -\sum_{n=1}^{\infty}\frac{1}{n}$ РЕД ДИВЕРГИРА

32. $\sum_{n=1}^{\infty}\frac{1}{\sqrt{n}(\sqrt{n^2+1}-n)}\cdot\frac{\sqrt{n^2+1}+n}{\sqrt{n^2+1}+n} \sim$

$\sim \sum_{n=1}^{\infty}\frac{n(\sqrt{1+1/n^2}+1)}{\sqrt{n}(n^2+1-n^2)} \sim \sum_{n=1}^{\infty}\frac{2n}{n^{1/2}} \sim 2\sum_{n=1}^{\infty}\frac{1}{n^{-1/2}}$ РЕД ДИВЕРГИРА

33. $\sum_{n=1}^{\infty}\frac{3}{n^2(\sqrt{2n-1}-\sqrt{2n+1})}\cdot\frac{\sqrt{2n-1}+\sqrt{2n+1}}{\sqrt{2n-1}+\sqrt{2n+1}} \sim$

$\sim \sum_{n=1}^{\infty}\frac{3\sqrt{2}n^{1/2}(\sqrt{1-1/2n}+\sqrt{1+1/2n})}{n^2(2n-1-2n-1)} \sim \sum_{n=1}^{\infty}\frac{6\sqrt{2}n^{1/2}}{-2n^2} \sim$

$\sim -3\sqrt{2}\sum_{n=1}^{\infty}\frac{1}{n^{3/2}}$ РЕД КОНВЕРГИРА

34. $\sum_{n=1}^{\infty}\frac{\sqrt{n+1}+\sqrt{n}}{\sqrt{n+1}-\sqrt{n}}\cdot\frac{\sqrt{n+1}-\sqrt{n}}{\sqrt{n+1}-\sqrt{n}}\cdot\frac{\sqrt{n+1}+\sqrt{n}}{\sqrt{n+1}+\sqrt{n}} \sim$

$\sim \sum_{n=1}^{\infty}\frac{(n+1-n)\cdot\sqrt{n}(1-1)}{(n+1-n)\cdot\sqrt{n}(1+1)} \sim$

35. $\sum_{n=1}^{\infty}(\sqrt{n+1}-\sqrt{n})^2 \sim \sum_{n=1}^{\infty}(n+1-2\sqrt{n^2+n}+n) \sim$

$\sim \sum_{n=1}^{\infty}(2n+1-2\sqrt{n^2+n})\cdot\frac{2n+1+2\sqrt{n^2+n}}{2n+1+2\sqrt{n^2+n}} \sim$

$\sim \sum_{n=1}^{\infty}\frac{4n^2+4n+1-4n^2-4n}{2n(1+1/2n+\sqrt{1+1/n})} \sim \frac{1}{4}\sum_{n=1}^{\infty}\frac{1}{n}$ РЕД ДИВЕРГИРА

36. $\sum_{n=1}^{\infty}n(\sqrt{n+5}-\sqrt{n+2})^4 \sim \sum_{n=1}^{\infty}n(\sqrt{n+5}-\sqrt{n+2})^4\cdot\frac{(\sqrt{n+5}+\sqrt{n+2})^4}{(\sqrt{n+5}+\sqrt{n+2})^4}$

$\sim \sum_{n=1}^{\infty}n\left(\frac{n+5-n-2}{\sqrt{n}(1+1)}\right)^4 \sim \sum_{n=1}^{\infty}n\frac{3^4}{2^4 n^2} \sim \frac{81}{16}\sum_{n=1}^{\infty}\frac{1}{n}$

РЕД ДИВЕРГИРА

37. $\sum_{n=1}^{\infty}(\sqrt{2n^2+n+1}-\sqrt{2n})^3\cdot\left(\frac{\sqrt{2n^2+n+1}+\sqrt{2n}}{\sqrt{2n^2+n+1}+\sqrt{2n}}\right)^3 \sim$

$\sim \sum_{n=1}^{\infty}\frac{(2n^2+n+1-2n)^3}{\left(\sqrt{2}n(\sqrt{1+1/2n+1/2n^2}+1/\sqrt{n})\right)^3} \sim \sum_{n=1}^{\infty}\frac{(2n^2-n+1)^3}{2\sqrt{2}n^3} \sim$

$\sim \sum_{n=1}^{\infty}\frac{8n^6(1-1/2n+1/2n^2)^3}{2\sqrt{2}n^3} \sim 2\sqrt{2}\sum_{n=1}^{\infty}\frac{1}{n^{-3}}$ РЕД ДИВЕРГЕР

38. $\sum_{n=1}^{\infty}\frac{\sqrt{n+3}-\sqrt{n}}{\sqrt{n^3}}\cdot\frac{\sqrt{n+3}+\sqrt{n}}{\sqrt{n+3}+\sqrt{n}} \sim \sum_{n=1}^{\infty}\frac{n+3-n}{n^{3/2}\cdot n^{1/2}(1+1)} \sim$

$\sim \frac{3}{2}\sum_{n=1}^{\infty}\frac{1}{n^2}$ РЕД КОНВЕРГИРА

39. $\sum_{n=1}^{\infty}(\sqrt[3]{n^2+1}-\sqrt[3]{n^2}) \sim \sum_{n=1}^{\infty}\frac{n^2+1-n^2}{n^{4/3}(1+1+1)} \sim \frac{1}{3}\sum_{n=1}^{\infty}\frac{1}{n^{4/3}}$

РЕД КОНВЕРГИРА

40. $\sum_{n=1}^{\infty}\frac{\sqrt[3]{n^3+n^2}-\sqrt[3]{n^3+1}}{n} \sim \sum_{n=1}^{\infty}\frac{n^3+n^2-n^3-1}{n\cdot n^2(1+1+1)} \sim$

$\sim \frac{1}{3}\sum_{n=1}^{\infty}\frac{1}{n}$ РЕД ДИВЕРГИРА

$\sum_{n=1} \frac{1}{(\sqrt[4]{n^2+1}-\sqrt{n})\,n^2} \sim \sum_{n=1} \frac{1}{(\sqrt{n^2+1}-n)\,n^2} \sim \sum_{n=1} (n^2+1-n^2)\cdot n^3 \sim$

$\sim 4\sum_{n=1}^{\infty} \frac{1}{n^{1/2}}$ РЕД ДИВЕРГИРА

42. $\sum_{n=1}^{\infty} \frac{\sqrt[3]{n^2+1}-\sqrt[3]{n^2}}{\sqrt[3]{n^2+1}+\sqrt[3]{n^2}} \sim \sum_{n=1}^{\infty} \frac{(n^2+1-n^2)\, n^{4/3}(1+1+1)}{(n^2+1+n^2)\, n^{4/3}(1-1+1)} \sim$

$\sim \frac{1}{2}\sum_{n=1}^{\infty} \frac{1}{n^2}$ РЕД КОНВЕРГИРА

43. $\sum_{n=1}^{\infty} e^{1/n}\cdot\frac{\sqrt{n}}{n+2} \sim 1\sum_{n=1}^{\infty} \frac{1}{n^{1/2}}$ РЕД ДИВЕРГИРА

44. $\sum_{n=1}^{\infty} \frac{e^{1/\sqrt{n}}+\sqrt{n}}{n^2-2} \sim \sum_{n=1}^{\infty} \frac{1}{n^{3/2}}$ РЕД КОНВЕРГИРА

45. $\sum_{n=1}^{\infty} (e^{3/n}-1) \sim 3\sum_{n=1}^{\infty} \frac{1}{n}$ РЕД ДИВЕРГИРА

46. $\sum_{n=1}^{\infty} \sqrt{n}\,(e^{1/n^2}-1) \sim \sum_{n=1}^{\infty} \sqrt{n}\cdot\frac{1}{n^2} \sim \sum_{n=1}^{\infty} \frac{1}{n^{3/2}}$ РЕД КОНВЕРГИРА

47. $\sum_{n=1}^{\infty} e^{(\sqrt{n+1}-\sqrt{n})}\cdot\frac{n^3+2n}{\sqrt{n^2+n}} \sim \sum_{n=1}^{\infty} e^{\frac{n+1-n}{\sqrt{n}(1+1)}}\cdot\frac{n^3(1+2/n^2)}{n^{7/2}\sqrt{1+2/n}} \sim$

$\sim \sum_{n=1}^{\infty} e^{\frac{1}{2\sqrt{n}}}\cdot\frac{1}{n^{1/2}} \sim \sum_{n=1}^{\infty} \frac{1}{2\sqrt{n}}\cdot\frac{1}{n^{1/2}} \sim \frac{1}{2}\sum_{n=1}^{\infty}\frac{1}{n}$

РЕД ДИВЕРГИРА

48. $\sum_{n=1}^{\infty} \frac{e^{2/n^2}-1}{n^2} \sim \sum_{n=1}^{\infty}\frac{\frac{2}{n^2}-1}{n^2} \sim \sum_{n=1}^{\infty}\frac{2-n^2}{n^4} \sim$

$\sim -\sum_{n=1}^{\infty} \frac{1}{n^2}$ РЕД КОНВЕРГИРА

50. $\sum_{n=1}^{\infty} (e^{1/n}-1)\cdot\sin\frac{1}{\sqrt{n+2}} \sim \sum_{n=1}^{\infty} \frac{1}{n}\cdot\frac{1}{\sqrt{n}\cdot(\sqrt{1+2/n})} \sim$

$\sim \sum_{n=1}^{\infty}\frac{1}{n^{3/2}}$ РЕД КОНВЕРГИРА

51. $\sum_{n=1} \frac{1}{n\sqrt{n}} \sim \sum_{n=1}\frac{1}{n\sqrt{n}} \sim \sum_{n=1}\frac{1}{n^{3/2}}$

$\frac{3}{2}>1$ РЕД КОНВЕРГИРА

52. $\sum_{n=1}^{\infty}\frac{\sin(1/n)}{\sqrt{n+2}} \sim \sum_{n=1}^{\infty}\frac{1}{n(\sqrt{n+2})} \sim \sum_{n=1}^{\infty}\frac{1}{n^{3/2}}$ РЕД КОНВЕРГИРА

53. $\sum_{n=1}^{\infty}\frac{\sin(1/\sqrt{n})}{\sqrt{n}+3} \sim \sum_{n=1}^{\infty}\frac{1}{\sqrt{n}(\sqrt{n}+3)} \sim \sum_{n=1}^{\infty}\frac{1}{n}$ РЕД ДИВЕРГИРА

54. $\sum_{n=1}^{\infty}\frac{\sin^2(1/n)}{n+5} \sim \sum_{n=1}^{\infty}\frac{(1/n)^2}{n+5} \sim \sum_{n=1}^{\infty}\frac{1}{n^3}$ РЕД КОНВЕРГИРА

50. $\sum_{n=1}^{\infty}\sin\left(\frac{2\pi}{n^3+1}\right) \sim \sum_{n=1}^{\infty}\frac{2\pi}{n^3+1} \sim 2\pi\sum_{n=1}^{\infty}\frac{1}{n^3}$ РЕД КОНВЕРГИРА

55. $\sum_{n=1}^{\infty}\frac{\sin^2(1/\sqrt{2n})}{n} \sim \sum_{n=1}^{\infty}\frac{1}{2n^2} \sim \frac{1}{2}\sum_{n=1}^{\infty}\frac{1}{n^2}$ РЕД КОНВЕРГИРА

56. $\sum_{n=1}^{\infty}\sin\left(\frac{\pi}{3\sqrt{n}}\right) \sim \sum_{n=1}^{\infty}\frac{\pi}{3\sqrt{n}} \sim \frac{\pi}{3}\sum_{n=1}^{\infty}\frac{1}{n^{1/2}}$ РЕД ДИВЕРГИРА

57. $\sum_{n=1}^{\infty} n\cdot\left(\cos\frac{1}{n^3}-1\right) \sim \sum_{n=1}^{\infty} n\cdot\frac{-1}{n^6\cdot 2} \sim -\frac{1}{2}\sum_{n=1}^{\infty}\frac{1}{n^5}$ РЕД КОНВЕРГИРА

58. $\sum_{n=1}^{\infty}\frac{1-\cos(2/n)}{\sqrt{n}} \sim \sum_{n=1}^{\infty}\frac{4}{n^2\cdot 2\cdot\sqrt{n}} \sim 2\sum_{n=1}^{\infty}\frac{1}{n^{5/2}}$ РЕД КОНВЕРГИРА

59. $\sum_{n=1}^{\infty} n^3\left(1-\cos\frac{1}{n^2}\right) \sim \sum_{n=1}^{\infty} n^3\frac{1}{2n^4} \sim \frac{1}{2}\sum_{n=1}^{\infty}\frac{1}{n}$ РЕД ДИВЕРГИРА

60. $\sum_{n=1}^{\infty} tg\left(\frac{\pi}{n}\right) \sim \sum_{n=1}^{\infty}\frac{\pi}{n} \sim \pi\sum_{n=1}^{\infty}\frac{1}{n}$ РЕД ДИВЕРГИРА

61. $\sum_{n=1}^{\infty}\frac{1}{n}\cdot tg\frac{5}{n} \sim \sum_{n=1}^{\infty}\frac{1}{n}\cdot\frac{5}{n} \sim 5\sum_{n=1}^{\infty}\frac{1}{n^2}$ РЕД КОНВЕРГИРА

62. $\sum_{n=1}^{\infty}\frac{n^2+3}{n^3+2}\cdot tg\frac{1}{\sqrt{n}} \sim \sum_{n=1}^{\infty}\frac{1}{n}\cdot\frac{1}{\sqrt{n}} \sim \sum_{n=1}^{\infty}\frac{1}{n^{3/2}}$ РЕД КОНВЕРГИРА

63. $\sum_{n=1}^{\infty} n\cdot\left(\cos\frac{1}{n}-1\right)\cdot tg\frac{1}{n} \sim \sum_{n=1}^{\infty} n\cdot\frac{1}{2n^2}\cdot\frac{1}{n} \sim$

$\sim \frac{1}{2}\sum_{n=1}^{\infty}\frac{1}{n^2}$ РЕД КОНВЕРГИРА

64. $\sum_{n=1}^{\infty} n\, tg\frac{\sqrt{n}+3}{n^2+2} \sim \sum_{n=1}^{\infty} n\frac{1}{n^{3/2}} \sim \sum_{n=1}^{\infty}\frac{1}{n^{1/2}}$ РЕД ДИВЕРГИРА

65. $\sum_{n=1}^{\infty} arctg\frac{n}{n^2+1} \sim \sum_{n=1}^{\infty}\frac{1}{n2}$ провериши конвергира

66. $\sum_{n=1}^{\infty}\frac{1}{\sqrt{n}}\cdot arctg\frac{1}{3\sqrt{n}} \sim \sum_{n=1}^{\infty}\frac{1}{\sqrt{n}}\cdot\frac{1}{3\sqrt{n}} \sim \frac{1}{3}\sum_{n=1}^{\infty}\frac{1}{n}$

РЕД ДИВЕРГИРА

67. $\sum_{n=1}^{\infty}\sqrt{\frac{n}{n^2+1}}\cdot arctg\frac{n+1}{\sqrt[3]{n^4}} \sim \sum_{n=1}^{\infty}\frac{\sqrt{n}}{n}\cdot\frac{n+1}{n^{4/3}} \sim$

$\sim \sum_{n=1}^{\infty}\frac{n^{3/2}+n^{1/2}}{n^{7/3}} \sim \sum_{n=1}^{\infty}\frac{1}{n^{\frac{5}{6}}}$ РЕД ДИВЕРГИРА

68. $\sum_{n=1}^{\infty}\frac{1}{\sqrt[4]{n}}\, arcsin\frac{1}{\sqrt[5]{n^4}} \sim \sum_{n=1}^{\infty}\frac{1}{n^{1/4}}\cdot\frac{1}{n^{4/5}} \sim$

$\sim \sum_{n=1}^{\infty}\frac{1}{n^{21/20}}$ РЕД КОНВЕРГИРА

69. $\sum_{n=1}^{\infty}\ln\left(1+\frac{2}{n}\right) \sim \sum_{n=1}^{\infty}\frac{2}{n} \sim 2\sum_{n=1}^{\infty}\frac{1}{n}$ РЕД ДИВЕРГИРА

70. $\sum_{n=1}^{\infty}\ln\left(1+\frac{1}{n\sqrt[3]{n}}\right) \sim \sum_{n=1}^{\infty}\frac{1}{n^{4/3}}$, РЕД КОНВЕРГИРА

71. $\sum_{n=1}^{\infty}\frac{1}{\sqrt[3]{n}}\ln\left(1+\frac{1}{\sqrt{n+1}}\right) \sim \sum_{n=1}^{\infty}\frac{1}{n^{1/3}}\cdot\frac{1}{n^{1/2}} \sim$

$\sim \sum_{n=1}^{\infty}\frac{1}{n^{7/10}}$ РЕД ДИВЕРГИРА

72. $\sum_{n=1}^{\infty}\frac{1}{n}\ln\left(1+\frac{1}{\sqrt{n}}\right) \sim \sum_{n=1}^{\infty}\frac{1}{n}\cdot\frac{1}{n^{1/2}} \sim \sum_{n=1}^{\infty}\frac{1}{n^{3/2}}$ КОНВЕРГИРА

73. $\sum_{n=1}^{\infty}\ln\left(1+\sin\frac{1}{n^2}\right) \sim \sum_{n=1}^{\infty}\ln\left(1+\frac{1}{n^2}\right) \sim \sum_{n=1}^{\infty}\frac{1}{n^2}$ РЕД КОНВЕРГИРА

74. $\sum_{n=1}^{\infty}\frac{1}{\sqrt{n}}\ln\left(1+\sin\frac{1}{\sqrt{n}}\right) \sim \sum_{n=1}^{\infty}\frac{1}{\sqrt{n}}\ln\left(1+\frac{1}{\sqrt{n}}\right) \sim$

$\sim \sum_{n=1}^{\infty}\frac{1}{\sqrt{n}}\cdot\frac{1}{\sqrt{n}} \sim \sum_{n=1}^{\infty}\frac{1}{n}$ РЕД ДИВЕРГИРА

75. $\sum_{n=1}^{\infty}\ln\left(\frac{n+3}{n}\right)\cdot\sin\frac{n}{n^2+3} \sim \sum_{n=1}^{\infty}\ln\left(1+\frac{3}{n}\right)\cdot\sin\frac{1}{n} \sim$

$\sim \sum_{n=1}^{\infty}\frac{3}{n}\cdot\frac{1}{n} \sim 3\sum_{n=1}^{\infty}\frac{1}{n^2}$ РЕД КОНВЕРГИРА

76. $\sum_{n=1}^{\infty}\sqrt[5]{n^2}\left(\ln(n^3+3)-\ln(n^3)\right) \sim \sum_{n=1}^{\infty} n^{2/5}\cdot\ln\frac{n^3+3}{n^3} \sim$

$\sim \sum_{n=1}^{\infty} n^{2/5}\cdot\ln\left(1+\frac{3}{n^3}\right) \sim \sum_{n=1}^{\infty} n^{2/5}\cdot\frac{3}{n^3} \sim 3\sum_{n=1}^{\infty}\frac{1}{n^{13/5}} \sim$

РЕД КОНВЕРГИРА

77. $\sum_{n=1}^{\infty} n\left(\ln(n^3+2)-3\ln n\right) \sim \sum_{n=1}^{\infty} n\left(\ln(n^3+2)-\ln n^3\right) \sim$

$\sim \sum_{n=1}^{\infty} n\ln\left(\frac{n^3+2}{n^3}\right) \sim \sum_{n=1}^{\infty} n\ln\left(1+\frac{2}{n^3}\right) \sim \sum_{n=1}^{\infty} n\cdot\frac{2}{n^3} \sim$

$\sim 2\cdot\sum_{n=1}^{\infty}\frac{1}{n^2}$ РЕД КОНВЕРГИРА

78. $\sum_{n=1}^{\infty}\sqrt[3]{n^2}\left(\ln(n^3+2n)-3\ln n\right) = \sum_{n=1}^{\infty} n^{2/3}\cdot\left(\ln(n^3+2n)-\ln n^3\right)$

$\sim \sum_{n=1}^{\infty} n^{2/3}\ln\left(\frac{n^3+2n}{n^3}\right) \sim \sum_{n=1}^{\infty} n^{2/3}\cdot\ln\left(1+\frac{2n}{n^3}\right) \sim$

$\sim \sum_{n=1}^{\infty} n^{2/3}\cdot\frac{2n}{n^3} \sim 2\sum_{n=1}^{\infty}\frac{1}{n^{4/3}}$ РЕД КОНВЕРГИРА

79. $\sum_{n=1}^{\infty} n\left(\ln\left(n^{3/2}+\frac{2}{n}\right)-\ln n^{3/2}\right) \sim \sum_{n=1}^{\infty} n\left(\frac{n^{5/2}+2}{n}\right)-\ln n$

$\sim \sum_{n=1}^{\infty} n\ln\frac{n^{5/2}+2}{n^{5/2}} \sim \sum_{n=1}^{\infty} n\ln\left(1+\frac{2}{n^{5/2}}\right) \sim 2\sum_{n=1}^{\infty}\frac{1}{n^{3/2}}$

РЕД КОНВЕРГИРА

80. $\sim \sum_{n=1}^{\infty} \frac{1}{n} \cdot \ln\left(1+\frac{1}{2n}\right) \sim \sum_{n=1}^{\infty} \frac{1}{n} \cdot \frac{1}{2n} \sim \frac{1}{2} \sum_{n=1}^{\infty} \frac{1}{n^2}$

РЕД КОНВЕРГИРА

81. $\sum_{n=1}^{\infty} \frac{\ln(1+\sqrt{n}) - \ln\sqrt{n}}{\sqrt{n^3}} \sim \sum_{n=1}^{\infty} n^{3/2} \cdot \ln\left(\frac{1+\sqrt{n}}{\sqrt{n}}\right) \sim$

$\sim \sum_{n=1}^{\infty} n^{3/2} \cdot \ln\left(1+\frac{1}{\sqrt{n}}\right) \sim \sum_{n=1}^{\infty} n^{3/2} \cdot \frac{1}{n^{1/2}} \sim$

$\sim \sum_{n=1}^{\infty} \frac{1}{n}$ РЕД ДИВЕРГИРА

82. $\sum_{n=1}^{\infty} \frac{\ln(n^2+4n) - 2\ln n}{\sqrt{n}} \sim \sum_{n=1}^{\infty} \frac{1}{\sqrt{n}} \left(\ln(n^2+4n) - \ln n^2\right) \sim$

$\sim \sum_{n=1}^{\infty} \frac{1}{\sqrt{n}} \ln \frac{n^2+4n}{n^2} \sim \sum_{n=1}^{\infty} \frac{1}{\sqrt{n}} \ln\left(1+\frac{4n}{n^2}\right) \sim$

$\sim \sum_{n=1}^{\infty} \frac{1}{\sqrt{n}} \cdot \frac{4n}{n^2} \sim 4 \sum_{n=1}^{\infty} \frac{1}{n^{3/2}}$ РЕД КОНВЕРГИРА

83. $\sum_{n=1}^{\infty} \frac{\ln(3n+2) - \ln 3 - \ln n}{\sqrt{n}} \sim \sum_{n=1}^{\infty} \frac{1}{\sqrt{n}} \ln \frac{3n+2}{3n} \sim$

$\sim \sum_{n=1}^{\infty} \frac{1}{\sqrt{n}} \ln\left(1+\frac{2}{3n}\right) \sim \sum_{n=1}^{\infty} \frac{1}{\sqrt{n}} \cdot \frac{2}{3n} \sim$

$\sim \frac{2}{3} \sum_{n=1}^{\infty} \frac{1}{n^{3/2}}$ РЕД КОНВЕРГИРА

84. $\sum_{n=1}^{\infty} \frac{\ln n + \ln(n+2) - \ln n^2}{\sqrt{n}} \sim \sum_{n=1}^{\infty} \frac{1}{\sqrt{n}} \ln\left(\frac{n^2+2n}{n^2}\right) \sim$

$\sim \sum_{n=1}^{\infty} \frac{1}{\sqrt{n}} \ln\left(1+\frac{2n}{n^2}\right) \sim \sum_{n=1}^{\infty} \frac{1}{\sqrt{n}} \cdot \frac{2n}{n^2} \sim$

$\sim 2 \sum_{n=1}^{\infty} \frac{1}{n^{3/2}}$ РЕД КОНВЕРГИРА

85. $\sim \sum_{n=1}^{\infty} \frac{1}{n^{3/2}} \cdot \ln\left(1+\frac{2n}{n^3}\right) \sim \sum_{n=1}^{\infty} \frac{1}{n^{3/2}} \cdot \frac{2n}{n^3} \sim 2 \sum_{n=1}^{\infty} \frac{1}{n^{7/2}}$

РЕД КОНВЕРГИРА

II) ОДРЕДИТИ ЗА КОЈУ ВРЕДНОСТ ПАРАМЕТРА p ДАТИ РЕД КОНВЕРГИРА:

1. $\sum_{n=2}^{\infty} \frac{n^3-1}{n^p-2} \sim \sum_{n=2}^{\infty} \frac{n^3(1-1/n^3)}{n^p(1-2/n^p)} \sim \sum_{n=2}^{\infty} \frac{n^3}{n^p} \sim \sum_{n=2}^{\infty} \frac{1}{n^{p-3}}$

$p-3>1$, $p>3+1$, $p \in (4, +\infty)$

2. $\sum_{n=1}^{\infty} \frac{n^{2p+3}}{2+n} \sim \sum_{n=1}^{\infty} \frac{1}{n^{1-2p}}$; $1-2p>1$, $-2p>0$, $p<0$; $\cancel{p \in (-\infty, 1/2)}$ $p \in (-\infty, 0)$

3. $\sum_{n=1}^{\infty} \frac{n^3+1}{(n^2+2n)^p} \sim \sum_{n=1}^{\infty} \frac{1}{n^{2p-3}}$; $2p-3>1$, $2p>4$, $p>2$; $\cancel{p \in (\frac{3}{2}, +\infty)}$ $p \in (2, +\infty)$

4. $\sum_{n=1}^{\infty} \frac{n+1}{(n^p+2)^3} \sim \sum_{n=1}^{\infty} \frac{1}{n^{3p-1}}$; $3p-1>1$, $3p>2$; $\cancel{p \in (\frac{1}{3}, +\infty)}$ $p \in (2/3, +\infty)$

5. $\sum_{n=1}^{\infty} \frac{\sqrt{n}-1}{n^p+2} \sim \sum_{n=1}^{\infty} \frac{1}{n^{p-1/2}}$; $p-1/2>1$, $p>3/2$; $\cancel{p \in (\frac{1}{2}, +\infty)}$ $p \in (3/2, +\infty)$

6. $\sum_{n=1}^{\infty} \frac{\sqrt{n}+1}{(n^2+3)^p} \sim \sum_{n=1}^{\infty} \frac{1}{n^{2p-1/2}}$; $2p-1/2>1$; $2p>3/2 \;/:2$; $p>3/4$; $\cancel{p \in (\frac{1}{4}, +\infty)}$ $p \in (3/4, +\infty)$

7. $\sum_{n=1}^{\infty} \frac{(n^2-3n)^p}{2n+\sqrt{n}} \sim \frac{1}{2} \sum_{n=1}^{\infty} \frac{1}{n^{1-2p}}$; $1-2p>1$, $-2p>0$, $p<0$; $p \in (-\infty, 0)$ $\cancel{p \in (-\infty, \frac{1}{2})}$

8. $\sum_{n=1}^{\infty} \frac{\sqrt[3]{n+1}}{(n^{2p}+2)^3} \sim \sum_{n=1}^{\infty} \frac{1}{n^{6p-1/3}}$; $6p-1/3>0$; $6p>1/3 \;/:6$; $p>1/18$; $\cancel{p \in (\frac{1}{18}, +\infty)}$

9. $\sum_{n=1}^{\infty} \frac{\sqrt[3]{2n-1}}{(n^2+1)^p} \sim \sqrt[3]{2} \sum_{n=1}^{\infty} \frac{1}{n^{2p-1/3}}$; $2p-1/3>1$; $2p>4/3 \;/:2$; $p>2/3$; $\cancel{p \in (4/6, +\infty)}$ $p \in (2/3, +\infty)$

10. $\sum_{n=1}^{\infty} \frac{(\sqrt{n+1}-1)^p}{\sqrt{n^3}} \sim \sum_{n=1}^{\infty} \frac{1}{n^{3/2-p/2}}$

$3/2-p/2>1$

$-p/2>-1/2 \;/\cdot(-2)$

$p<1$; $\cancel{p \in (-\infty, 3)}$ $p \in (-\infty, 1)$

11. $\sum_{n=1}^{\infty} \frac{n+1}{(\sqrt{n^2+1}+n)^3} \sim 8 \sum_{n=1}^{\infty} \frac{1}{n^{3-p}}$ $-p > -2 /\cdot(-1)$ $p < 2$ ~~$p \in (-\infty, 8)$~~ $p \in (-\infty, 2)$

12. $\sum_{n=1}^{\infty} \frac{(\sqrt{n^2+1}+n)^p}{n^7 - 7n^3 + 1} \sim 2^p \sum_{n=1}^{\infty} \frac{1}{n^{7-p}}$ $7-p>1$ $-p > -6 /:(-1)$ $p<6$ ~~$p \in (-\infty, 0)$~~ $p \in (-\infty, 6)$

13. $\sum_{n=1}^{\infty} \frac{n^3-1}{(\sqrt{n^2+1}+n)^p} \sim \sum_{n=1}^{\infty} \frac{1}{n^{p-3}}$ $p-3>1$ $p>4$ ~~$p \in (3, +\infty)$~~ $p \in (4, +\infty)$

14. $\sum_{n=1}^{\infty} \frac{(\sqrt{n^3+2}+n)^p}{n^3+2n-2} \sim \sum_{n=1}^{\infty} \frac{1}{n^{3-3p/2}}$ $3-3p/2>1$ $-3p/2 > -2 /\cdot(-2/3)$ $p < 4/3$ $p \in (-\infty, 4/3)$

15. $\sum_{n=1}^{\infty} \frac{(\sqrt{n^2-1})^p + 3}{(\sqrt{n+3})^3} \sim \sum_{n=1}^{\infty} \frac{1}{n^{3/2-p}}$ $3/2-p>1$ $-p > -1/2 /:(-1)$ $p < 1/2$ $p \in (-\infty, 1/2)$

16. $\sum_{n=1}^{\infty} \frac{(\sqrt[3]{n^2+1})^p}{\sqrt[5]{n^2+2}} \sim \sum_{n=1}^{\infty} \frac{1}{n^{2/5-2p/3}}$ $2/5-2p/3>1 /\cdot 15$ $6-10p>15$ $-10p > 9 /:(-10)$ $p < 8/12$ ~~$p \in (-\infty, \frac{3}{5})$~~

17. $\sum_{n=1}^{\infty} \frac{\sqrt{n}+1}{(\sqrt[3]{n^2+1}+3)^p} \sim \sum_{n=1}^{\infty} \frac{1}{n^{2p/3-1/2}}$ $\frac{2p}{3} - \frac{1}{2} > 1 /\cdot 6$ $4p-3>6$ $4p>9 /:4$ $p > 9/4$ ~~$p \in (\frac{3}{4}, +\infty)$~~ $p \in (\frac{9}{4}, +\infty)$

18. $\sum_{n=1}^{\infty} \frac{n^2-5n+6}{(\sqrt[3]{n+2})^p+1} \sim \sum_{n=1}^{\infty} \frac{1}{n^{p/3-2}}$ $\frac{p}{3}-2>1$ $\frac{p}{3}>3$ $p \in (9, +\infty)$ ~~$p \in (6, +\infty)$~~

19. $\sum_{n=1}^{\infty} \frac{\sqrt[3]{2n-1}}{(\sqrt[4]{n-2}-5)^p} \sim \sqrt[3]{2} \sum_{n=1}^{\infty} \frac{1}{n^{p/4-1/3}}$ $p \in (\frac{16}{3}, +\infty)$

$\frac{p}{4} - \frac{1}{3} > 1 /\cdot 12$ $3p-4>12$ $3p>16 /:3$ $p > \frac{16}{3}$ ~~$p \in (\frac{4}{3}, +\infty)$~~ $p \in \frac{16}{3}$

20. $\sum_{n=2}^{\infty} \frac{n^p}{\sqrt[3]{n^2-2}} \sim \sum_{n=2}^{\infty} \frac{1}{n^{2/3-p}}$ $\frac{2}{3}-p>1$ $-p > 1 - \frac{2}{3}$ $-p > \frac{1}{3}$ $p < \frac{1}{3}$ $p \in (-\infty, \frac{2}{3})$

21. $\sum_{n=1}^{\infty} \frac{n^{2p}+3}{(\sqrt{n+1})^p} \sim \sum_{n=1}^{\infty} \frac{1}{n^{p/2-2p}}$ $\frac{p}{2} - 2p > 1 /\cdot 2$ $p-4p>2$ $-3p > 2 /:(-3)$ $p < -2/3$ $p \in (-\infty, 0)$

22. $\sum_{n=1}^{\infty} \frac{\sqrt[3]{n^p-2}}{n^2-n+1} \sim \sum_{n=1}^{\infty} \frac{1}{n^{2-p/3}}$ $2 - \frac{p}{3} > 1$ $-\frac{p}{3} > -1 /:(-3)$ $p < 3$ $p \in (-\infty, 3)$

23. $\sum_{n=1}^{\infty} \frac{(n^2+1)^p\sqrt{n}}{(2n^p)^4} \sim \frac{1}{16} \sum_{n=1}^{\infty} \frac{1}{n^{4p-2p-\frac{1}{2}}}$

$4p-2p-\frac{1}{2}>0$ $2p > \frac{1}{2} /\cdot 2$ $p > 1/4$ $p \in (1/4, +\infty)$

24. $\sum_{n=1}^{\infty} \frac{n+13}{\sqrt[p]{n^3+n+1}} \sim \sum_{n=1}^{\infty} \frac{1}{n^{3/p-1}}$ $\frac{3}{p}-1>1$ $\frac{3}{p}>2$ $\frac{3}{p} > 1+1$

p	$-$	$+$	$+$
$3-p$	$+$	$+$	$-$
	$-$	$+$	$-$

$p \in (0,3)$ $p \in (0,3)$ $p \in (-\infty, -3) \cup (0, \infty)$

25. $\sum_{n=1}^{\infty} \frac{(n^p+2)^2}{\sqrt[3]{n^{2p}+3}} \sim \sum_{n=1}^{\infty} \frac{1}{n^{2p/3-2p}}$ $\frac{2p}{3} - 2p > 1 /\cdot 3$ $2p-6p>3$ $-4p > 3 /:(-4)$ $p<0$

~~$p \in (-\infty, -3/4)$~~ $p \in (-\infty, 0)$

26. $\sum_{n=1}^{\infty} \frac{n^2\sqrt{n}+n+2}{\sqrt[p]{n^2+1}} \sim \sum_{n=1}^{\infty} \frac{1}{n^{2/p-5/2}}$ $\frac{2}{p} - \frac{5}{2} > 1$ $\frac{4-5p}{2p} > 1$ $\frac{2}{p} > 1 + \frac{5}{2}$ $\frac{4-7p}{2p} > 0$

$p \in (0, \frac{4}{5})$ $p \in (-4/7, 0)$

$\sum_{n=1}^{\infty} n^p(\sqrt{n+2}-\sqrt{n+1}) \cdot \frac{\sqrt{n+2}+\sqrt{n+1}}{\sqrt{n+2}+\sqrt{n+1}} \sim$

$\sim \sum_{n=1}^{\infty} n^p \frac{n+2-n-1}{\sqrt{n}(1+1)} \sim \frac{1}{2}\sum_{n=1}^{\infty} \frac{1}{n^{\frac{1}{2}-p}}$ $\quad p \in (-\infty, -\frac{1}{2})$

$\frac{1}{2} - p > 1 \qquad -p > 1 - \frac{1}{2} \qquad -p > \frac{1}{2} \qquad p < -\frac{1}{2}$

28. $\sum_{n=1}^{\infty} \sqrt[3]{n^p}(\sqrt{n+1}-\sqrt{n}) \cdot \frac{\sqrt{n+1}+\sqrt{n}}{\sqrt{n+1}+\sqrt{n}} \sim$

$\sim \sum_{n=1}^{\infty} n^{p/3} \frac{n+1-n}{n^{1/2}(1+1)} \sim \frac{1}{2}\sum_{n=1}^{\infty} \frac{1}{n^{1/2-p/3}}$

$\frac{1}{2} - \frac{p}{3} > 1 / \cdot 6 \qquad 3-2p > 6 \qquad -2p > 6-3 \qquad -2p > 3 \qquad p < -3/2$

$p \in (-\infty, -3/2)$

29. $\sum_{n=1}^{\infty} n^{2p}(n-\sqrt{n^2-n}) \cdot \frac{n+\sqrt{n^2-n}}{n+\sqrt{n^2-n}} \sim$

$\sim \sum_{n=1}^{\infty} n^{2p} \frac{n^2-n^2+n}{n(1+1)} \sim \frac{1}{2}\sum_{n=1}^{\infty} \frac{1}{n^{-2p}}$

$-2p > 1 / : (-2) \qquad p < -\frac{1}{2} \qquad p \in (-\infty, -\frac{1}{2})$

30. $\sum_{n=1}^{\infty} \frac{\sqrt{n^2+3}-\sqrt{n^2-n}}{n^p} \cdot \frac{\sqrt{n^2+3}+\sqrt{n^2-n}}{\sqrt{n^2+3}+\sqrt{n^2-n}} \sim$

$\sim \sum_{n=1}^{\infty} \frac{n^2+3-n^2+n}{n^{p+1} \cdot 2} \sim \frac{3}{2}\sum_{n=1}^{\infty} \frac{1}{n^p} \qquad p > 1$

$p \in (1, +\infty)$

31. $\sum_{n=2}^{\infty} \frac{\sqrt{n+2}-\sqrt{n-2}}{(n+1)^p} \cdot \frac{\sqrt{n+2}+\sqrt{n-2}}{\sqrt{n+2}+\sqrt{n-2}} \sim \qquad p+\frac{1}{2} > 1 \qquad p > \frac{1}{2}$

$\sum_{n=1}^{\infty} \frac{n+2-n+2}{n^{p+1/2} \cdot 2} \sim 2\sum_{n=1}^{\infty} \frac{1}{n^{p+1/2}} \qquad p \in (\frac{1}{2}, +\infty)$

32. $\sum_{n=1}^{\infty} \frac{\sqrt{n+1}-\sqrt{n-1}}{(n^2+1)^p} \cdot \frac{\sqrt{n+1}+\sqrt{n-1}}{\sqrt{n+1}+\sqrt{n-1}} \sim \sum_{n=1}^{\infty} \frac{1}{2n^{2p+1/2}} \sim$

$\sim \sum_{n=1}^{\infty} \frac{1}{n^{2p+1/2}} \qquad 2p+1/2 > 1 \qquad 2p > 1/2 / : 2 \qquad p > 1/4 \qquad p \in (1/4, +\infty)$

33. $\sum_{n=1}^{\infty} \frac{1}{n^p(\sqrt{n^2+2}-n)} \cdot \frac{\sqrt{n^2+2}+n}{\sqrt{n^2+2}+n} \sim \sum_{n=1}^{\infty} \frac{n^2+2-n^2}{n^{p+1} \cdot 2} \sim \sum_{n=1}^{\infty} \frac{1}{n^{p+1}}$

$p+1 > 1 \qquad p > 1-1 \qquad p > 0 \qquad p \in (0, +\infty)$

34. $\sum_{n=1}^{\infty} \frac{n^p}{\sqrt{n^2+2n}-\sqrt{n^2+1}} \cdot \frac{\sqrt{n^2+2n}+\sqrt{n^2+1}}{\sqrt{n^2+2n}+\sqrt{n^2+1}} \sim$

$\sim \sum_{n=1}^{\infty} \frac{n^{p+1} \cdot 2}{n^2+2n-n^2-1} \sim \sum_{n=1}^{\infty} \frac{1}{n^{-p}} \qquad -p > 1 / : (-1) \qquad p < -1 \qquad p \in (-\infty, -1)$

35. $\sum_{n=1}^{\infty} n(\sqrt{n+1}-\sqrt{n})^p \cdot \frac{(\sqrt{n+1}+\sqrt{n})^p}{(\sqrt{n+1}+\sqrt{n})^p} \sim \qquad \frac{p}{2} - 1 > 1$

$\sim \sum_{n=1}^{\infty} n \frac{(n+1-n)^p}{\sqrt{n}^p(1+1)^p} \sim \frac{1}{2^p}\sum_{n=1}^{\infty} \frac{1}{n^{p/2-1}} \qquad \frac{p}{2} > 2 / \cdot 2 \qquad p > 4$

$p \in (4, +\infty)$

36. $\sum_{n=1}^{\infty} (\sqrt{2n^2+1}-\sqrt{2}n)^p \cdot \left(\frac{\sqrt{2n^2+1}+\sqrt{2}n}{\sqrt{2n^2+1}+\sqrt{2}n}\right)^p \sim$

$\sim \sum_{n=1}^{\infty} \frac{(2n^2+1-2n^2)^p}{(\sqrt{2}n)^p(1+1)^p} \sim \frac{1}{\sqrt{8}^p}\sum_{n=1}^{\infty} \frac{1}{n^p} \qquad p > 1 \qquad p \in (1, +\infty)$

37. $\sum_{n=1}^{\infty} (\sqrt[3]{n^3+1}-n)^p \sim \sum_{n=1}^{\infty} \frac{(n^3+1-n^3)^p}{n^{2p} \cdot 3^p} \sim \frac{1}{3^p}\sum_{n=1}^{\infty} \frac{1}{n^{2p}}$

$2p > 1 / : 2 \qquad p > 1/2 \qquad p \in (1/2, +\infty)$

38. $\sum_{n=1}^{\infty} \frac{n^{-2}}{(\sqrt[3]{n^3+n}-\sqrt[3]{n^3-n})^p} \sim \sum_{n=1}^{\infty} \frac{n^{-2} \cdot n^{2p} \cdot 3^p}{(n^3+n-n^3+n)^p} \sim$

$\sim \left(\frac{3}{2}\right)^p \sum_{n=1}^{\infty} \frac{1}{n^{-2p}} \qquad -2p > 1 / : (-2) \qquad p < -1/2$

$p \in (-\infty, -1/2)$

39. $\sum_{n=1}^{\infty} \frac{(\sqrt{n+1}-\sqrt{n-1})^p}{\sqrt[3]{n}} \sim \sum_{n=1}^{\infty} \frac{(n+1-n+1)^p}{\sqrt[3]{n} \cdot n^{p/2} \cdot 2^p} \sim$

$\sim \sum_{n=1}^{\infty} \frac{1}{n^{p/2+1/3}}$ $\quad \frac{p}{2}+\frac{1}{3} > 1 \,/\cdot 6$

$3p+2 > 6$

$3p > 4 \,/ : 3$ $\quad p \in (\frac{4}{3}, +\infty)$

$p > 4/3$

40. $\sum_{n=1}^{\infty} \frac{(\sqrt{4n^2+1}-2n)^p}{4n^2+1} \sim \sum_{n=1}^{\infty} \frac{(4n^2+1-4n^2)^p}{4n^2 \cdot 2^p n^p \cdot 2^p} \sim$

$\sim \frac{1}{4^{p+1}} \sum_{n=1}^{\infty} \frac{1}{n^{p+2}}$ $\quad p+2 > 1$ $\quad p > -1$ $\quad p \in (-1, +\infty)$

41. $\sum_{n=1}^{\infty} \frac{(\sqrt{n+2}-\sqrt{n})^p}{\sqrt[4]{n-1}} \sim \sum_{n=1}^{\infty} \frac{(n+2-n)^p}{n^{1/4} \cdot n^{p/2} \cdot 2^p} \sim$

$\sim \sum_{n=1}^{\infty} \frac{1}{n^{p/2+1/4}}$ $\quad \frac{p}{2}+\frac{1}{4} > 1 \,/\cdot 4$

$2p+1 > 4$ $\quad p \in (-1/2, +\infty)$

$2p > 3 \,/ : 2$

$p > 3/2$

42. $\sum_{n=1}^{\infty} \left(\frac{\sqrt[4]{n^2+1}-\sqrt[4]{n^2-1}}{\sqrt{n+1}} \right)^p \sim \sum_{n=1}^{\infty} \left(\frac{\sqrt{n^2+1}-\sqrt{n^2-1}}{\sqrt{n}\sqrt{n} \cdot 2} \right)^p \sim$

$\sim \sum_{n=1}^{\infty} \left(\frac{n^2+1-n^2+1}{2n \cdot n \cdot 2} \right)^p \sim \frac{1}{2^p} \sum_{n=1}^{\infty} \frac{1}{n^{2p}}$

$p \in (1/2, +\infty)$

$2p > 1 \,/ : 2$ $\quad$ ~~$p \in (1, +\infty)$~~

$p > 1/2$

43. $\sum_{n=1}^{\infty} \sqrt{n} (e^{1/n}-1)^p \sim \sum_{n=1}^{\infty} \sqrt{n} \cdot \frac{1}{n^p} \sim \sum_{n=1}^{\infty} \frac{1}{n^{p-1/2}}$

$p-1/2 > 1$ $\quad p > 3/2$ $\quad p \in (3/2, +\infty)$

44. $\sum_{n=1}^{\infty} \frac{1}{n^2-1} (e^{1/\sqrt{n}}-1)^p \sim \sum_{n=1}^{\infty} \frac{1}{n^2} \cdot \frac{1}{n^{p/2}} \sim \sum_{n=1}^{\infty} \frac{1}{n^{p/2+2}}$

$\frac{p}{2}+2 > 1 \,/\cdot 2$

$p+4 > 2$ $\quad p \in (-2, +\infty)$

$p > -2$

45. $\sum_{n=1} n^p (e^{\frac{1}{n^2}}-1)^p \sim \sum_{n=1} n^p \frac{1}{n^{2p}} \sim \sum_{n=1} \frac{1}{n^p}$

$p > 1$ $\quad p \in (1, +\infty)$

46. $\sum_{n=1}^{\infty} \frac{1}{n^3} (e^{2/\sqrt{n}}-1)^{p/2} \sim \sum_{n=1}^{\infty} \frac{1}{n^3} \cdot \left(\frac{2}{\sqrt{n}}\right)^{p/2} \sim$

$\sim 2^{p/2} \sum_{n=1}^{\infty} \frac{1}{n^{p/4+3}}$ $\quad p/4+3 > 1 \,/\cdot 4$

$p+12 > 4$ $\quad p \in (-8, +\infty)$

$p > -8$

47. $\sum_{n=1}^{\infty} (\sqrt{n+2})^p \cdot \sin\frac{1}{n^2} \sim \sum_{n=1}^{\infty} n^{p/2} \cdot \frac{1}{n^2} \sim$

$\sim \sum_{n=1}^{\infty} \frac{1}{n^{2-p/2}}$ $\quad 2-p/2 > 1 \,/\cdot 2$

$4-p > 2$ $\quad p \in (-\infty, 2)$

$-p > -2 \,/ : (-1)$

$p < 2$

48. $\sum_{n=1}^{\infty} \sqrt{n^p+1} \cdot \sin\frac{1}{n+1} \sim \sum_{n=1}^{\infty} n^{p/2} \cdot \frac{1}{n+1} \sim$

$\sim \sum_{n=1}^{\infty} \frac{1}{n^{1-p/2}}$ $\quad 1-\frac{p}{2} > 1 \,/\cdot 2$

$2-p > 2$ $\quad p \in (-\infty, 0)$

$-p > 0 \,/ : (-1)$

$p < 0$

49. $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n+2}} \cdot \left(\sin\frac{1}{\sqrt{n}}\right)^p \sim \sum_{n=1}^{\infty} \frac{1}{\sqrt{n}} \cdot \frac{1}{\sqrt{n}^p} \sim \sum_{n=1}^{\infty} \frac{1}{n^{p/2+1/2}}$

$\frac{p}{2}+\frac{1}{2} > 1 \,/\cdot 2$ $\quad p+1 > 2$, $p > 1$ $\quad p \in (1, +\infty)$

50. $\sum_{n=1}^{\infty} (\sqrt{n+1})^p \cdot \left(\sin\frac{2}{n^2}\right)^{-1} \sim \sum_{n=1}^{\infty} n^{p/2} \cdot \frac{n^2}{2} \sim \frac{1}{2} \sum \frac{1}{n^{-p/2-2}}$

$-p/2-2 > 1 \,/\cdot (-2)$ $\quad p \in (-6, +\infty)$

$p+4 > 2$

$p > -6$

51. $\sim \sum_{n=1} \frac{n^3+3}{\ldots} \sim \sum_{n=1} \frac{\ldots}{n^3} \cdot \frac{\ldots}{2^p n^p} \sim$

$\sim 2^p \sum_{n=1}^{\infty} \frac{1}{n^{p+3}}$ $\quad p+3>1 \quad p>-2 \quad p\in(-2,+\infty)$

52. $\sum_{n=1}^{\infty} \sqrt{n^p}\cdot\left(1-\cos\frac{1}{2\sqrt{n}}\right) \sim \sum_{n=1}^{\infty} n^{p/2}\cdot\frac{1}{2n^{1/2}} \sim$

$\sim \frac{1}{2}\sum_{n=1}^{\infty}\frac{1}{n^{1/2-p/2}}$ $\quad 1/2-p/2>1 \,/\cdot 2$

$1-p>2$

$-p>1 \,/:(-1)$

$p<-3$ $\quad p\in(-\infty,-3)$

53. $\sum_{n=1}^{\infty}(n^p+1)\operatorname{tg}\frac{3}{\sqrt[3]{n}} \sim \sum_{n=1}^{\infty} n^p\cdot\frac{3}{n^{1/3}} \sim 3\sum_{n=1}^{\infty}\frac{1}{n^{1/3-p}}$

$1/3-p>1$

$-p>2/3 \,/:(-1)$ $\quad p\in\left(-\infty,-\frac{2}{3}\right)$

$p<-2/3$

54. $\sum_{n=1}^{\infty}\frac{(n^2+1)^p}{\operatorname{tg}\frac{1}{n^2}} \sim \sum_{n=1}^{\infty}\frac{n^{2p}}{\frac{1}{n^2}} \sim \sum_{n=1}^{\infty}\frac{1}{n^{-p-2}}$

$-p-2>1 \,/\cdot(-1)$

$p+2<-1$ $\quad p\in(-\infty,-3)$

$p<-3$

55. $\sum_{n=1}^{\infty} n\ln^p\left(1+\frac{1}{\sqrt{n}}\right) \sim \sum_{n=1}^{\infty} n\,\frac{1}{n^{p/2}} \sim \sum_{n=1}^{\infty}\frac{1}{n^{p/2-1}}$

$\frac{p}{2}-1>1 \,/\cdot 2$ $\quad p\in(4,+\infty)$

$p-2>2$

$p>4$

56. $\sum_{n=1}^{\infty}\frac{1}{\sqrt[3]{n^p+3}}\cdot\ln\left(1+\frac{2}{n^2}\right) \sim$

$\sim \sum_{n=1}^{\infty}\frac{1}{n^{p/3}}\cdot\frac{2}{n^2} \sim 2\sum_{n=1}^{\infty}\frac{1}{n^{p/3+2}}$

$\frac{p}{3}+2>1 \,/\cdot 3$ $\quad p+6>3 \quad p>-3 \quad p\in(-3,+\infty)$

57. $\sum_{n=1} \frac{\ldots}{\ln(n^3+3)-\ln n^3} \sim \sum_{n=1} \frac{\ldots}{\ln\left(\frac{n^3+3}{n^3}\right)} \sim \sum_{n=1} \frac{\ldots}{\ln\left(1+\frac{3}{n^3}\right)}$

$\sim \sum_{n=1}^{\infty}\frac{n^p}{\frac{3}{n^3}} \sim \frac{1}{3}\sum_{n=1}^{\infty}\frac{1}{n^{-p-3}}$ $\quad -p-3>1 \,/\cdot(-1)$

$p+3<-1$

$p<-4$ $\quad p\in(-\infty,-4)$

58. $\sum_{n=1}^{\infty}\frac{\left(\ln(\sqrt{n^2+1}-\ln\sqrt{n^2})\right)^p}{n^{2p}} \sim$

$\sim \sum_{n=1}^{\infty}\frac{\left(\ln\left(\frac{\sqrt{n^2+1}}{\sqrt{n^2}}\right)\right)^p}{n^{2p}} \sim$

$\sim \sum_{n=1}^{\infty}\frac{\left(\ln\left(1+\frac{2}{n^{1/3}}\right)\right)^p}{n^{2p}} \sim \sum_{n=1}^{\infty}\frac{\frac{2^p}{n^{p/3}}}{n^{2p}} \sim 2^p\sum\frac{1}{n^{2p+p/3}}$

$p\in(3/7,\infty)$

$2p+\frac{p}{3}>1 \cdot 3$

$6p+p>3$

$7p>3 \,/:7$

$p>3/7$

59. $\sum_{n=1}^{\infty}\frac{(\ln(n+1)-\ln n)^p}{(2n-1)^2} \sim \sum_{n=1}^{\infty}\frac{\left(\ln\frac{n+1}{n}\right)^p}{4n^2-4n+1} \sim$

$\sim \sum_{n=1}^{\infty}\frac{\left(\ln\left(1+\frac{1}{n}\right)\right)^p}{4n^2} \sim \sum_{n=1}^{\infty}\frac{\frac{1}{n^p}}{4n^2} \sim$

$\sim \frac{1}{4}\sum_{n=1}\frac{1}{n^{p+2}}$ $\quad p+2>1 \quad p\in(-1,+\infty)$

$p>-1$

60. $\sum_{n=1}^{\infty}\frac{\sqrt{\ln(n^2+3)-2\ln n}}{(n+1)^p} \sim \sum_{n=1}^{\infty}\frac{\sqrt{\ln\left(\frac{n^2+3}{n^2}\right)}}{n^p} \sim$

$\sim \sum_{n=1}^{\infty}\frac{\sqrt{\ln\left(1+\frac{3}{n^2}\right)}}{n^p} \sim \sum_{n=1}^{\infty}\frac{\frac{\sqrt{3}}{n}}{n^p} \sim$

$\sim \sqrt{3}\sum_{n=1}^{\infty}\frac{1}{n^{p+1}}$ $\quad p+1>1 \quad p\in(0,+\infty)$

$p>0$

III) ИСПИТАТИ КОНВЕРГЕНЦИЈУ РЕДА:

1. $\sum_{n=1}^{\infty} \frac{n^3}{3^n}$ Даламберов тест

$$\lim_{n\to\infty} \frac{(n+1)^3}{3^{n+1}} \cdot \frac{3^n}{n^3} = \lim_{n\to\infty} \frac{n^3+3n^2+3n+1}{3n^3} = \frac{1}{3}$$

$\frac{1}{3} < 1$ РЕД КОНВЕРГИРА

2. $\sum_{n=1}^{\infty} \frac{n^2+2}{3^n}$ $L = \lim_{n\to\infty} \frac{(n+1)^2+2}{3^{n+1}} \cdot \frac{3^n}{n^2+2} =$

Даламберов тест
$L = 1/3$
РЕД КОНВЕРГИРА

$$= \lim_{n\to\infty} \frac{n^2+2n+1+2}{3n^2+6} = \lim_{n\to\infty} \frac{n^2+2n+3}{3n^2+6} = \frac{1}{3}$$

3. $\sum_{n=1}^{\infty} \frac{n^4}{4^n+1}$ $L = \lim_{n\to\infty} \frac{(n+1)^4}{4^{n+1}+1} \cdot \frac{4^n+1}{n^4} \Rightarrow$

$$L = \lim_{n\to\infty} \frac{n^4+\dots}{n^4} \cdot \frac{4^n(1+(\frac{1}{4})^n)}{4^{n+1}(1+(\frac{1}{4})^{n+1})} \Rightarrow$$

$$L = \lim_{n\to\infty} 1 \cdot \frac{4^n}{4^{n+1}} = \frac{1}{4} \quad \text{(C)}$$

4. $\sum_{n=1}^{\infty} \frac{2^n}{n^2+2n}$ $L = \lim_{n\to\infty} \frac{2^{n+1}}{(n+1)^2+2(n+1)} \cdot \frac{n^2+2n}{2^n}$

$$L = \lim_{n\to\infty} \frac{2n^2+4n}{n^2+2n+1+2n+2} = 2 \quad \text{(D)}$$

5. $\sum_{n=1}^{\infty} \frac{4^n}{n^2+2n}$ $L = \lim_{n\to\infty} \frac{4^{n+1}}{(n+1)^2+2n} \cdot \frac{n^2+2n}{4^n}$

$$L = \lim_{n\to\infty} \frac{4n^2+8n}{n^2+2n+1+2n} = 4 \quad \text{(D)}$$

6. $\sum_{n=1}^{\infty} \frac{2^{2n+1}}{n \cdot 3^n}$ $L = \lim_{n\to\infty} \frac{2^{2n+3}}{(n+1)3^{n+1}} \cdot \frac{n \cdot \cancel{3^n}}{\cancel{2^{2n+1}}}$

$$L = \lim_{n\to\infty} \frac{4n}{3n+3} = \frac{4}{3} \quad \text{(D)}$$

7. $\sum_{n=1}^{\infty} \frac{n^2}{4^n+3^n}$ $L = \lim_{n\to\infty} \frac{(n+1)^2}{4^{n+1}+3^{n+1}} \cdot \frac{4^n+3^n}{n^2}$

$$L = \lim_{n\to\infty} \frac{n^2+2n+1}{n^2} \cdot \frac{4^n(1+(\frac{3}{4})^n)}{4^{n+1}(1+(\frac{3}{4})^{n+1})}$$

$$L = \lim_{n\to\infty} 1 \cdot \frac{1}{4} = \frac{1}{4} \quad \text{(C)}$$

8. $\sum_{n=1}^{\infty} \frac{n^3+3}{2^n+3^n}$ $L = \lim_{n\to\infty} \frac{(n+1)^3+3}{2^{n+1}+3^{n+1}} \cdot \frac{2^n+3^n}{n^3+3}$

$$L = \lim_{n\to\infty} \frac{n^3+3n^2+3n+4}{n^3+3} \cdot \frac{3^n((\frac{2}{3})^n+1)}{3^{n+1}((\frac{2}{3})^{n+1}+1)} = 1 \cdot \frac{1}{3} = \frac{1}{3} \quad \text{(C)}$$

9. $\sum_{n=1}^{\infty} \frac{n^5}{3^n+5^n}$ $L = \lim_{n\to\infty} \frac{(n+1)^5}{3^{n+1}+5^{n+1}} \cdot \frac{3^n+5^n}{n^5}$

$$L = \lim_{n\to\infty} \frac{n^5+\dots}{n^5} \cdot \frac{5^n((\frac{3}{5})^n+1)}{5^{n+1}((\frac{3}{5})^{n+1}+1)} = 1 \cdot \frac{1}{5} = \frac{1}{5} \quad \text{(C)}$$

10. $\sum_{n=1}^{\infty} \frac{3^n+7^n}{n^4+2n^3}$ $L = \lim_{n\to\infty} \frac{3^{n+1}+7^{n+1}}{(n+1)^4+2(n+1)^3} \cdot \frac{n^4+2n^3}{3^n+7^n}$

$$L = \lim_{n\to\infty} \frac{7^{n+1}((3/7)^{n+1}+1)}{7^n((3/7)^n+1)} \cdot \frac{n^4+2n^3}{n^4+\dots} = 7 \quad \text{(D)}$$

11. $\sum_{n=1}^{\infty} \frac{5^{n+1}+3}{7^n+n^2}$ $L = \lim_{n\to\infty} \frac{5^{n+2}+3}{7^{n+1}+(n+1)^2} \cdot \frac{7^n+n^2}{5^{n+1}+3}$

$$L = \lim_{n\to\infty} \frac{5^{n+2}(1+0)}{5^{n+1}(1+0)} \cdot \frac{7^n(1+\frac{n^2}{7^n})}{7^{n+1}(1+0)} = \frac{5}{7} \quad \text{(C)}$$

12. $\sum_{n=1}^{\infty} \frac{n^5+4^{n+1}}{3^n+6}$ $L = \lim_{n\to\infty} \frac{(n+1)^5+4^{n+2}}{3^{n+1}+6} \cdot \frac{3^n+6}{n^5+4^{n+1}}$

$$L = \lim_{n\to\infty} \frac{4^{n+2}(\dots+1)}{3^{n+1}(1+0)} \cdot \frac{3^n(1+0)}{4^{n+1}(\dots)} = \frac{4}{3} \quad \text{(D)}$$

13. $\sum_{n=1}^{\infty} \frac{2^n}{n!}$ $L=\lim_{n\to\infty} \frac{2^{n+1}}{(n+1)!}\cdot\frac{n!}{2^n} = \lim_{n\to\infty}\frac{2}{n+1} = 0$ (C)

14. $\sum_{n=1}^{\infty} \frac{n^3}{n!}$ $\quad L=\lim_{n\to\infty} \frac{(n+1)^3}{(n+1)!}\cdot\frac{n!}{n^3} = \lim_{n\to\infty}\frac{n^3+\dots}{(n+1)\cdot n^3}$

$= \lim_{n\to\infty}\frac{n^3+\dots}{n^4+n^3} = 0$ (C)

15. $\sum_{n=1}^{\infty} \frac{5^n+1}{n!}$ (D)

$L=\lim_{n\to\infty}\frac{5^{n+1}+1}{(n+1)!}\cdot\frac{n!}{5^n+1} = \lim_{n\to\infty}\frac{5^{n+1}(1+0)}{5^n(1+0)}\cdot\frac{n!}{(n+1)\cdot n!}\cdot 5$

16. $\sum_{n=1}^{\infty} \frac{n^2(2^n+1)}{(\sqrt5+\sqrt3)^n}$ $\quad L=\lim_{n\to\infty}\frac{(n+1)^2(2^{n+1}+1)}{(\sqrt5+\sqrt3)^{n+1}}\cdot\frac{(\sqrt5+\sqrt3)^n}{n^2(2^n+1)}$

$L=\lim_{n\to\infty}\frac{(n^2+2n+1)\cdot 2^{n+1}(1+0)}{n^2\cdot 2^n(1+0)}\cdot\frac{1}{\sqrt5+\sqrt3}$

$L=\lim_{n\to\infty}\frac{2(n^2+2n+1)}{(\sqrt5+\sqrt3)\cdot n^2} = \frac{2(\sqrt5-\sqrt3)}{5-3} = \sqrt5-\sqrt3 = 2 > 1$ (D)

17. $\sum_{n=1}^{\infty} \frac{n(n+1)\cdot(n+8)}{3^n}$ $\quad L=\lim_{n\to\infty}\frac{(n+1)\cdot(n+3)\cdot(n+8)}{3^{n+1}}\cdot\frac{3^n}{n(n+1)(n+8)}$

$L=\lim_{n\to\infty}\frac{n^3+\dots}{3n^3+\dots} = \frac{1}{3}$ (C)

18. $\sum_{n=1}^{\infty} \frac{n^4(\sqrt2+\sqrt3)^n}{5^n}$ $\quad L=\lim_{n\to\infty}\frac{(n+1)^4(\sqrt2+\sqrt3)^{n+1}}{5^{n+1}}\cdot\frac{5^n}{n^4(\sqrt2+\sqrt3)^n}$

$L=\lim_{n\to\infty}\frac{n^4+\dots\cdot(\sqrt2+\sqrt3)}{5n^4} = \frac{\sqrt2+\sqrt3}{5} = \frac{1{,}41+1{,}73}{5} < 1$ (C)

19. $\sum_{n=1}^{\infty} \frac{n!}{n^3 3^n}$ $\quad L=\lim_{n\to\infty}\frac{(n+1)!}{(n+1)^3\cdot 3^{n+1}}\cdot\frac{n^3 3^n}{n!}$

$L=\lim_{n\to\infty}\frac{n^4+n^3}{3n^3+\dots} = +\infty$ ДИВЕРГИРА РЕД

20. $\sum_{n=1}^{\infty} \frac{(n!)^2}{4^{n^2}}$ $\quad L=\lim_{n\to\infty}\frac{((n+1)!)^2}{4^{(n+1)^2}}\cdot\frac{4^{n^2}}{(n!)^2}$

$L=\left(\frac{(n+1)\cdot n!}{n!}\right)^2\cdot\frac{4^{n^2}}{4^{n^2+2n+1}}$

$L=(n+1)^2\cdot\frac{1}{4^{2n+1}} = 0$ РЕД КОНВЕРГИРА

21. $\sum_{n=1}^{\infty} \frac{(2n)!}{4^n n^2}$ $\quad L=\lim_{n\to\infty}\frac{(2n+2)!}{4^{n+1}(n+1)^2}\cdot\frac{4^n n^2}{(2n)!}$ РЕД ДИВЕРГИРА

$L=\lim_{n\to\infty}\frac{(2n+2)\cdot(2n+1)\cdot n^2}{4n^2+8n+4} = \lim_{n\to\infty}\frac{4n^4+\dots}{4n^2+8n+4} = +\infty$

22. $\sum_{n=1}^{\infty} \frac{n^3+2}{(3n)!}$ $\quad L=\lim_{n\to\infty}\frac{(n+1)^3+2}{(3n+3)!}\cdot\frac{(3n)!}{n^3+2}$

$L=\lim_{n\to\infty}\frac{n^3+3n^2+3n+3}{n^3+2}\cdot\frac{1}{(3n+3)\cdot(3n+2)\cdot(3n+1)}$

$L=\lim_{n\to\infty}\frac{1}{27n^3+\dots} = 0$ РЕД КОНВЕРГИРА

23. $\sum_{n=1}^{\infty} \frac{n!(2n+1)!}{(3n)!}$ $\quad L=\lim_{n\to\infty}\frac{(n+1)!(2n+3)!}{(3n+3)!}\cdot\frac{(3n)!}{n!(2n+1)!}$ РЕД КОНВЕРГИРА

$L=\lim_{n\to\infty}\frac{(n+1)\cdot(2n+3)\cdot(2n+2)}{(3n+3)\cdot(3n+2)\cdot(3n+1)} = \lim_{n\to\infty}\frac{4n^3+\dots}{27n^3+\dots} = \frac{4}{27}$

24. $\sum_{n=1}^{\infty} \frac{(4n)!}{(2n+1)!(n!)^2}$ $\quad L=\lim_{n\to\infty}\frac{(4n+4)!}{(2n+3)!((n+1)!)^2}\cdot\frac{(2n+1)!(n!)^2}{(4n)!}$

$L=\lim_{n\to\infty}\frac{(4n+4)\cdot(4n+3)\cdot(4n+2)\cdot(4n+1)}{(2n+3)\cdot(2n+2)\cdot(n+1)^2}$ РЕД ДИВЕРГИРА

$L=\lim_{n\to\infty}\frac{4^4 n^4+\dots}{2^2\cdot n^4+\dots} = \frac{4^4}{4} = 4^3 = 64$

25. $\sum_{n=1}^{\infty} \frac{(n!)}{(2n)!}$ $L = \lim_{n\to\infty} \frac{((n+1)!)^2}{(2n+2)!} \cdot \frac{(2n)!}{(n!)^2}$ РЕД ДИВЕРГИРА

$L = \lim_{n\to\infty} \frac{(n+1)^2}{2n+2} = \lim_{n\to\infty} \frac{(n+1)^2}{2(n+1)} = \lim_{n\to\infty} \frac{n+1}{2} = +\infty$

26. $\sum_{n=1}^{\infty} \frac{(n!)^3}{(3n)!}$ $L = \lim_{n\to\infty} \frac{((n+1)!)^3}{(3n+3)!} \cdot \frac{(3n)!}{(n!)^3}$ РЕД КОНВЕРГИРА

$L = \lim_{n\to\infty} \frac{n^3+3n^2+3n+1}{(3n+3)\cdot(3n+2)\cdot(3n+1)} = \lim_{n\to\infty} \frac{n^3+\dots}{27n^3+\dots} = \frac{1}{27}$

27. $\sum_{n=1}^{\infty} \frac{(2n+1)!}{\pi^n (n!)^2}$ $L = \lim_{n\to\infty} \frac{(2n+3)!}{\pi^{n+1}((n+1)!)^2} \cdot \frac{\pi^n (n!)^2}{(2n+1)!}$

$L = \lim_{n\to\infty} \frac{(2n+4)\cdot(2n+3)}{\pi(n+1)^2} = \lim_{n\to\infty} \frac{4n^2+\dots}{\pi n^2+\dots} = \frac{4}{\pi}$ РЕД ДИВЕРГИРА

28. $\sum_{n=1}^{\infty} \frac{(2n+1)!}{e^n ((n+1)!)^2}$ $L = \lim_{n\to\infty} \frac{(2n+3)!}{e^{n+1}((n+2)!)^2} \cdot \frac{e^n((n+1)!)^2}{(2n+1)!}$

$L = \lim_{n\to\infty} \frac{(2n+3)\cdot(2n+2)}{e\,(n+2)^2} = \lim_{n\to\infty} \frac{4n^2+\dots}{e n^2+\dots} = \frac{4}{e}$

РЕД ДИВЕРГИРА

29. $\sum_{n=1}^{\infty} \frac{(n!)^3}{((2n)!)^2}$ $L = \lim_{n\to\infty} \frac{((n+1)!)^3}{((2n+2)!)^2} \cdot \frac{((2n)!)^2}{(n!)^3}$

$L = \lim_{n\to\infty} \frac{n^3+\dots}{(2n+2)^2\cdot(2n+1)^2} = \lim_{n\to\infty} \frac{n^3+\dots}{4n^4+\dots} = 0$

РЕД КОНВЕРГИРА

30. $\sum_{n=1}^{\infty} \frac{((n+1)!)^2}{(2n+2)!}$ $L = \lim_{n\to\infty} \frac{((n+2)!)^2}{(2n+4)!} \cdot \frac{(2n+2)!}{((n+1)!)^2}$

$L = \lim_{n\to\infty} \frac{n^2+4n+4}{(2n+4)\cdot(2n+3)} = \lim_{n\to\infty} \frac{n^2+\dots}{4n^2+\dots} = \frac{1}{4}$ РЕД КОНВЕРГИРА

31. $\sum_{n=1}^{\infty} \frac{n!}{n^n}$ $L = \lim_{n\to\infty} \frac{(n+1)!}{(n+1)^{n+1}} \cdot \frac{n^n}{n!}$

$L = \lim_{n\to\infty} \frac{(n+1)\,n^n}{(n+1)^n\cdot(n+1)} = \lim_{n\to\infty} \left(\frac{n}{n+1}\right)^n = \lim_{n\to\infty} \frac{1}{\left(\frac{n+1}{n}\right)^{n}}$

$= \lim_{n\to\infty} \frac{1}{\left(1+\frac{1}{n}\right)^{n\cdot\frac{1}{n}\cdot n}} = \lim_{n\to\infty} \frac{1}{e^{+1}} < 1$ КОНВЕРГИРА

32. $\sum_{n=1}^{\infty} \frac{n^n}{n!}$ $L = \lim_{n\to\infty} \frac{(n+1)^{n+1}}{(n+1)!} \cdot \frac{n!}{n^n}$

$L = \lim_{n\to\infty} \frac{(n+1)^n\cdot(n+1)}{(n+1)\cdot n^n} = \lim_{n\to\infty} \left(\frac{n+1}{n}\right)^n =$

$= \lim_{n\to\infty} \left(1+\frac{1}{n}\right)^{n\cdot\frac{1}{n}\cdot n} = e^1 = e$ РЕД ДИВЕРГИРА

33. $\sum_{n=1}^{\infty} \frac{2^n\cdot n!}{n^n}$ $L = \lim_{n\to\infty} \frac{2^{n+1}(n+1)!}{(n+1)^{n+1}} \cdot \frac{n^n}{2^n\cdot n!}$

$L = \lim_{n\to\infty} \frac{2(n+1)\,n^n}{(n+1)^n\cdot(n+1)} = \lim_{n\to\infty} \frac{2n^n}{(n+1)^n} = \lim_{n\to\infty} \frac{2}{e} = \frac{2}{e}$

РЕД КОНВЕРГИРА

34. $\sum_{n=1}^{\infty} \frac{3^n n!}{n^n}$ $L = \lim_{n\to\infty} \frac{3^{n+1}(n+1)!}{(n+1)^{n+1}} \cdot \frac{n^n}{3^n\cdot n!}$

$L = \lim_{n\to\infty} \frac{3(n+1)\,n^n}{(n+1)^n\cdot(n+1)} = \lim_{n\to\infty} \frac{3}{e}$ РЕД ДИВЕРГИРА

35. $\sum_{n=1}^{\infty} \frac{(4n)!\,n^n}{(3n)!}$ $L = \lim_{n\to\infty} \frac{(4n+4)!\,(n+1)^{n+1}}{(3n+3)!} \cdot \frac{(3n)!}{(4n)!\cdot n^n}$

$L = \lim_{n\to\infty} \frac{(4n+4)\cdot(4n+3)\cdot(4n+2)\cdot(4n+1)\,(n+1)^n\cdot(n+1)}{(3n+3)\cdot(3n+2)\cdot(3n+1)\cdot n^n}$

$L = \lim \frac{4^4 n^5+\dots}{3^3 n^3+\dots}\, e = +\infty$ РЕД ДИВЕРГИРА

36. $\sum_{n=1} \frac{(3n)!\, n^n}{(4n)!}$ $\quad L = \lim\limits_{n\to\infty} \frac{(3n+3)!\,(n+1)^{n+1}}{(4n+4)!} \cdot \frac{(4n)!}{(3n)!\, n^n}$

$$L = \lim_{n\to\infty} \frac{(3n+3)\cdot(3n+2)\cdot(3n+1)\,(n+1)^n\,(n+1)}{(4n+4)\cdot(4n+3)\cdot(4n+2)\cdot(4n+1)\cdot n^n}$$

$$L = \lim_{n\to\infty} \frac{27n^4+\dots}{256n^4+\dots}\, e = \frac{27e}{256}$$ РЕД КОНВЕРГИРА

37. $\sum_{n=1}^{\infty} \frac{2\cdot 6\cdot 10\cdots(4n-2)}{4\cdot 7\cdot 10\cdots(3n+1)} =$

$$L = \lim_{n\to\infty} \frac{6\cdot 10\cdot 14\cdots(4n+2)}{7\cdot 10\cdot 13\cdots(3n+4)} \cdot \frac{4\cdot 7\cdot 10\cdots(3n+1)}{2\cdot 6\cdot 10\cdots(4n-2)}$$

$$L = \lim_{n\to\infty} \frac{(4n+2)\cdot 4}{(3n+4)\cdot 2} = \frac{4\cdot 4}{3\cdot 2} = \frac{16}{6} = \frac{8}{3}$$ РЕД ДИВЕРГИРА

38. $\sum_{n=1}^{\infty} \frac{2\cdot 5\cdot 8\cdots(3n-1)}{1\cdot 6\cdot 11\cdots(5n-4)}$

$$L = \lim_{n\to\infty} \frac{5\cdot 8\cdot 11\cdots(3n+2)}{6\cdot 11\cdot 16\cdots(5n+1)} \cdot \frac{1\cdot 6\cdot 11\cdots(5n-4)}{2\cdot 5\cdot 8\cdots(3n-1)}$$

$$L = \lim_{n\to\infty} \frac{(3n+2)\cdot 1}{(5n+1)\cdot 2} = \frac{3\cdot 1}{5\cdot 2} = \frac{3}{10}$$ РЕД КОНВЕРГИРА

39. $\sum_{n=1}^{\infty} \frac{2\cdot 6\cdot 10\cdots(4n+2)}{2\cdot 5\cdot 8\cdots(3n-2)}$

$$L = \lim_{n\to\infty} \frac{10\cdot 14\cdots(4n+6)}{8\cdot 11\cdots(3n+5)} \cdot \frac{2\cdot 5\cdot 8\cdots(3n-2)}{2\cdot 6\cdot 10\cdots(4n+2)}$$

$$L = \lim_{n\to\infty} \frac{20n+30}{18n+30} = \frac{20}{18} = \frac{10}{9}$$ РЕД ДИВЕРГИРА

40. $\sum_{n=1} \frac{100\cdot 102\cdots(98+2n)}{1\cdot 4\cdot 7\cdots(3n-2)}$

$$L = \lim_{n\to\infty} \frac{102\cdot 104\cdots(100+2n)}{4\cdot 7\cdots(3n+1)} \cdot \frac{1\cdot 4\cdot 7\cdots(3n-2)}{100\cdot 102\cdots(98+2n)}$$

$$L = \lim_{n\to\infty} \frac{1\cdot(100+2n)}{(3n+1)\cdot 100} = \lim_{n\to\infty} \frac{2\cdot 1}{100\cdot 3} = \frac{1}{150}$$ РЕД КОНВЕРГИРА

41. $\sum_{n=1}^{\infty} \frac{2\cdot 5\cdot 8\cdots(3n-1)}{(2n)!!}$

$$L = \lim_{n\to\infty} \frac{5\cdot 8\cdots(3n+2)}{(2n+2)!!} \cdot \frac{(2n)!!}{2\cdot 5\cdot 8\cdots(3n-1)}$$

$$L = \lim_{n\to\infty} \frac{3n+2}{(2n+2)\cdot 2} = \frac{3}{2\cdot 2} = \frac{3}{4}$$ РЕД КОНВЕРГИРА

42. $\sum_{n=1}^{\infty} \frac{100\cdot 103\cdots(97+3n)}{(2n+1)!!}$

$$L = \lim_{n\to\infty} \frac{103\cdot 106\cdots(100+3n)}{(2n+3)!!} \cdot \frac{(2n+1)!!}{100\cdot 103\cdots(97+3n)}$$

$$L = \lim_{n\to\infty} \frac{100+3n}{(2n+3)\cdot 100} = \frac{3}{2\cdot 100} = \frac{3}{200}$$ РЕД КОНВЕРГИРА

43. $\sum_{n=0}^{\infty} \frac{(2n+1)!!}{1\cdot 6\cdot 11\cdots(5n+1)}$

$$L = \lim_{n\to\infty} \frac{(2n+3)!!}{11\cdot 16\cdots(5n+6)} \cdot \frac{1\cdot 6\cdot 11\cdots(5n+1)}{(2n+1)!!}$$

$$L = \lim_{n\to\infty} \frac{(2n+3)\cdot 6}{5n+6} = \frac{2\cdot 6}{5} = \frac{12}{5}$$ РЕД ДИВЕРГИРА

44. $\sum_{n=0}^{\infty} \frac{(2n)!!}{1\cdot4\cdot7\cdots(3n+1)}$

$L=\lim_{n\to\infty} \frac{(2n+2)!!}{\cancel{7\cdot10}\cdots(3n+4)}\cdot\frac{\cancel{1\cdot4\cdot7\cdots(3n+1)}}{\cancel{(2n)!!}}$

$L=\lim_{n\to\infty} \frac{4(2n+2)}{3n+4}=\frac{8}{3}$ РЕД ДИВЕРГИРА

45. $\sum_{n=1}^{\infty} \frac{1\cdot5\cdot9\cdots(4n-3)}{3^n(2n+1)!!}$

$L=\lim_{n\to\infty} \frac{\cancel{5\cdot9}\cdots(4n+1)}{3^{n+1}(2n+3)!!}\cdot\frac{\cancel{3^n}\cdot\cancel{(2n+1)!!}}{\cancel{1\cdot5\cdot9}\cdots(4n-3)}$

$L=\lim_{n\to\infty} \frac{4n+1}{6n+9}=\frac{4}{6}=\frac{2}{3}$ РЕД КОНВЕРГИРА

46. $\sum_{n=1}^{\infty} \frac{5^n(2n)!!}{1\cdot7\cdot13\cdots(6n-5)}$

$L=\lim_{n\to\infty} \frac{5^n\cdot5\cdot(2n+2)!!}{7\cdot13\cdots(6n+1)}\cdot\frac{1\cdot7\cdot13\cdots(6n-5)}{5^n\cdot(2n)!!}$

$L=\lim_{n\to\infty} \frac{5(2n+2)}{6n+1}=\frac{5\cdot2}{6}=\frac{5}{3}$ РЕД ДИВЕРГИРА

47. $\sum_{n=1}^{\infty} \frac{2\cdot7\cdot12\cdots(5n-3)}{4^n\cdot(2n-1)!!}$

$L=\lim_{n\to\infty} \frac{\cancel{7\cdot12}\cdots(5n+2)}{\cancel{4^n}\cdot4(2n+1)!!}\cdot\frac{\cancel{4^n}\cdot\cancel{(2n-1)!!}}{2\cdot\cancel{7\cdot12}\cdots(5n-3)}$

$L=\lim_{n\to\infty} \frac{5n+2}{8(2n+1)}=\frac{5}{8\cdot2}=\frac{5}{16}$ РЕД КОНВЕРГИРА

48. $\sum_{n=1}^{\infty} \frac{5^n(2n)!!}{1\cdot8\cdot15\cdots(7n-6)}$

$L=\lim_{n\to\infty} \frac{5^n\cdot5(2n+2)!!}{8\cdot15\cdots(7n+1)}\cdot\frac{\cancel{1\cdot8\cdot15\cdots(7n-6)}}{5^n\cdot(2n)!!}$

$L=\lim_{n\to\infty} \frac{10n+10}{7n+1}=\frac{10}{7}$ РЕД ДИВЕРГИРА

49. $\sum_{n=1}^{\infty} \frac{(2n+1)!!}{3^n n!}$

$L=\lim_{n\to\infty} \frac{(2n+3)!!}{3^n\cdot3(n+1)!}\cdot\frac{\cancel{3^n}\cdot\cancel{n!}}{\cancel{(2n+1)!!}}$

$L=\lim_{n\to\infty} \frac{2n+3}{3n+3}=\frac{2}{3}$ РЕД КОНВЕРГИРА

50. $\sum_{n=1}^{\infty} \frac{4^n(n+2)!!}{3^n(2n)!!}$

$L=\lim_{n\to\infty} \frac{\cancel{4^n}\cdot4(n+3)!!}{\cancel{3^n}\,3(2n+2)!!}\cdot\frac{\cancel{3^n}\cdot\cancel{(2n)!!}}{\cancel{4^n}\cdot\cancel{(n+2)!!}}$

$L=\lim_{n\to\infty} \frac{4n+12}{6n+6}=\frac{4}{6}=\frac{2}{3}$ РЕД КОНВЕРГИРА

51. $\sum_{n=1}^{\infty} \frac{2^n(2n-1)!!}{5^n}$

$L=\lim_{n\to\infty} \frac{\cancel{2^n}\cdot2(2n+1)!!}{\cancel{5^n}\cdot5}\cdot\frac{\cancel{5^n}}{\cancel{2^n}\cdot\cancel{(2n-1)!!}}$

$L=\lim_{n\to\infty} \frac{4n+2}{5}=+\infty$ РЕД ДИВЕРГИРА

52. $\sum\limits_{N=1} \frac{3^n (2n)!!}{(2n+1)!}$

$L = \lim\limits_{n\to\infty} \frac{3^n \cdot 3(2n+2)!!}{(2n+3)!} \cdot \frac{(2n+1)!}{3^n (2n)!!}$

$L = \frac{6n+6}{2n+3} = \frac{6}{2} = 3$ РЕД ДИВЕРГИРА

53. $\sum\limits_{N=1}^{\infty} \frac{4^n (2n)!!}{(3n)!}$

$L = \lim\limits_{n\to\infty} \frac{\cancel{4^n} \cdot 4(2n+2)!!}{(3n+3)!} \cdot \frac{\cancel{(3n)!}}{\cancel{4^n} \cdot (2n)!!}$

$L = \lim\limits_{n\to\infty} \frac{8n+8}{(3n+3)\cdot(3n+2)\cdot(3n+1)} = 0$ РЕД КОНВЕРГИРА

54. $\sum\limits_{N=1}^{\infty} \frac{(2n+2)!}{3^n \cdot (2n+2)!!}$

$L = \lim\limits_{n\to\infty} \frac{(2n+4)!}{3^n \cdot 3(2n+4)!!} \cdot \frac{3^n \cdot (2n+2)!!}{(2n+2)!}$

$L = \lim\limits_{n\to\infty} \frac{(2n+5)\cdot(2n+3)}{3(2n+4)} = +\infty$ РЕД ДИВЕРГИРА

55. $\sum\limits_{N=1}^{\infty} \frac{(2n-1)!!}{(2n)!!} \cdot \frac{1}{n}$

$L = \lim\limits_{n\to\infty} \frac{(2n+1)!!}{(2n+2)!!} \cdot \frac{1}{n+1} \cdot \frac{\cancel{(2n)!!}}{\cancel{(2n-1)!!}} \cdot \frac{n}{1}$

$L = \lim\limits_{n\to\infty} \frac{(2n+1)\cdot 2n^2}{2n^2+4n+2} = \lim\limits_{n\to\infty} \frac{4n^3+2n^2}{2n^2+4n} = +\infty$

РЕД ДИВЕРГИРА

56. $\sum\limits_{N=1} \frac{(2n)!!}{(2n+1)!!} \cdot n$

$L = \lim\limits_{n\to\infty} \frac{(2n+2)!!}{(2n+3)!!} (n+1) \cdot \frac{\cancel{(2n+1)!!}}{(2n)!!} \cdot \frac{1}{n}$

$L = \lim\limits_{n\to\infty} \frac{2n^2+2n+2n+2}{(2n+3)\cdot(2n+2)\cdot n} = 0$ РЕД КОНВЕРГИРА

57. $\sum\limits_{N=1}^{\infty} \frac{(2n-1)!!}{(2n)!!} \cdot \frac{1}{3^n}$

$L = \lim\limits_{n\to\infty} \frac{(2n+1)!!}{(2n+2)!!} \cdot \frac{1}{\cancel{3^n} \cdot 3} \cdot \frac{\cancel{(2n)!!}}{\cancel{(2n-1)!!}} \cdot \frac{\cancel{3^n}}{1}$

$L = \lim\limits_{n\to\infty} \frac{2n+1}{3(2n+2)} = \frac{2}{3\cdot 2} = \frac{1}{3}$ РЕД КОНВЕРГИРА

58. $\sum\limits_{N=1}^{\infty} \frac{(2n+2)!!}{(2n+1)!!} \cdot \frac{1}{7^n}$

$L = \lim\limits_{n\to\infty} \frac{(2n+4)!!}{(2n+3)!!} \cdot \frac{1}{\cancel{7^n} \cdot 7} \cdot \frac{\cancel{(2n+1)!!}}{\cancel{(2n+2)!!}} \cdot \frac{\cancel{7^n}}{1}$

$L = \lim\limits_{n\to\infty} \frac{2n+4}{7(2n+3)} = \frac{2}{7\cdot 2} = \frac{1}{7}$ РЕД КОНВЕРГИРА

59. $\sum\limits_{N=2}^{\infty} \left(\frac{n-2}{n+2}\right)^{n^2}$ $L = \lim\limits_{n\to\infty} \left(\frac{n+2-4}{n+2}\right)^{\frac{n^2}{n}}$

$L = \lim\limits_{n\to\infty} \left(1 - \frac{4}{n+2}\right)^{\frac{n+2}{4} \cdot n \cdot \left(-\frac{4}{n+2}\right)}$

$L = e^{-4} = \frac{1}{e^4}$ РЕД КОНВЕРГИРА

60. $\sum_{N=3}^{\infty}\left(\frac{n-3}{n+3}\right)^{n^2}$ $L=\lim_{n\to\infty}\left(\frac{n+3-6}{n+3}\right)^{\frac{n^2}{n}}$

$L=\lim_{n\to\infty}\left(1-\frac{6}{n+3}\right)^{\frac{n+3}{6}\cdot n\cdot\left(\frac{6}{n+3}\right)}$

$L=e^{-6}=\frac{1}{e^6}$ РЕД КОНВЕРГИРА

61. $\sum_{N=1}^{\infty}\left(\frac{3n}{n+2}\right)^{3n}\cdot\left(\frac{2n}{2n+1}\right)^{n^2}$

$L=\lim_{n\to\infty}\left(\frac{3n}{n+2}\right)^{3n}\cdot\left(\frac{2n}{2n+1}\right)^{n^2}$

$L=\lim_{n\to\infty}\left(\frac{n+2+2n}{n+2}\right)^{\frac{3n}{n}}\cdot\left(\frac{2n+1-1}{2n+1}\right)^{\frac{n^2}{n}}$

$L=\lim_{n\to\infty}\left(1+\frac{2n}{n+2}\right)^{\frac{n+2}{2n}\cdot3\cdot\frac{2n}{n+2}}\cdot\left(1-\frac{1}{2n+1}\right)^{(2n+1)\cdot\left(\frac{n}{2n+1}\right)}$

$L=e^{6}\cdot e^{-1/2}=e^{11/2}$ РЕД ДИВЕРГИРА

62. $\sum_{N=1}^{\infty}\left(\frac{3n-1}{2n+1}\right)^{n^2}$ $L=\lim_{n\to\infty}\left(\frac{3n-1}{2n+1}\right)^{\frac{n^2}{n}}$

$L=\lim_{n\to\infty}\left(\frac{2n+1+n-2}{2n+1}\right)^{n}=\lim_{n\to\infty}\left(1+\frac{n-2}{2n+1}\right)^{\frac{2n+1}{n-2}\cdot n\cdot\frac{n-2}{2n+1}}$

$L=\lim_{n\to\infty}e^{+\infty}=+\infty$ РЕД ДИВЕРГИРА

63. $\sum_{N=2}^{\infty}\left(\frac{n-1}{n+1}\right)^{n(n-1)}$ $L=\lim_{n\to\infty}\left(\frac{n-1}{n+1}\right)^{\frac{n(n-1)}{n}}$

$L=\lim_{n\to\infty}\left(\frac{n+1-2}{n+1}\right)^{n-1}=\lim_{n\to\infty}\left(1-\frac{2}{n+1}\right)^{\frac{n+1}{2}\cdot(n-1)\cdot\left(-\frac{2}{n+1}\right)}$

$L=e^{-2}=\frac{1}{e^2}$ РЕД ШОНВЕРГИРА

64. $\sum_{N=1}^{\infty}\left(\frac{3n-1}{3n+1}\right)^{n(n-1)}$ $L=\lim_{n\to\infty}\left(\frac{3n-1}{3n+1}\right)^{n-1}$

$L=\lim_{n\to\infty}\left(\frac{3n+1-2}{3n+1}\right)^{n-1}=\lim_{n\to\infty}\left(1-\frac{2}{3n+1}\right)^{\frac{3n+1}{2}\cdot\left(-\frac{2(n-1)}{3n+1}\right)}$

$L=e^{-2/3}=1/e^{2/3}$ РЕД КОНВЕРГИРА

65. $\sum_{N=1}^{\infty}\left(\frac{n^2+5}{n^2+6}\right)^{n^3}$ $L=\lim_{n\to\infty}\left(\frac{n^2+5}{n^2+6}\right)^{\frac{n^3}{n}}$

$L=\lim_{n\to\infty}\left(\frac{n^2+6-1}{n^2+6}\right)^{n^2}=\lim_{n\to\infty}\left(1-\frac{1}{n^2+6}\right)^{\frac{n^2+6}{1}\cdot\left(-\frac{n^2}{n^2+6}\right)}$

$L=e^{-1}=1/e$ РЕД КОНВЕРГИРА

66. $\sum_{N=1}^{\infty}\left(\frac{2n}{3-4n}\right)^{2n}\left(\frac{2n^2}{2n^2+1}\right)^{n^3}$

$L=\lim_{n\to\infty}\left(\frac{2n}{3-4n}\right)^{\frac{2n}{n}}\cdot\left(\frac{2n^2}{2n^2+1}\right)^{\frac{n^3}{n}}$

$L=\lim_{n\to\infty}\left(\frac{3-4n+6n-3}{3-4n}\right)^{2}\cdot\left(\frac{2n^2+1-1}{2n^2+1}\right)^{n^2}$

$L=\lim_{n\to\infty}\left(1+\frac{6n-3}{3-4n}\right)^{\frac{3-4n}{6n-3}\cdot\frac{6n-3}{3-4n}\cdot2}\cdot\left(1-\frac{1}{2n^2+1}\right)^{(2n^2+1)\cdot\left(-\frac{1}{2n^2+1}\right)}$

$L=e^{-\frac{12}{4}}\cdot e^{-1/2}=e^{14/4}=1/e^{14/4}$ РЕД КОНВЕР

67. $\sum_{N=1}^{\infty}\left(\frac{3n^2+5}{3n^2+n}\right)^{n^2(n-1)}$ $L=\lim_{n\to\infty}\left(\frac{3n^2+5}{3n^2+n}\right)^{\frac{n^2(n-1)}{n}}$

$L=\lim_{n\to\infty}\left(\frac{3n^2+n-2}{3n^2+n}\right)^{n^2+n}=\lim_{n\to\infty}\left(1-\frac{2}{3n^2+n}\right)^{(n^2-n)\cdot\frac{3n^2+n}{2}\cdot\left(-\frac{2}{3n^2+n}\right)}$

$=\lim_{n\to\infty}e^{-2/3}=1/e^{2/3}$ РЕД ШОНВЕРГИРА

68. $\sum_{n=1}^{\infty}\left(\frac{3n^2-2}{3n^2+n+3}\right)^{n+1}$ $L=\lim_{n\to\infty}\left(\frac{3n^2-2}{3n^2+n+3}\right)^{n+1}$

$L=\lim_{n\to\infty}\left(\frac{3n^2+n+3-n-3}{3n^2+n+3}\right)^{n+1}$

$L=\lim_{n\to\infty}\left(1-\frac{n+5}{3n^2+n+3}\right)^{\frac{3n^2+n+3}{n+5}\cdot(n+1)\cdot\left(-\frac{n+5}{3n^2+n+3}\right)}$

$L=e^{-1/3}=1/e^{1/3}$ РЕД КОНВЕРГИРА

69. $\sum_{n=1}^{\infty}\left(\frac{\sqrt{n}+2}{3+\sqrt{n}}\right)^{n^{3/2}}$ $L=\lim_{n\to\infty}\left(\frac{\sqrt{n}+2}{\sqrt{n}+3}\right)^{n^{3/2-2/2}}$

$L=\lim_{n\to\infty}\left(\frac{\sqrt{n}+3-1}{\sqrt{n}+3}\right)^{n^{1/2}}$

$L=\lim_{n\to\infty}\left(1-\frac{1}{\sqrt{n}+3}\right)^{n^{1/2}\cdot(\sqrt{n}+3)\cdot\left(-\frac{1}{\sqrt{n}+3}\right)}$

$L=e^{-1}=1/e$ РЕД КОНВЕРГИРА

70. $\sum_{n=1}^{\infty}\left(\frac{\sqrt{n}+5}{\sqrt{n}+2}\right)^{n\sqrt{n}}$ $L=\lim_{n\to\infty}\left(\frac{\sqrt{n}+5}{\sqrt{n}+2}\right)^{\frac{n\sqrt{n}}{n}}$

$L=\lim_{n\to\infty}\left(\frac{\sqrt{n}+2+3}{\sqrt{n}+2}\right)^{\sqrt{n}}=\lim_{n\to\infty}\left(1+\frac{3}{\sqrt{n}+2}\right)^{\frac{\sqrt{n}+2}{3}\cdot\frac{3\sqrt{n}}{\sqrt{n}+2}}$

$L=e^{3}$ НИЗ ДИВЕРГИРА

71. $\sum_{n=1}^{\infty}n^2\cdot\left(\frac{3n+4}{4n+5}\right)^{n}$ $L=\lim_{n\to\infty}\sqrt[n]{n^2}\cdot\left(\frac{3n+4}{4n+5}\right)^{n/n}$

$L=\lim_{n\to\infty}\left(\sqrt[n]{n}\right)^2\cdot\frac{3}{4}=1\cdot\frac{3}{4}=\frac{3}{4}$ РЕД КОНВЕРГИРА

72. $\sum_{n=1}^{\infty}n\cdot\left(\frac{3n-2}{n+5}\right)^{n}$ $L=\lim_{n\to\infty}\sqrt[n]{n}\cdot\left(\frac{3n-2}{n+5}\right)$

$L=\lim_{n\to\infty}1\cdot 3=3$ РЕД ДИВЕРГИРА

73. $\sum_{n=1}^{\infty}n^3\cdot\left(\frac{5n-2}{3n+2}\right)^{n}$ $L=\lim_{n\to\infty}\sqrt[n]{n^3}\left(\frac{5n-2}{3n+2}\right)^{n/n}$

$L=1\cdot\frac{5}{3}=\frac{5}{3}$ РЕД ДИВЕРГИРА

74. $\sum_{n=1}^{\infty}\left(\frac{n+1}{2n-1}\right)^{2n}\cdot\left(\frac{n+3}{n+5}\right)^{n^2}$

$L=\lim_{n\to\infty}\left(\frac{n+1}{2n-1}\right)^{2n/n}\cdot\left(\frac{n+3}{n+5}\right)^{n^2/n}$

$L=\left(\frac{1}{2}\right)^2\cdot\left(\frac{n+5-2}{n+5}\right)^{n}=\frac{1}{4}\cdot\left(1-\frac{2}{n+5}\right)^{\frac{n+5}{2}\cdot\left(\frac{-2n}{n+5}\right)}$

$=\frac{1}{4}\cdot e^{-2}=\frac{1}{4e^2}$ РЕД КОНВЕРГИРА

75. $\sum_{n=1}^{\infty}n\cdot\left(\frac{2n-1}{2n+2}\right)^{n^2}$ $L=\lim_{n\to\infty}\sqrt[n]{n}\left(\frac{2n-1}{2n+2}\right)^{n^2/n}$

$L=\lim_{n\to\infty}1\cdot\left(\frac{2n+2-3}{2n+2}\right)^{n}=\lim_{n\to\infty}\left(1-\frac{3}{2n+2}\right)^{\frac{2n+2}{3}\cdot\frac{-3n}{2n+2}}$

$L=e^{-3/2}=1/e^{3/2}$ РЕД КОНВЕРГИРА

76. $\sum_{n=1}^{\infty}n^2\cdot\left(\frac{3n^2-1}{3n^2+1}\right)^{n^2(n-1)}$ $L=\lim_{n\to\infty}\sqrt[n]{n^2}\left(\frac{3n^2-1}{3n^2+1}\right)^{\frac{n^2(n-1)}{n}}$

$L=\lim_{n\to\infty}1\cdot\left(\frac{3n^2+1-2}{3n^2+1}\right)^{n^2-n}=\lim_{n\to\infty}\left(1-\frac{2}{3n^2+1}\right)^{\frac{3n^2+1}{2}\cdot\frac{-2}{3n^2+1}\cdot(n^2-n)}$

$L=e^{-2/3}=1/e^{2/3}$ РЕД КОНВЕРГИРА

77. $\sum_{n=1} 2^n \cdot \left(\frac{\ }{3n-2}\right)$ $\quad L=\lim_{n\to\infty} \sqrt[n]{2^n \cdot \left(\frac{\ }{3n-2}\right)^{n}}$

$= 2\cdot\left(\frac{3n-2+2}{3n-2}\right)^{n+1}$

$L = 2\cdot\left(1+\frac{2}{3n-2}\right)^{\frac{3n-2}{2}\cdot\frac{2(n+1)}{3n-2}}$

$L=2\cdot e^{2/3}$ РЕД ДИВЕРГИРА

78. $\sum_{n=1}^{\infty} 2^{-n}\cdot\left(\frac{n^2+1}{n^2}\right)^{n^3}$ $\quad L=\lim_{n\to\infty} 2^{-n/n}\cdot\left(\frac{n^2+1}{n^2}\right)^{n^3/n}$

$L = 2^{-1}\cdot\left(1+\frac{1}{n^2}\right)^{n^2\cdot n^2\cdot\frac{1}{n^2}}$

$L=\frac{1}{2}\cdot e = \frac{e}{2}$ РЕД ДИВЕРГИРА

79. $\sum_{n=1}^{\infty} 3^{n+1}\cdot\left(\frac{n+2}{n+3}\right)^{n^2}$ $\quad L=\lim 3^{\frac{n+1}{n}}\left(\frac{n+2}{n+3}\right)^{n^2/n}$

$L=3\cdot\left(\frac{n+3-1}{n+3}\right)^n = 3\cdot\left(1-\frac{1}{n+3}\right)^{n+3\cdot\frac{-n}{n+3}}$

$L=3\cdot e^{-1}=\frac{3}{e}$ РЕД ДИВЕРГИРА

80. $\sum_{n=1}^{\infty} 3^{-n}\cdot\left(\frac{2n+3}{2n}\right)^{n^2}$ $\quad L=\lim_{n\to\infty}\left(3^{-n/n}\cdot\left(\frac{2n+3}{2n}\right)^{n^2/n}\right)$

$L=3^{-1}\left(1+\frac{3}{2n}\right)^{n\cdot\frac{2n}{3}\cdot\frac{3}{2n}}=\frac{e^{2/3}}{3}$ РЕД КОНВЕРГИРА

81. $\sum_{n=1}^{\infty}\left(\frac{6n+1}{5n-3}\right)^{n/2}\left(\frac{5}{6}\right)^{2n/3}$

$L=\lim_{n\to\infty}\left(\frac{6n+1}{5n-3}\right)^{n/2n}\left(\frac{5}{6}\right)^{2n/3n}$

$L=\frac{\sqrt{6}}{\sqrt{5}}\cdot\frac{\sqrt[3]{5^2}}{\sqrt[3]{6^2}}=\frac{\sqrt[6]{6^3\cdot5^4}}{\sqrt[6]{5^3\cdot6^4}}=\sqrt[6]{\frac{5}{6}}$

РЕД КОНВЕРГИРА

82. $\sum_{n=1}^{\infty}\left(\frac{3n+5}{4n-2}\right)^{n/3}\left(\frac{4}{3}\right)^{4n/5}$ РЕД ДИВЕРГИРА

$L=\lim_{n\to\infty}\left(\frac{3n+5}{4n-2}\right)^{n/3n}\cdot\left(\frac{4}{3}\right)^{4n/5n}$

$L=\sqrt[3]{\frac{3}{4}}\cdot\sqrt[5]{\frac{4^4}{3^4}}=\sqrt[15]{\frac{3^5}{4^5}\cdot\frac{4^{12}}{3^{12}}}=\sqrt[15]{\frac{4^7}{3^7}}=\left(\frac{4}{3}\right)^{7/15}$

83. $\sum_{n=1}^{\infty}\ln^n\frac{2n+1}{n}$ $\quad L=\lim_{n\to\infty}\sqrt[n]{\ln^n\frac{2n+1}{n}}=\lim\ln\frac{2n+1}{n}$

$L=\lim_{n\to\infty}\ln\frac{n+n+1}{n}=\lim\ln\left(1+\frac{n+1}{n}\right)^{\frac{n}{n+1}\cdot\frac{n+1}{n}}$ шешо $n\to\infty$ не важи

$L=\lim\ln e^1=1$

84. $\sum_{n=1}^{\infty}\ln^n\frac{3n^2+2}{3n^2}$ $\quad L=\lim\sqrt[n]{\ln^n\frac{3n^2+2}{3n^2}}=\lim_{n\to\infty}\ln\left(\frac{3n^2}{3n^2}\right)$

$L=\lim_{n\to\infty}\ln\left(1+\frac{2}{3n^2}\right)^{\frac{3n^2}{2}\cdot\frac{2}{3n^2}}$

$L=\lim_{n\to\infty}\frac{2}{3n^2}=0$ РЕД КОНВЕРГИРА

85. $\sum_{n=2}^{\infty}\frac{\ln n}{n!}$ $\quad L=\lim\sqrt[n]{\frac{\ln n}{n!}}=0$ РЕД КОНВЕРГИРА

86. $\sum_{n=2}^{\infty}\frac{\ln^3 n}{n!}$ $\quad L=\lim\sqrt[n]{\frac{\ln n^3}{n!}}=0$ РЕД КОНВЕРГИРА

87. $\sum_{n=1}^{\infty}\frac{\ln n}{(2n)!}$ $\quad L=\lim\sqrt[n]{\frac{\ln(n)}{(2n)!}}=0$ РЕД КОНВЕРГИРА

88. $\sum_{n=1}^{\infty}\frac{n^3}{\ln^n(n+1)}$ $\quad L=\lim_{n\to\infty}\sqrt[n]{\frac{n^3}{\ln^3(n+1)}}=\lim_{n\to\infty}\frac{n^{3/n}\cdot 1}{\ln(n+1)}$

$\left(\frac{n}{\ln(n+1)}\right)^{3n}=\left(\frac{\ln(n+1)}{n}\right)^{-3n}=0$

РЕД КОНВЕРГИРА

$\sum_{N=1} \frac{}{(n^2+1)\cdot \ln(n+1)}$ $L = \lim_{n\to\infty} \sqrt[n]{\frac{}{(n^2+1)\ln(n+1)}}$

$L = \frac{1}{\ln(n+1)^{\frac{n^2+1}{n}}} = L = \ln(n+1)^{\frac{-n}{n^2+1}} = \ln(n+1)^0 = \ln 1 = 0$

90. $\sum_{N=2}^{\infty} \frac{3}{(5n+2)\cdot \ln^2 n}$ $L = \lim_{n\to\infty} \sqrt[n]{\frac{3}{(5n+2)\cdot \ln^2 n}} = \frac{1}{1\cdot \ln n^{2n\to\infty}} = 0$

91. $\sum_{N=2}^{\infty} \frac{\ln^2 n}{(n+1)!}$ $L = \lim_{n\to\infty} \sqrt[n]{\frac{\ln^2 n}{(n+1)!}} = 0$ РЕД КОНВЕРГИРА

92. $\sum_{N=2}^{\infty} \frac{n}{(\ln n)^n}$ $L = \lim_{n\to\infty} \sqrt[n]{\frac{n}{(\ln(n))^n}} = \frac{1}{\ln n} = 0$ C

93. $\sum_{N=1}^{\infty} \ln\left(\frac{2^n+1}{2^n}\right)$ $L = \lim_{n\to\infty} \ln^{1/n}\left(\frac{2^n+1}{2^n}\right)$

$L = \lim \ln^{1/n}\left(1+\frac{1}{2^n}\right)^{2^n\cdot\frac{1}{2^n}}$ $n\to\infty$ C

$L = \lim_{n\to\infty} \left(\ln e^{\frac{1}{2^n}}\right)^{1/n} = \lim_{n\to\infty} \left(\frac{1}{2^n}\right)^{1/n} = \frac{1}{2^{n/n}} = \frac{1}{2}$

94. $\sum_{N=1}^{\infty} \frac{\sin^2(2n)}{2^n}$ $L = \lim_{n\to\infty} \left(\frac{\sin^2(2n)}{2^n}\right)$

$L = \lim_{n\to\infty} \frac{1}{2^{n/n}} = \frac{1}{2}$ C

95. $\sum_{N=1}^{\infty} 2^n \cdot \sin^2 \frac{1}{2^n}$ $L = \lim_{n\to\infty} \sqrt[n]{2^n \sin^2 \frac{1}{2^n}}$ C

$L = \lim_{n\to\infty} 2^{n/n} \sin^{2/n} \frac{1}{2^n} = 2 \frac{1}{2^2} = 2\cdot\frac{1}{4} = \frac{1}{2}$

96. $\sum_{N=1}^{\infty} \frac{1}{3^n+\cos n}$ $L = \lim_{n\to\infty} \sqrt[n]{\frac{1}{3^n+\cos n}}$

$L = \lim_{n\to\infty} \frac{1}{3^{n/n}\sqrt[n]{1+\frac{\cos n}{3^n}}} = \frac{1}{3} = \frac{1}{3}$ C

97. $\sum_{N=1} n^2 \cdot \sin\frac{\pi}{2^n}$ $L = \lim \sqrt[n]{n^2 \sin\frac{\pi}{2^n}}$

$L = \lim_{n\to\infty} (\sqrt[n]{n})^2 \left(\frac{\pi}{2^n}\right)^{1/n \; n\to\infty}$

$L = 1\cdot 1\cdot \frac{\pi}{2^{n/n}} = 1\cdot\frac{1}{2} = \frac{1}{2}$ C

98. $\sum_{N=1}^{\infty} \left(1-\cos\left(\frac{\pi}{2^n}\right)\right)$

$L = \lim_{n\to\infty} \left(1-\cos\frac{\pi}{2^n}\right)^{1/n} = \lim_{n\to\infty} \left(\frac{\pi^2}{2^{2n}\cdot 2}\right)^{1/n}$

$L = \lim_{n\to\infty} \frac{\pi^{2/n}}{2^{\frac{2n+1}{n}}} = \lim_{n\to\infty} \frac{1}{2^2} = \frac{1}{4}$ C

99. $\sum_{N=1}^{\infty} \frac{n+2}{4^n+\sin n}$ $L = \lim_{n\to\infty} \sqrt[n]{\frac{n+2}{4^n+\sin n}}$

$L = \lim \frac{\sqrt[n]{n+2}}{4^{n/n}\sqrt[n]{1+\frac{\sin n}{4^n}}} = \frac{1}{4}$ C

100. $\sum_{N=1}^{\infty} \frac{3^n}{2^{n+1}} \cdot \sin^n \frac{1}{n}$

$L = \lim_{n\to\infty} \frac{3^{n/n}}{2^{n/n}\left(1+\left(\frac{1}{2}\right)\right)^{1/n}} \cdot \left(\sin\frac{1}{n}\right)^{n/n}$

$L = \frac{3}{2} \cdot \frac{1}{n} = 0$ C

101. $\sum_{n=1}^{n} (e^n+1)\cdot \operatorname{arctg}\frac{1}{n^2}$

$L = \lim_{n\to\infty} \sqrt[n]{(e^n+1)\cdot \operatorname{arctg}^n \frac{1}{n^2}}$

$L = e\cdot\frac{1}{n^2} = 0$ C

$\sum_{N=1} \frac{\dots}{\sqrt[4]{n^3+2n}}$ $\sum_{N=1}' \left|\frac{\dots}{\sqrt[4]{n^3+2n}}\right| \sim \sum_{N=1}' \frac{1}{n^{3/4}}$ $3/4 < 1$

РЕД УСЛОВНО КОНВЕРГИРА

$\lim_{n\to\infty} \frac{1}{n^{3/4}} = 0$

$\sqrt[4]{n^3+2n} < \sqrt[4]{(n+1)^3+2(n+1)}$

$\frac{1}{\sqrt[4]{n^3+2n}} > \frac{1}{\sqrt[4]{(n+1)^3+2n+2}}$

6. $\sum_{N=1}^{\infty} \frac{(-1)^{n-1}}{\sqrt[4]{n^4+3n}}$ $\sum_{N=1}^{\infty} \left|\frac{(-1)^{n-1}}{\sqrt[4]{n^4+3n}}\right| \sim \sum_{N=1}^{\infty} \frac{1}{n}$ $p = 1 \leq 1$

РЕД УСЛОВНО КОНВЕРГИРА

$\lim_{n\to\infty} \frac{1}{n} = 0$

$\sqrt[4]{n^4+3n} < \sqrt[4]{(n+1)^4+3(n+1)}$

$\frac{1}{\sqrt[4]{n^4-3n}} > \frac{1}{\sqrt[4]{(n+1)^4+3n+3}}$

7. $\sum_{N=1}^{\infty} (-1)^n \frac{n}{\sqrt{n^3+2n}}$ $\sum_{N=1}^{\infty} \left|(-1)^n \frac{n}{\sqrt{n^3+2n}}\right| \sim \sum_{N=1}^{\infty} \frac{1}{n^{1/2}}$

$1/2 < 1$

РЕД УСЛОВНО КОНВЕРГИРА

$\lim_{n\to\infty} \frac{1}{n^{1/2}} = 0$

$f'(x) = \frac{n}{\sqrt{n^3+2n}}' < 0$ ЗА ДОВОЛНО ВЕЛИКО $n \in N$

$$\frac{\sqrt{n^3+2n} - n \cdot \frac{1}{2} \frac{1}{\sqrt{n^3+2n}} (3n^2+2)}{n^3+2n}$$

$$= \frac{2n^3+4n-3n^3-2n}{2(n^3+2n)\sqrt{n^3+2n}} = \frac{-n^3+2n}{2(n^3+2n)^{3/2}} = \frac{-n(n^2-2)}{2\sqrt{n^3(n^2+2)^3}} < 0 \quad (T)$$

8. $\sum_{N=1}^{\infty} (-1)^n \frac{n^2}{\sqrt{n^3+1}}$ $\sum_{N=1}^{\infty} \left|(-1)^n \frac{n^2}{\sqrt{n^3+1}}\right| \sim \sum_{N=1}^{\infty} \frac{1}{n^{-1/2}}$ $-1/2 < 1$

РЕД УСЛОВНО КОНВЕРГИРА

$\lim_{n\to\infty} \frac{1}{n^{-1/2}} = 0$

$$f'(x) = \left(\frac{n^2}{\sqrt{n^3+1}}\right)' = \frac{2n\sqrt{n^3+1} + n^2 \cdot \frac{1}{2} \frac{3n^2}{\sqrt{n^3+1}}}{n^3+1} =$$

$$= \frac{2n^4+2n-3n^4}{2(n^3+1)^{3/2}} = \frac{-n^4+2n}{2(n^3+1)^{3/2}} < 0$$ ЗА ДОВОЛНО ВЕЛИКЕ $n \in N$

9. $\sum_{N=1}^{\infty} (-1)^{n-1} \frac{n-1}{\sqrt{n+2}}$ $\sum_{N=1}^{\infty} \left|(-1)^{n-1} \frac{n-1}{\sqrt{n+2}}\right| \sim \sum_{N=1}^{\infty} \frac{1}{n^{-1/2}}$

РЕД УСЛОВНО КОНВЕРГИРА

$\lim_{n\to\infty} \frac{1}{n^{-1/2}} = 0$

$f'(x) < 0$ ЗА ДОВОЉНО ВЕЛИКЕ ВРЕДНОСТИ $n \in N$

$$\left(\frac{n-1}{\sqrt{n+2}}\right)' = \frac{\sqrt{n+2} - (n-1) \cdot \frac{1}{2} \frac{1}{\sqrt{n+2}}}{n+2} =$$

$$= \frac{2n+4-n+1}{2(n+2)^{3/2}} = \frac{n+5}{2(n+2)^{3/2}}$$

102. $\sum_{N=1} (\sqrt{2}-\sqrt[3]{2})\cdot(\sqrt{2}-\sqrt[5]{2})\cdots(\sqrt{2}-\sqrt[2n+1]{2})$

$\lim_{n\to\infty} \sqrt[n]{a_n} = \lim \sqrt{2}\left(1-\frac{2^{1/3}}{2^{1/2}}\right)\cdot \sqrt{2}\left(1-\frac{2^{1/5}}{2^{1/2}}\right)\cdot \sqrt{2}\left(1-\frac{2^{1/2n+1}}{2^{1/2n+2}}\right)$

$= \lim_{n\to\infty} (\sqrt{2})^{n/n} = \sqrt{2}$ ред дивергира

103. $\sum_{N=1}^{\infty} (\sqrt{3}-\sqrt[3]{3})\cdot(\sqrt{3}-\sqrt[5]{3})\cdots(\sqrt{3}-\sqrt[2n+1]{3})$

$L = \lim_{n\to\infty} a_n^{1/n} \quad L = (\sqrt{3})^{n/n}\cdot(1-0)^{n/n} = \sqrt{3}$

ред дивергира

104. $\sum_{N=1}^{\infty} (\sqrt{5}-\sqrt[3]{5})(\sqrt{5}-\sqrt[5]{5})\cdots(\sqrt{5}-\sqrt[2n+1]{5})$

$L = \lim_{n\to 0} (a_n)^{1/n} \quad L = \lim 5^{n/2n}\cdot(1-0)^{n/n}$

$= \sqrt{5}$ ред дивергира

105. $\sum_{N=1}^{\infty} (\sqrt{7}-\sqrt[3]{7})\cdot(\sqrt{7}-\sqrt[5]{7})\cdots(\sqrt{7}-\sqrt[2n+1]{7})$

$\lim_{n\to\infty} a_n^{1/n} = \lim \sqrt{7}^{\frac{2n+1}{n}}\cdot(1-0)^{n} = \sqrt{7}$

ред дивергира

IV) ИСПИТАТИ КОНВЕРГЕНЦИЈУ РЕДА:

1. $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n^3+2}} \quad \sum_{N=1}^{\infty} \left|\frac{(-1)^n}{\sqrt{n^3+2}}\right| \sim \sum_{N=1}^{\infty} \frac{1}{n^{3/2}} \quad p=\frac{3}{2}>1$

РЕД АПСОЛУТНО КОНВЕРГИРА

2. $\sum_{N=1}^{\infty} \frac{(-1)^n}{\sqrt[3]{n^4+3}} \quad \sum_{N=1}^{\infty} \left|\frac{(-1)^n}{\sqrt[3]{n^4+3}}\right| \sim \sum_{N=1}^{\infty} \frac{1}{n^{4/3}} \quad p=4/3>1$

РЕД АПСОЛУТНО КОНВЕРГИРА

3. $\sum_{N=1}^{\infty} \frac{(-1)^n}{\sqrt[5]{n^4+2}} \quad \sum_{N=1}^{\infty} \left|\frac{(-1)^n}{\sqrt[5]{n^4+2}}\right| \sim \sum_{N=1}^{\infty} \frac{1}{n^{4/5}} \quad p=4/5<1$

ЛАЈБНИЦОВ ТЕСТ

$\therefore$ УСЛОВНО КОНВЕРГИРА

(1) $\lim_{n\to\infty} \frac{1}{n^{4/5}} = 0$

(2) $\sqrt[5]{n^4+2} < \sqrt[5]{(n+1)^4+2}$

$\frac{1}{\sqrt[5]{n^4+2}} > \frac{1}{\sqrt[5]{(n+1)^4+2}}$

4. $\sum_{N=1}^{\infty} \frac{(-1)^n}{\sqrt[3]{n^2+1}} \quad \sum_{N=1}^{\infty} \left|\frac{(-1)^n}{\sqrt[3]{n^2+1}}\right| \sim \sum_{N=1}^{\infty} \frac{1}{n^{2/3}} \quad 2/3<1$

РЕД УСЛОВНО КОНВЕРГИРА

$\lim_{n\to\infty} \frac{1}{n^{2/3}} = 0$

$\sqrt[3]{n^2+1} < \sqrt[3]{(n+1)^2+1}$

$\frac{1}{\sqrt[3]{n^2+1}} > \frac{1}{\sqrt[3]{(n+1)^2+1}}$

10. $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{n+3}{\sqrt{n^3+3n}}$ $\quad \sum_{n=1}^{\infty}\left|(-1)^{n-1}\frac{n+3}{\sqrt{n^3+3n}}\right| \sim \sum_{n=1}^{\infty}\frac{1}{n^{3/2}}$

$3/2>1$ РЕД КОНВЕРГИРА (АПСОЛУТНО)!

11. $\sum_{n=1}^{\infty}(-1)^{n}\frac{n^2}{\sqrt[3]{n^{10}-2n^5}}$ $\quad \sum_{n=1}^{\infty}\left|(-1)^{n}\frac{n^2}{\sqrt[3]{n^{10}+2n^5}}\right| \sim \sum_{n=1}^{\infty}\frac{1}{n^{4/3}}$ $\quad 4/3>1$

$\boxed{\text{РЕД АПСОЛУТНО КОНВЕРГИРА}}$

12. $\sum_{n=1}^{\infty}(-1)^{n}\frac{n+1}{\sqrt[3]{n^5+2}}$ $\quad \sum_{n=1}^{\infty}\left|(-1)^{n}\frac{n+1}{\sqrt[3]{n^5+2}}\right| \sim \sum_{n=1}^{\infty}\frac{1}{n^{2/3}}$ $\quad 2/3<1$

ЛАЈБНИЦОВ ТЕСТ

1 $\lim_{n\to\infty}\frac{1}{n^{2/3}}=0$

2 $f'(x)<0$ ЗА ДОВОЛНО ГОЛЕМИ ВРЕДНОСТИ x

$\left(\frac{x+1}{\sqrt[3]{x^5+2}}\right)' = \frac{\sqrt[3]{x^5+2}-(x+1)\cdot\frac{1}{3}\frac{1}{\sqrt[3]{(x^5+2)^2}}\cdot 5x^4}{\sqrt[3]{(x^5+2)^2}}$

$\frac{3x^5+6-x-1}{3\sqrt[3]{(x^5+2)^4}}<0$ $\quad 3x^5-x+5$ $\quad 3\sqrt[3]{(x^5+2)^4}>0$

$x\frac{2}{3}-\frac{5}{2}<0$ $\quad \frac{4-15}{6}$

$\boxed{\text{РЕД УСЛОВНО КОНВЕРГИРА}}$

13. $\sum_{n=1}^{\infty}(-1)^{n}\frac{\sqrt{n^5}}{\sqrt[3]{n^2+2}}$ $\quad \sum_{n=1}^{\infty}\left|(-1)^{n}\frac{\sqrt{n^5}}{\sqrt[3]{n^2+2}}\right| \sim \sum_{n=1}^{\infty}\frac{1}{n^{-11/6}}$

$-11/6<1$ ЛАЈБНИЦОВ ТЕСТ

1 $\lim_{n\to\infty}\frac{1}{n^{-11/6}}=0$

2 $f'(x)<0$

$\frac{\frac{5}{2}n^{3/2}\sqrt[3]{n^2+2}-\frac{1}{3}\frac{2n}{\sqrt[3]{(n^2+2)^2}}\sqrt{n^5}}{\sqrt[3]{(n^2+2)^2}}$ $\quad \frac{\frac{15}{2}n^{3/2}(n^2+2)^{4/3}-2n\cdot n^{5/2}}{3\sqrt[3]{(n^2+2)^4}}$ >0

$\frac{15}{2}n^{3/2}(n^2+2)^{2/3}-2n^{7/2}<0$

$\frac{15}{2}\sqrt{n^3}\cdot\sqrt[3]{n^4+4n^2+4}-2\sqrt{n^7}<0$

$\frac{15}{2}\sqrt[3]{n^{\frac{9}{2}+4}+4n^{2+\frac{9}{2}}+4n^{\frac{9}{2}}}-2\sqrt{n^7}<0$

14. $\sum_{n=1}^{\infty}(-1)^{n-1}\frac{\sqrt{n^4+1}}{\sqrt[3]{n^4}}$ $\quad \sum_{n=1}^{\infty}\left|(-1)^{n}\frac{\sqrt{n^4+1}}{\sqrt[3]{n^4}}\right| \sim \sum_{n=1}^{\infty}\frac{1}{n^{4/3}}$

$\sim\sum_{n=1}^{\infty}\frac{1}{n^{-2/3}}$ $\quad -2/3<1$ ЛАЈБНИЦОВ ТЕСТ

1 $\lim_{n\to\infty}\frac{1}{n^{-2/3}}=0$

2 $\frac{\sqrt{n^4+1}}{\sqrt[3]{n^4}}-\frac{\sqrt{(n+1)^4+1}}{\sqrt[3]{(n+1)^4}}>0$

$\frac{\sqrt{n^4+1}\cdot\sqrt[3]{(n+1)^4}-\sqrt{(n+1)^4+1}\cdot\sqrt[3]{n^4}}{\sqrt[3]{n^4(n+1)^4}}>0$ $\quad \sqrt[3]{n^4(n+1)^4}>0$

$f'(x)<0$ $\quad \frac{1}{2}\frac{4x^3}{\sqrt{x^4+1}}\cdot x^{4/3}-\sqrt{x^4+1}\cdot\frac{4}{3}x^{1/3}$ $\quad x^{8/3}$

$\frac{12x^3-8x^{13/3}\ 8x^{1/3}}{6x^{8/3}\sqrt{x^4+1}}<0$

$12x^3-8x^{1/3}(x^4+1)<0 \;/:4$

$3x^3-2\sqrt[3]{x}(x^4+1)<0$ $\quad \sqrt[3]{x}=a$ $\quad \frac{4}{3}$

$3a^9-2a(a^{12}+1)<0$

$3a^9-2a^{13}-2a<0 \;/:a$

$-2a^{12}+3a^8-2<0$ $\quad a$

$$\sum_{n=1}(-1)\frac{\cdots}{\sqrt[4]{n^5+3}} \quad \sum_{n=1}\left|(-1)^n\frac{\cdots}{\sqrt[4]{n^5+3}}\right| \sim \sum_{n=1}\frac{1}{n^{3/4}}$$

3/4 < 1 ЛАЈБНИЦОВ ТЕСТ

1 $\lim_{n\to\infty}\frac{1}{n^{3/4}}=0$

2 $f'(x)<0 \quad \dfrac{\frac{1}{2\sqrt{x+2}}-\frac{5n^4\cdot\sqrt{n+2}}{4\sqrt[4]{(n^5+3)^3}}}{\sqrt[4]{(n^5+3)^2}}<0$

$$\sqrt[4]{(n^5+3)^5} \qquad \frac{\sqrt{n+2}}{\sqrt[4]{n^5+3}} > \frac{\sqrt{n+3}}{\sqrt[4]{(n+1)^5+3}}$$

$$\frac{\sqrt{n+2}}{\sqrt[4]{n^5+3}}-\frac{\sqrt{n+3}}{\sqrt[4]{(n+1)^5+3}}>0$$

$$\frac{\sqrt[4]{((n+1)^5+3)\cdot(n+2)^2}-\sqrt[4]{(n+3)^2\cdot(n^5+3)}}{\sqrt[4]{(n^5+3)\cdot((n+1)^5+3)}}>0$$

>0

$$\sqrt[4]{n^5+3}<\sqrt[4]{(n+1)^5+3}$$

$$\frac{1}{\sqrt[4]{n^5+3}}>\frac{1}{\sqrt[4]{(n+1)^5+3}}$$

$$\sqrt{n+2}<\sqrt{n+3}$$

16. $\sum_{n=1}(-1)^n\frac{\sqrt{n+1}}{\sqrt{n+2}} \quad \sum_{n=1}\frac{1}{n^0}$ 0<1 МОДУЛАРНИ РЕД ДИВЕРГИРА

ЛАЈБНИЦОВ ТЕСТ

1 $\lim_{n\to\infty}\frac{1}{n^0}=1$

17. $\sum_{n=1}^{\infty}\frac{(-1)^{n-1}n}{\sqrt{n}-\sqrt{n-1}} \sim \sum_{n=1}^{\infty}\frac{(-1)^{n-1}n(\sqrt{n}+\sqrt{n-1})}{n-n+1} \sim$

$$\sim\sum_{n=1}^{\infty}(-1)^{n-1}n(\sqrt{n}+\sqrt{n-1})$$

МОДУЛАРНИ РЕД $\sum_{n=1}^{\infty}n(\sqrt{n}+\sqrt{n-1})$

$$\lim_{n\to\infty}n(\sqrt{n}+\sqrt{n-1})=\lim_{n\to\infty}2n^{3/2}=+\infty$$

18. $\sum_{n=1}^{\infty}\frac{(-1)^n}{n^2(\sqrt{n}-\sqrt{n-1})}\cdot\frac{\sqrt{n}+\sqrt{n-1}}{\sqrt{n}+\sqrt{n-1}} \sim$

$$\sim\sum_{n=1}^{\infty}\frac{(-1)^n\cdot(\sqrt{n}+\sqrt{n-1})}{n^2(n-n+1)} \sim \sum_{n=1}^{\infty}\frac{(-1)^n(\sqrt{n}+\sqrt{n-1})}{n^2}$$

МОДУЛАРНИ РЕД $\sum_{n=1}^{\infty}\frac{1}{n^{3/2}}$ 3/2 > 1 МОДУЛАРНИ РЕД КОНВЕРГИРА

$\boxed{\text{РЕД АПСОЛУТНО КОНВЕРГИРА}}$

19. $\sum_{n=1}^{\infty}\frac{(-1)^n(n+3)}{\sqrt{n^2+n}-n}\cdot\frac{\sqrt{n^2+n}+n}{\sqrt{n^2+n}+n} \sim$

$$\sim\sum_{n=1}^{\infty}(-1)^n\frac{(n+3)\cdot(\sqrt{n^2+n}+n)}{\cancel{n^2}+n-\cancel{n^2}} \sim \sum_{n=1}^{\infty}(-1)^n\frac{(n+3)(\sqrt{n^2+n}+n)}{n}$$

МОДУЛАРНИ РЕД $\sum_{n=1}^{\infty}\frac{1}{n^{-1}}$

20. $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n(\sqrt{n^2+n}-n)} \cdot \frac{\sqrt{n^2+n}+n}{\sqrt{n^2+n}+n} \sim$

$\sum_{n=1}^{\infty} \frac{(-1)^{n-1}\cdot(n+\sqrt{n^2+n})}{n(n^2+n-n^2)} \sim \sum_{n=1}^{\infty} \frac{(-1)^{n-1}(n+\sqrt{n^2+n})}{n^2}$

модуларни ред $\quad 1\sum \frac{1}{n} \qquad 1=1$ модуларни ред дивергира

лајбницов тест

1 $\lim_{n\to\infty} \frac{1}{n} = 0$

за доволно големи произволни вредности

2 $f'(x)<0$ променливе x

$\left(\frac{x+\sqrt{x^2+x}}{x^2}\right)' = \frac{\left(1+\frac{2x+1}{2\sqrt{x^2+x}}\right)x^2 - 2x\left(x+\sqrt{x^2+x}\right)}{x^4} =$

$= \frac{2\sqrt{x^2+x}\,x^2+2x^3-4x^2-4x\sqrt{x^2+x}}{2\sqrt{x^2+x}\cdot x^4} < 0$

>0

$2x^3-4x^2+2x^2\sqrt{x^2+x}-4x\sqrt{x^2+x}<0$

$2x^3-4x^2<4x\sqrt{x^2+x}-2x^2\sqrt{x^2+x} \quad /:2$

$x^3-2x^2<2x\sqrt{x^2+x}-x^2\sqrt{x^2+x} \quad /()^2$

$\cancel{x^6}-4x^5+\cancel{4x^4}<\cancel{4x^4}+4x^3-4x^3(x^2+x)+\cancel{x^6}+x^5$

$-\cancel{4x^5}<4x^3-\cancel{4x^5}-4x^4+x^5$

$-x^5+4x^4-4x^3<0 \quad /:(-1)$

$x^5-4x^4+4x^3>0$

$x^3(x^2-4x+4)>0$

$x^3(x-2)^2>0$

		0		2	
x	$-$		$+$		$+$
$x-2$	$-$		$-$		$+$
	$+$		$-$		$+$

$x\in(-\infty,0)\cup(2,+\infty)$

$\boxed{\text{Ред условно конвергира}}$

20. $\sum_{n=1} \frac{(-1)^{n}}{n(\sqrt{n^2+n}-n)} \cdot \frac{\sqrt{n^2+n}+n}{\sqrt{n^2+n}+n}$

21. $\sum_{n=1}^{\infty} (-1)^n \frac{\sqrt{3n}-\sqrt{3n-2}}{n^2+n} \cdot \frac{\sqrt{3n}+\sqrt{3n-2}}{\sqrt{3n}+\sqrt{3n-2}} \sim$

$\sim \sum_{n=1}^{\infty} (-1)^n \frac{\cancel{3n}-\cancel{3n}+2}{(n^2+n)(\sqrt{3n}+\sqrt{3n-2})}$

модуларни ред конвергира $\quad \frac{2}{2\sqrt{3}}\sum_{n=1}^{\infty}\frac{1}{n^{5/2}} \sim \frac{\sqrt{3}}{3}\sum_{n=1}^{\infty}\frac{1}{n^{5/2}}$

$5/2>1$ ред апсолутно конвергира

22. $\sum_{n=1}^{\infty} (-1)^n \frac{\sqrt{2n+1}-\sqrt{2n}}{n} \cdot \frac{\sqrt{2n+1}+\sqrt{2n}}{\sqrt{2n+1}+\sqrt{2n}} \sim$

$\sim \sum_{n=1}^{\infty} (-1)^n \frac{\cancel{2n}+1-\cancel{2n}}{n(\sqrt{2n+1}+\sqrt{2n})}$ модуларни ред конвергира $\frac{\sqrt{2}}{4}\sum\frac{1}{n^{3/2}}$

ред апсолутно конвергира

23. $\sum_{n=1}^{\infty} (-1)^n \frac{n^2+3}{2^n+1}$ модуларни ред дивергира $\sum_{n=1}^{\infty}\frac{n^2+3}{2^n+1} \sim \sum_{n=1}^{\infty}\frac{n^2}{2^n}$

$\sim \sum_{n=1}\frac{1}{n^{-2}} \qquad \frac{(n+1)^2}{2^{n+1}} \cdot \frac{2^n}{n^2}$

лајбницов тест 1 $\lim_{n\to\infty}\frac{n^2}{2^n}=0$ ред условно конвергира

2 $\frac{n^2}{2^n} > \frac{(n+1)^2}{2^{n+1}}$

$\frac{n^2}{2^n} - \frac{n^2+2n+1}{2^{n+1}} > 0$

$\frac{2n^2-n^2-2n-1}{2^n\cdot 2} > 0 \quad /\cdot 2$

$\frac{n^2-2n-1}{2^n} > 0$

$\frac{(n-1)^2}{2^n} > 0 \qquad 2^n>0$

$(n-1)^2>0$ T

$\sum_{N=1}^{\infty} (-1) \frac{}{3^n+1}$ дивергира $\sum \frac{}{3^n-1}$

$\sim \sum_{N=1}^{\infty} \frac{2^n n^2}{3^n(1+(\frac{1}{3})^n)} \sim \sum_{N=1}^{\infty} \left(\frac{2}{3}\right)^n n^2 \sim \sum_{N=1}^{\infty} \frac{1}{n^{-2}}$

$-2<0$

Лајбницов тест

$1. \lim_{n\to\infty} \frac{n^2 2^n}{3^n+1} = 0$

$\frac{(n+1)^2 \cdot \cancel{2^n} \cdot 2}{3^n \cdot 3+1} \cdot \frac{3^n+1}{n^2 \cdot \cancel{2^n}} = \left(\frac{n+1}{n}\right)^2 \cdot \frac{\cancel{3^n}\left(\left(\frac{1}{3}\right)^n+1\right)}{\cancel{3^n}\left(3+\left(\frac{1}{3}\right)^n\right)} =$

$= 1^2 \cdot \frac{1}{3} = 1 \cdot \frac{1}{3} = \frac{1}{3}$

25. $\sum_{N=1}^{\infty} (-1)^{n-1} \frac{3^n}{4^n+n^2}$

модуларни ред $\sum_{n=1}^{\infty} \frac{3^n}{4^n+n^2} \sim \sum_{N=1}^{\infty} \frac{3^n}{\;} \sim$

$\sim \sum_{N=1}^{\infty} \frac{3^n}{4^n} \sim \sum_{N=1}^{\infty} 0$

26. $\sum_{N=1}^{\infty} (-1)^n \frac{n^2+2008n}{2^n}$

модуларни ред $\sum_{N=1}^{\infty} \frac{n^2\left(1+\frac{2008}{n}\right)}{2^n} \sim \sum_{N=1}^{\infty} \frac{n^2}{2^n} = 0$

27. $\frac{n^2+2008n}{2^n} > \frac{(n+1)^2+2008(n+1)}{2^{n+1}}$

$\frac{2n^2+2008n \cdot 2 - (n+1)^2 - 2008n - 2008}{\underbrace{2^{n+1}}_{>0}} > 0$

$n^2 + 2006n - 2009 > 0$

27. $\sum_{N=1}^{\infty} (-1)^n \frac{2^n+10^5}{n!}$ модуларни ред $\sum_{N=1}^{\infty} \frac{2^n+10^5}{n!}$

$\frac{2^n\left(1+\frac{10^5}{2^n}\right)}{n!} = \frac{2^n}{n!} = 0$ дивергира

$\lim_{n\to\infty} \frac{2^n+10^5}{n!} = 0$ ред условно конвергира

$\frac{2^n+10^5}{n!} > \frac{2^{n+1}+10^5}{(n+1)!}$

$\frac{2^n+10^5}{n!} - \frac{2^n \cdot 2+10^5}{(n+1)n!} > 0$

$\frac{2^n n + 2^n + 10^5 n + \cancel{10^5} - 2^n \cdot 2 - \cancel{10^5}}{\underbrace{\underbrace{(n+1)}_{>0} \cdot \underbrace{n!}_{>0}}_{>0}} > 0$

$2^n n + 10^5 n - 2^n > 0$

$\underbrace{2^n}_{>0}(n-1) + \underbrace{10^5 n}_{>0} > 0$

28. $\sum_{N=1}^{\infty} (-1)^{n-1} \frac{n!}{2^n+3^n+4^n}$ модуларни ред $\sum_{N=1}^{\infty} \frac{n!}{2^n+3^n+4^n}$

$\lim_{n\to\infty} \frac{n!}{2^n+3^n+4^n} = \lim_{n\to\infty} \frac{n!}{4^n\left(\left(\frac{2}{4}\right)^n+\left(\frac{3}{4}\right)^n+1\right)} =$

$= \lim_{n\to\infty} \frac{n!}{4^n} = +\infty$ конвергира (ред апсолутно конвергира)

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$$29.\ \sum_{n=1} (-1)^n \frac{3^n+n}{n!}$$

МОДУЛАРНИ РЕД $\sum_{n=1}^{\infty} \frac{3^n+n^{2008}}{n!}$

$$\lim_{n\to\infty} \frac{3^n+n^{2008}}{n!} = \lim_{n\to\infty} \frac{n^{2008}\left(\frac{3^n}{n^{2008}}+1\right)}{n!} = 0$$

$$\frac{3^n+n^{2008}}{n!} > \frac{3^n \cdot 3+(n+1)^{2008}}{(n+1)!}$$

$$\frac{3^n \cdot n+3^n \cdot n^{2008} \cdot n^{2008} - 3^n \cdot 3 - n^{2008} \ldots + 1}{\underbrace{(n+1)}_{>0}\underbrace{n!}_{>0}} > 0$$

$$30.\ \sum_{n=1}^{\infty} (-1)^n \frac{n!+2^n}{(2n)!}$$

МОДУЛАРНИ РЕД $\sum_{n=1}^{\infty} \frac{n!+2^n}{(2n)!} \sim \sum_{n=1}^{\infty} \frac{n!\left(1+\frac{2^n}{n!}\right)}{2^n \cdot n!} \sim$

$$\sim \sum_{n=1}^{\infty} \frac{n!}{2^n \cdot n!} \sim \sum_{n=1}^{\infty} \frac{1}{2^n}$$ дивергира

$$\lim_{n\to\infty} \frac{1}{2^n} = 0$$

$$\frac{1}{2^n} > \frac{1}{2^{n+1}} \qquad 2^n < 2^{n+1}$$

$$2^n < 2^n \cdot 2 \ /:2^n$$

$$\frac{1}{2^n} > \frac{1}{2^{n+1}} \qquad 1 < 2 \quad T$$

ред условно конвергира

V) ЗА КОЈУ ВРЕДНОСТ ПАРАМЕТРА p ДАТИ РЕД
а) АПСОЛУТНО КОНВЕРГИРА
б) КОНВЕРГИРА ?

$$1.\ \sum_{n=1}^{\infty} \frac{(-1)^n}{(n^p+1)^2}$$ МОДУЛАРНИ РЕД $\sum_{n=1}^{\infty} \frac{1}{n^{2p}}$

апсолутно конвергира $2p>1 /:2$ $\quad p>1/2 \quad p\in\left(\frac{1}{2},+\infty\right)$

конвергира $p\in(-\infty,2]$ $\quad 2p\le 1/:2 \quad p\le 1/2$

$$2.\ \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{(n^{3p}+4)^4}$$ МОДУЛАРНИ РЕД $\sum_{n=1}^{\infty} \frac{1}{n^{12p}}$

АПСОЛУТНО КОНВЕРГИРА $12p>1 /:12$
$p\in\left(\frac{1}{12},+\infty\right)$ $\quad p>1/12$

УСЛОВНО КОНВЕРГИРА
$p\in(-\infty, 1/12]$

$$3.\ \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{\sqrt[3]{n^p+1}}$$ МОДУЛАРНИ РЕД $\sum_{n=1}^{\infty} \frac{1}{n^{p/3}}$

АПСОЛУТНО КОНВЕРГИРА $p/3>1 /\cdot 3$
$p\in(3,+\infty)$ $\quad p>3$

РЕЛАТИВНО КОНВЕРГИРА
$p\in(-\infty,3]$

$$4.\ \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{\sqrt[3]{n^{2p}+3}}$$ МОДУЛАРНИ РЕД $\sum_{n=1}^{\infty} \frac{1}{n^{2p/3}}$

АПСОЛУТНО КОНВЕРГИРА $2p/3>1 /\cdot 3/2$
$p\in(3/2,+\infty)$ $\quad p>3/2$

РЕЛАТИВНО КОНВЕРГИРА
$p\in(-\infty,3/2]$

$$5.\ \sum_{n=1}^{\infty} \frac{(-1)^n}{\left(\sqrt[4]{n^3+2}\right)^p}$$ МОДУЛАРНИ РЕД $\sum_{n=1}^{\infty} \frac{1}{n^{3p/4}}$

АПСОЛУТНО КОНВЕРГИРА $3p/4>1 /\cdot 4/3$
$p\in(4/3,+\infty)$ $\quad p>4/3$

РЕЛАТИВНО КОНВЕРГИРА
$p\in(-\infty,4/3]$

6. $\sum_{n=1} \frac{(-1)^{\dots}}{(\sqrt[3]{n^5+4})^p}$ модуларни ред $\sum_{n=1} \frac{1}{n^{5p/3}}$

Апсолутно конвергира $5p/3>1 \,/\cdot 3/5$, $p>3/5$

$p\in(3/5,+\infty)$

Релативно конвергира

$p\in(-\infty,3/5]$

7. $\sum_{n=1}^{\infty} (-1)^{n-1}\frac{n^p}{n^3+n}$ модуларни ред $\sum_{n=1}^{\infty} \frac{1}{n^{3-p}}$

Апсолутно конвергира $3-p>1$, $-p>1-3$, $-p>-2$, $p<2$

$p\in(-\infty,2)$

Релативно конвергира

$p\in[2,+\infty)$

8. $\sum_{n=1}^{\infty} (-1)^n \frac{(2n^3+n)^p}{n^2-n}$ модуларни ред $2^p\sum_{n=1}^{\infty} \frac{1}{n^{2-3p}}$

Апсолутно конвергира $2-3p>1$, $-3p>1-2$, $-3p>-1 \,/:(-3)$, $p<1/3$

$p\in(-\infty,1/3)$

Релативно конвергира

$p\in[1/3,+\infty)$

9. $\sum_{n=1}^{\infty} (-1)^n \frac{(n^2+3n)^p}{n^3+1}$ модуларни ред $\sum_{n=1}^{\infty} \frac{1}{n^{3-2p}}$

Апсолутно конвергира $3-2p>1$, $-2p>1-3$, $-2p>-2 \,/:(-2)$, $p<1$

$p\in(-\infty,1)$

Релативно конвергира

$p\in[1,+\infty)$

10. $\sum_{n=1}^{\infty} (-1)^n \frac{(n+1)^p}{\sqrt{n^3+1}}$ модуларни ред $\sum_{n=1}^{\infty} \frac{1}{n^{\frac{3}{2}-p}}$

Апсолутно конвергира $\frac{3}{2}-p>1$, $-p>1-\frac{3}{2}$, $-p>-\frac{1}{2}$, $p<1/2$

$p\in(-\infty,1/2)$

Релативно конвергира

$p\in[1/2,+\infty)$

11. $\sum_{n=1}^{\infty} \frac{(-1)^{n-1} n^p}{\sqrt{n^2+1}+n}$ модуларни ред $\frac{1}{2}\sum_{n=1}^{\infty} \frac{1}{n^{1-p}}$

$1-p>1$, $-p>1-1$, $-p>0$, $p<0$

Апсолутно конвергира $p\in(-\infty,0)$

Релативно конвергира $p\in[0,+\infty)$

12. $\sum_{n=1} (-1)^{n-1}\frac{\sqrt{n+1}}{(n+5)^p}$ модуларни ред $\sum_{n=1} \frac{1}{n^{p-3/2}}$

$p-\frac{3}{2}>1$, $p>1+\frac{3}{2}$, $p>\frac{5}{2}$

АС $p\in(2{,}5;+\infty)$

УС $p\in(-\infty,2{,}5]$

13. $\sum_{n=1}^{\infty} (-1)^n \frac{(\sqrt{n+1})^p}{n^3-3n-5}$ модуларни ред $\sum_{n=1}^{\infty} \frac{1}{n^{3-p/2}}$

$3-p/2>1$, $-p/2>1-3$, $-p/2>-2 \,/\cdot(-2)$, $p<4$

Апсолутно конвергира $p\in(-\infty,4)$

Конвергира $p\in[4,+\infty)$

14. $\sum_{n=1}^{\infty} (-1)^n \frac{(n+1)^2}{(\sqrt{n^2+1}+1)^p}$ модуларни ред $\sum_{n=1}^{\infty} \frac{1}{n^{p-2}}$

$p-2>1$, $p>1+2$, $p>3$

Апсолутно конвергира $p\in(3,+\infty)$

Конвергира $p\in(-\infty,3]$

15. $\sum_{n=1}^{\infty} (-1)^n \frac{(\sqrt{n}+2)^p}{n^3+n-1}$ модуларни ред $\sum_{n=1}^{\infty} \frac{1}{n^{3-p/2}}$

$3-p/2>1$, $-p/2>1-3$, $-p/2>-2 \,/\cdot(-2)$, $p<4$

Апсолутно конвергира $p\in(-\infty,4)$

Конвергира $p\in[4,+\infty)$

16. $\sum_{n=1}^{\infty} (-1)^{n-1}\frac{\sqrt[3]{n+1}}{(\sqrt{n^2+1})^p}$ модуларни ред $\sum_{n=1}^{\infty} \frac{1}{n^{p-2/3}}$

$p-2/3>1$, $p>1+2/3$, $p>5/3$

Апсолутно конвергира $p\in(5/3,+\infty)$

Конвергира $p\in(-\infty,5/3]$

17. $\sum_{n=1}^{\infty} (-1)^{n-1}\frac{(\sqrt{n}+2)^3}{\sqrt[3]{2n^{p}+1}}$ модуларни ред $\frac{1}{\sqrt[3]{2}}\sum_{n=1}^{\infty} \frac{1}{n^{p/3-3/2}}$

$\frac{p}{3}-\frac{3}{2}>1 \,/\cdot 6$, $2p-9>6$, $2p>6+9$, $2p>15 \,/:2$, $p>15/2$

Апсолутно конвергира $p\in(15/2,+\infty)$

Релативно конвергира $p\in(-\infty,15/2]$

18. $\sum_{n=1}^{\infty}(-1)^n \frac{n^p\sqrt{n}}{\sqrt{n+11}-\sqrt{n+1}}\cdot\frac{\sqrt{n+11}+\sqrt{n+1}}{\sqrt{n+11}+\sqrt{n+1}} \sim$

$\sim \sum_{n=1}^{\infty}(-1)^n \frac{n^{p+1}\cdot 2}{\cancel{n}+11-\cancel{n}-1}$

модулари ред $\frac{2}{11}\sum_{n=1}^{\infty} \frac{1}{n^{-p-1}}$ $\quad -p-1>1$, $-p>1+1$, $-p>2 \ /\cdot(-1)$ $\quad p<-2$

апсолутно конвергира $p\in(-\infty,-2)$ релативно конвергира $p\in[-2,+\infty)$

19. $\sum_{n=1}^{\infty} \frac{(-1)^n\cdot n^p}{\sqrt{n^2+n}-\sqrt{n^2+1}}\cdot\frac{\sqrt{n^2+n}+\sqrt{n^2+1}}{\sqrt{n^2+n}+\sqrt{n^2+1}}\sim$

$\sim\sum_{n=1}^{\infty} \frac{(-1)^n n^{p+1}\cdot 2}{\cancel{n^2}+n-\cancel{n^2}-1}$ модулари ред $2\sum_{n=1}^{\infty}\frac{1}{n^{-p}}$

$-p>1$ апсолутно конвергира $p\in(-\infty,-1)$

$p<-1$ конвергира $p\in[-1,+\infty)$

20. $\sum_{n=1}^{\infty}(-1)^n \frac{n^p}{(\sqrt{n^2+1}-n)^3}\cdot\frac{(\sqrt{n^2+1}+n)^3}{(\sqrt{n^2+1}+n)^3}\sim$

$\sim\sum_{n=1}^{\infty}(-1)^n\frac{n^{p+3}\cdot 8}{(n^2+1-n^2)^3}$

модулари ред $8\sum_{n=1}^{\infty}\frac{1}{n^{-p-3}}$ $\quad -p-3>1$, $-p>1+3$, $-p>4$, $p<-4$

апсолутно конвергира $p\in(-\infty,-4)$ релативно конвергира $p\in[-4,+\infty)$

21. $\sum_{n=1}^{\infty}(-1)^n\frac{\sqrt{n-2}}{(\sqrt{n^2+1}-n)^p}\cdot\frac{(\sqrt{n^2+1}+n)^p}{(\sqrt{n^2+1}+n)^p}\sim$

$\sim\sum_{n=1}^{\infty}(-1)^n\frac{\sqrt{n-2}\cdot n^p\cdot 2^p}{(n^2+1-n^2)^p}$ апсолутно конвергира $p\in(-\infty,-\frac{3}{2})$

модулари ред $2^p\sum_{n=1}^{\infty}\frac{1}{n^{-p-1/2}}$ $\quad -p-1/2>1$, $p<-3/2$, $-p>1+1/2$, конвергира, $-p>3/2$ $\quad p\in[-\frac32,+\infty)$

22. $\sum_{n=1}^{\infty}(-1)^n\frac{\sqrt{n^2+3n}-n}{n^p}\cdot\frac{\sqrt{n^2+3n}+n}{\sqrt{n^2+3n}+n}\sim$

$\sim\sum_{n=1}^{\infty}(-1)^n\frac{n^2+3n-n^2}{n^{p+1}\cdot 2}$ модулари ред $\frac32\sum_{n=1}^{\infty}\frac{1}{n^p}$

за $p>1$ $p\in(1,+\infty)$ апсолутно конвергира

за $p\le1$ $p\in(-\infty,1]$ конвергира

23. $\sum_{n=1}^{\infty}(-1)^n(\sqrt{n+3}-\sqrt{n})^p\cdot\frac{(\sqrt{n+3}+\sqrt n)^p}{(\sqrt{n+3}+\sqrt n)^p}\sim$

$\sim\sum_{n=1}^{\infty}(-1)^n\frac{(n+3-n)^p}{(\sqrt{n+3}+\sqrt n)^p}$ модулари ред $(\frac32)^p\sum_{n=1}^{\infty}\frac{1}{n^{p/2}}$

апсолутно конвергира $p\in(2,+\infty)$ $\quad p/2>1/\cdot2$, $p>2$ $\quad$ конвергира $p\in(-\infty,2]$

24. $\sum_{n=1}^{\infty}(-1)^{n-1}\frac{(\sqrt{n^2+3}-n)^p}{\sqrt[3]{n^2-1}}\cdot\left(\frac{\sqrt{n^2+3}+n}{\sqrt{n^2+3}+n}\right)^p\sim$

$\sim\sum_{n=1}^{\infty}(-1)^{n-1}\frac{(\cancel{n^2}+3-\cancel{n^2})^p}{\sqrt[3]{n^2-1}\,(\sqrt{n^2+3}+n)^p}$

модулари ред $(\frac32)^p\sum_{n=1}^{\infty}\frac{1}{n^{p+2/3}}$ $\quad p+\frac23>1$, $p>1-\frac23$, $p>\frac13$

апсолутно конвергира $p\in(\frac13,+\infty)$ $\quad$ конвергира $p\in(-\infty,\frac13]$

25. $\sum_{n=1}^{\infty}\frac{(-1)^n\cdot n^p}{\sqrt[3]{n^2+1}-\sqrt[3]{n^2+n}}$ модулари ред

$\sum_{n=1}^{\infty}\frac{n^p(n^{4/3}(1+1+1))}{\cancel{n^2}+1-\cancel{n^2}-n}\sim 3\sum_{n=1}^{\infty}\frac{1}{n^{-p-1/3}}$

$-p-1/3>1$, $-p>1+1/3$, $-p>4/3\ /\cdot(-1)$, $p<-4/3$

апсолутно конвергира $p\in(-\infty,-\frac43)$

конвергира $p\in[-\frac43,+\infty)$

$\sum_{N=1} (-1)^{\ldots} \frac{\ldots}{(\sqrt{n+1}-\sqrt{n})^p} \cdot \frac{\ldots}{\sqrt{n^2+2}\cdot n} \cdot \frac{\ldots}{1\cdot(\sqrt{n+1}+\sqrt{n})^p}$

модуларни ред $\sum_{N=1}^{\infty} \frac{(n^2+2-n^2)\cdot n^{p/2}\cdot 2}{(n+1-n)^p \cdot n \cdot 1} \sim 4\sum_{N=1}^{\infty}\frac{1}{n^{1-p/2}}$

$1-p/2>1$
$-p/2>1\ 1$
$-p/2>2 /\cdot(-2)$
$p<-4$

апсолутно конвергира $p\in(-\infty,-4)$
конвергира $p\in[-4,+\infty)$

27. $\sum_{N=1}^{\infty}(-1)^n \frac{\sqrt{n+2}-\sqrt{n+1}}{(\sqrt{n^2+1}-n)^p}\cdot\frac{\sqrt{n+2}+\sqrt{n+1}}{\sqrt{n+2}+\sqrt{n+1}}\cdot\left(\frac{\sqrt{n^2+1}+n}{\sqrt{n^2+1}+n}\right)^p$

модуларни ред $\sum_{N=1}^{\infty}\frac{(n+2-n-1)\cdot n^p\cdot 2^p}{(n^2+1-n^2)^p\cdot n^{1/2}\cdot 2} \sim 2^{p-1}\sum_{N=1}^{\infty}\frac{1}{n^{1/2-p}}$

$1/2-p>1$
$-p>1-1/2$
$-p>1/2 /\cdot(-1)$
$p<-1/2$

апсолутно конвергира $p\in(-\infty,-\frac{1}{2})$ конвергира $p\in[-\frac{1}{2},+\infty)$

28. $\sum_{N=1}^{\infty}(-1)^n\cdot\left(1-\cos\frac{1}{n}\right)^p$

модуларни ред $\sum_{N=1}^{\infty}\left(\frac{1^2}{2\cdot n^2}\right)^p \sim \frac{1}{2^p}\sum_{N=1}^{\infty}\frac{1}{n^{2p}}$

$2p>1 /:2$
$p>1/2$

апсолутно конвергира $p\in(1/2,+\infty)$

конвергира $p\in(-\infty,1/2]$

29. $\sum_{N=1}^{\infty}(-1)^n\cdot n^p\cdot\sin\frac{1}{\sqrt{n}}$

модуларни ред $\sum_{N=1}^{\infty} n^p\cdot\frac{1}{n^{1/2}} \sim \sum_{N=1}^{\infty}\frac{1}{n^{1/2-p}}$

$1/2-p>1$
$-p>1+1/2$
$-p>+1/2$

апсолутно конвергира $p\in(-\infty,-1/2)$ конвергира $p\in[-1/2,+\infty)$

30. $\sum_{N=1}(-1)^{n-1}\cdot\left(tg\frac{2}{\sqrt[3]{n^2}}\right)^p$

модуларни ред $\sum_{N=1}^{\infty}\frac{2^p}{n^{2p/3}} \sim 2^p\sum_{N=1}^{\infty}\frac{1}{n^{2p/3}}$

$\frac{2p}{3}>1 /\cdot 3/2$
$p>3/2$

апсолутно конвергира $p\in(\frac{3}{2},+\infty)$ конвергира $p\in(-\infty,\frac{3}{2}]$

31. $\sum_{N=1}^{\infty}(-1)^n\cdot n^p\cdot tg\frac{1}{(n+2)^2}$

модуларни ред $\sum_{N=1}^{\infty} n^p\frac{1}{(n+2)^2} \sim \sum_{N=1}^{\infty}\frac{1}{n^{2-p}}$

$2p>1$
$-p>1-2$
$-p>-1 /:(-1)$
$p<1$

апсолутно конвергира $p\in(-\infty,1)$ конвергира $p\in[1,+\infty)$

32. $\sum_{N=1}^{\infty}(-1)^n \ln^p\left(1+\frac{1}{n^2}\right)$

модуларни ред $\sum_{N=1}^{\infty}\frac{1}{n^{2p}}$

$2p>1 /:2$
$p>1/2$

апсолутно конвергира $p\in(1/2,+\infty)$
конвергира $p\in(-\infty,1/2]$

33. $\sum_{N=1}^{\infty}(-1)^n\cdot n^p\cdot\left(e^{1/\sqrt{n}}-1\right)$

модуларни ред $\sum_{N=1}^{\infty} n^p\cdot\frac{1}{\sqrt{n}} \sim \sum_{N=1}^{\infty}\frac{1}{n^{1/2-p}}$

$1/2-p>1$
$-p>1-1/2$
$-p>1/2 /\cdot(-1)$
$p<-1/2$

апсолутно конвергира $p\in(-\infty,-\frac{1}{2})$

конвергира $p\in[-\frac{1}{2},+\infty)$

$\sum_{n=1} (\sqrt{n+1}-\sqrt{n})^p \cdot \ln \frac{\dots}{2n-1} \sim$

$\sim \sum_{n=1}^{\infty} \frac{(n+1-n)^p}{n^{p/2}\cdot 2} \cdot \ln\left(\frac{2n-1+2}{2n-1}\right) \sim$

$\sim \sum_{n=1}^{\infty} \frac{1^p}{2n^{p/2}} \cdot \ln\left(1+\frac{2}{2n-1}\right)^{\frac{2n-1}{2}\cdot\frac{2}{2n-1}} \sim$

$\sim \sum_{n=1}^{\infty} \frac{1}{2n^{p/2}} \cdot \ln e^{\frac{2}{2n-1}} \sim$

$\sim \sum_{n=1}^{\infty} \frac{1}{2n^{p/2}} \cdot \frac{2}{2n-1} \sim \frac{1}{2}\sum_{n=1}^{\infty}\frac{1}{n^{p/2+1}}$

$\frac{p}{2}+1>1$ КОНВЕРГИРА $p\in(0,+\infty)$

$\frac{p}{2}>1-1$ ДИВЕРГИРА $p\in(-\infty,0]$

$\frac{p}{2}>0$

$p>0$

8. $\sum_{n=1}^{\infty}\sqrt{\ln\left(1+\frac{2}{n}\right)}\left(\ln\frac{\sqrt{n}-1}{\sqrt{n+1}}\right)^p \sim$

$\sim\sum_{n=1}^{\infty} \sqrt{\ln\left(1+\frac{2}{n}\right)^{\frac{n}{2}\cdot\frac{2}{n}}}\cdot\left(\ln\left(\frac{\sqrt{n}+1-2}{\sqrt{n}+1}\right)\right)^p \sim$

$\sim\sum_{n=1}^{\infty}\sqrt{\ln e^{\frac{2}{n}}}\cdot\left(\ln\left(1-\frac{2}{\sqrt{n}+1}\right)^{\frac{\sqrt{n}+1}{2}\cdot\frac{2}{\sqrt{n}+1}}\right)^p \sim$

$\sim\sum_{n=1}^{\infty}\sqrt{\frac{2}{n}}\left(\ln e^{-\frac{2}{\sqrt{n}+1}}\right)^p\sim$

$\sim\sum_{n=1}^{\infty}\frac{\sqrt{2}}{n^{1/2}}\left(\frac{-2}{\sqrt{n}+1}\right)^p\sim$

$\sim\sum_{n=1}^{\infty}\frac{\sqrt2\cdot(-2)^p}{n^{1/2+p/2}}\sim(-2)^p\cdot\sqrt2\cdot\sum_{n=1}^{\infty}\frac{1}{n^{\frac{p+1}{2}}}$

$\frac{p+1}{2}>1$ КОНВЕРГИРА

$p+1>2$ $p\in(1,+\infty)$

$p>2-1$ ДИВЕРГИРА

$p>1$ $p\in(-\infty,1]$

9. $\sum_{n=1}^{\infty}\left(1-\cos\frac1n\right)\cdot\frac{1}{\sqrt{n^p}}\sim\sum_{n=1}^{\infty}\frac{1^2}{2\cdot n^2}\cdot\frac{1}{n^{p/2}}\sim$

$\sim\frac12\sum_{n=1}^{\infty}\frac{1}{n^{2+p/2}}$

$2+\frac p2>1$ КОНВЕРГИРА $p\in(-2,+\infty)$

$\frac p2>1-2$ ДИВЕРГИРА $p\in(-\infty,-2]$

$\frac p2>-1/2$

$p>-2$

10. $\sum_{n=1}^{\infty}\left(1-\cos\frac{1}{\sqrt n}\right)\cdot\frac{1}{n^p}\sim\sum_{n=1}^{\infty}\frac{1^2}{2(\sqrt n)^2}\cdot\frac{1}{n^p}\sim$

$\sim\frac12\sum_{n=1}^{\infty}\frac{1}{n^{p+1}}$

$p+1>1$ КОНВЕРГИРА $p\in(0,+\infty)$

$p>1-1$ ДИВЕРГИРА

$p>0$ $p\in(-\infty,0]$

11. $\sum_{n=1}^{\infty}\left(\frac{\sqrt{n+\sqrt{n+\sqrt n}}-\sqrt n}{n}\right)^p\cdot\operatorname{tg}^2\frac{u}{\sqrt[3]{n}}\sim$

$\sim\sum_{n=1}^{\infty}\left(\frac{\cancel{n}+\sqrt{n+\sqrt n}-\cancel{n}}{n}\right)^p\cdot\frac{1}{n^{p/2}(1+1)}\cdot\left(\frac{u}{\sqrt[3]{n}}\right)^2\sim$

$\sim\sum_{n=1}^{\infty}\frac{n^{p/2}}{2\cdot n^{p/2+p}}\cdot\frac{u^2}{n^{2/3}}\sim\frac{u^2}{2}\sum_{n=1}^{\infty}\frac{1}{n^{p+2/3}}$

$p+\frac23>1$ КОНВЕРГИРА $p\in(1/3,+\infty)$

$p>1-\frac23$ ДИВЕРГИРА

$p>\frac13$ $p\in(-\infty,1/3]$

РЕДОВИ Испитати конвергенцију реда у зависности од параметра $p \in \mathbb{R}$:

1. $\sum_{n=1}^{\infty}\left(\sqrt[3]{n+2}-\sqrt[3]{n-2}\right)\cdot(n^2+1)^p \sim$

$\sum_{n=1}^{\infty}\frac{(\cancel{n}+2-\cancel{n}+2)\,n^{2p}}{n^{2/3}(1+1+1)} \sim \frac{4}{3}\sum_{n=1}^{\infty}\frac{1}{n^{2/3-2p}}$

$\frac{2}{3}-2p>1$ — конвергира за $p\in(-\infty,-2/3)$

$-2p>1-\frac{2}{3}$

$-2p>\frac{1}{3} /:(-2)$ — дивергира за $p\in[-2/3,+\infty)$

$p<-2/3$

2. $\sum_{n=1}^{\infty}\frac{\sqrt[3]{n^2+1}-\sqrt[3]{n^2-1}}{n^{2p}} \sim \sum_{n=1}^{\infty}\frac{\cancel{n^2}+1-\cancel{n^2}+1}{n^{2p}\cdot n^{4/3}(1+1+1)} \sim$

$\sim\frac{2}{3}\sum_{n=1}^{\infty}\frac{1}{n^{2p+4/3}}$ $\quad 2p+\frac{4}{3}>1$ $\quad p>-1/6$

$2p>1-\frac{4}{3}$

$2p>-1/3 /:2$

конвергира $p\in(-1/6,+\infty)$ дивергира $p\in(-\infty,-1/6]$

3. $\sum_{n=1}^{\infty}\frac{\sqrt[3]{n^p+2}-\sqrt[3]{n^p-2}}{(\sqrt{n}+2)^p} \sim \sum_{n=1}^{\infty}\frac{\cancel{n^p}+2-\cancel{n^p}+2}{n^{p/2}\cdot n^{2p/3}\cdot 3} \sim$

$\sim\frac{4}{3}\sum_{n=1}^{\infty}\frac{1}{n^{p/2+2p/3}}$ $\quad \frac{p}{2}+\frac{2p}{3}>1/\cdot 6$

$3p+4p>6$ — конвергира за $p\in(\frac{6}{7},\infty)$ — дивергира за $p\in(-\infty,\frac{6}{7}]$

$7p>6 /:7$

$p>6/7$

4. $\sum_{n=1}^{\infty}\frac{\sqrt[3]{n^{3p}+1}-\sqrt[3]{n^{3p}-1}}{n} \sim \sum_{n=1}^{\infty}\frac{\cancel{n^{3p}}+1-\cancel{n^{3p}}+1}{n\cdot n^{2p}\cdot 3} \sim$

$\sim\frac{2}{3}\sum_{n=1}^{\infty}\frac{1}{n^{2p+1}}$

$2p+1>1$ — КОНВЕРГИРА $p\in(0,\infty)$

$2p>1-1$

$2p>0 /:2$

$p>0$ — ДИВЕРГИРА $p\in(-\infty,0]$

5. $\sum_{n=1}^{\infty}\frac{2\cdot 6\cdots(4n-2)}{2^{pn}(2n+1)!!}$, $p\neq 1$ $\quad 4n+4-2=4n+2$

$\frac{\cancel{6\cdot 10\cdots(4n-2)}\cdot(4n+2)}{2^{pn}\cdot 2^p(2n+3)\cdot(2n+1)!!}\cdot\frac{2^{pn}\cdot(2n+1)!!}{\cancel{2\cdot 6\cdots(4n-2)}}=$

$=\frac{4n+2}{2^{p+1}(2n+3)}=\frac{4n+2}{2^{p+2}n+2^{p+1}\cdot 3}$ $\quad \frac{4}{2^{p+2}}=1$ не важи тест

за $\frac{4}{2^{p+2}}<1$ конвергира $\quad \frac{4}{2^{p+2}}=1$ $\quad \frac{4}{2^{p+2}}>1$ дивергира

$4<2^{p+2}$ $\quad 4=2^{p+2}$ $\quad 4>2^{p+2}$

$2^2<2^{p+2}$ $\quad 2^2=2^{p+2}$ $\quad 2^2>2^{p+2}$

$2<p+2$ $\quad 2=p+2$ $\quad 2>p+2$

$2-2<p$ $\quad 2-2=p$ $\quad 2-2>p$

$0<p$ $\quad 0=p$ $\quad 0>p$

$p>0$ $\quad p=0$ $\quad p<0$

$p\in(0,+\infty)$ $\quad p\in(-\infty,0)$

6. $\sum_{n=1}^{\infty}\frac{(6n-4)\cdot(6n-7)\cdots 8\cdot 5\cdot 2}{3^{pn}(2n)!}$ $\quad 6n+6-4=6n+2$

$\frac{(6n+2)\cdot(6n-1)\cdot\cancel{(6n-4)\cdot(6n-7)\cdots 8\cdot 5\cdot 2}}{\cancel{3^{pn}}\cdot 3^p(2n+1)\cdot\cancel{2n!}}\cdot\frac{\cancel{3^{pn}}\cdot\cancel{(2n)!}}{\cancel{(6n-4)\cdot(6n-7)\cdots 8\cdot 5\cdot 2}}=$

$=\frac{(6n+2)\cdot(6n-1)}{3^p(2n+1)}=\frac{36n^2+\cdots}{3^p\cdot 2n+3^p}$

12. $\sum_{n=1}^{\infty} \frac{e^{1/\sqrt{n}}-1}{(\sqrt{n+2}-\sqrt{n})^p} \sim \sum_{n=1}^{\infty} \frac{1/\sqrt{n}\cdot n^{p/2}\cdot 2}{(n+2-n)^p} \sim$

$\sim \frac{1}{2^{p-1}} \sum \frac{1}{n^{1/2-p/2}} \qquad \frac{1}{2}-\frac{p}{2}>1$

$\frac{-p}{2}>1-\frac{1}{2}$ конвергира $p\in(-\infty,-1)$

$\frac{-p}{2}>\frac{1}{2} \;/\cdot(-2)$ дивергира $p\in[-1,+\infty)$

$p \neq -1$

13. $\sum_{n=1}^{\infty} \left(\frac{n^2+pn-3}{n^2+2n+5}\right)^{n^2} \sim \sum_{n=1}^{\infty} \left(\frac{n^2+2n+5+(p-2)n-8}{n^2+2n+5}\right)^{n^2} \sim$

$\sim \sum_{n=1}^{\infty} \left(1+\frac{(p+2)n-8}{n^2+2n+5}\right)^{\frac{n^2+2n+5}{(p+2)n-8}\cdot\frac{(p+2)n^3-8n^2}{n^2+2n+5}} \sim$

$\sim \sum_{n=1}^{\infty} e^{\frac{(p+2)n^3-8n^2}{n^2+2n+5}}$

$$\frac{e^{\frac{(p+2)(n+1)^3-8(n+1)^2}{(n+1)^2+2(n+1)+5}}}{e^{\frac{(p+2)(n^3-8n^2)}{n^2+2n+5}}}=$$

$$=e^{\frac{(p+2)(n^3+3n^2+3n+1)-8(n^2+2n+1)}{n^2+2n+1+2n+2+5}-\frac{(p+2)n^3-8n^2}{n^2+2n+5}}=$$

$$=e^{\frac{(3+8+8)n^2+n}{4n+1}}=e^{\frac{3n^2+n}{4n+1}}$$

$\to\infty$ ред дивергира.

14. $\sum_{n=1}^{\infty}\left(\frac{3n^2-pn}{3n^2+2n+1}\right)^{n(n+1)} \sim \sum_{n=1}^{\infty}\left(\frac{3n^2+2n+1+(-p-2)n-1}{3n^2+2n+1}\right)^{n^2+n} \sim$

$\sim \sum_{n=1}^{\infty}\left(1+\frac{(-p-2)n-1}{3n^2+2n+1}\right)^{\frac{3n^2+2n+1}{(-p-2)n-1}\cdot(n^2+n)\cdot\frac{(-p-2)n-1}{3n^2+2n+1}} \sim$

$\sim \sum_{n=1}^{\infty} e^{\frac{(-p-2)n^3+(-p-2)n^2-n^2-n}{3n^2+2n+1}} \sim$

$$\frac{e^{\frac{(-p-2)(n+1)^3+(-p-3)(n+1)^2-(n+1)}{3(n+1)^2+2(n+1)+1}}}{e^{\frac{(-p-2)n^3+(-p-3)n^2-n}{3n^2+2n+1}}}$$

$$=e^{\frac{((-p-2)(n+1)^3+(-p-3)(n+1)^2-(n+1))(3n^2+2n+1)-((-p-2)n^3+(-p-3)n^2-n)(3(n^2+2n+1)+2n+2+1)}{(3(n^2+2n+1)+2n+2+1)(3n^2+2n+1)}}$$

$3n^2+8n+5$

$$=\frac{3(-p-2)n^5-3(-p-2)n^5}{9n^4+\dots}$$

$$\frac{3(-p-2)n^4+3(-p-3)n^4+2(-p-2)n^4}{}$$

$$=\frac{(8(-p-2)n^4+3(-p-3)n^4)+\dots}{}$$

$$=\frac{(-3p-6+3p-9-2p-4+8p+16+3p+9+1)n^4+\dots}{9n^4+\dots}=$$

$$=\frac{3p+6}{9}$$

$\frac{3p+6}{9}$ $\quad 3p+6=9$, $3p=9-6$, $3p=3/:3$, $p=1$

конвергира за $p<1$

дивергира за $p>1$

$$\sum_{n=1} \frac{n \cdot 3^n}{} \cdot \frac{(n+1)3^{n+1}}{} \cdot \frac{}{p^n} =$$

$$= \frac{p \cdot n}{3n+3} = \frac{p}{3} = 1/3 \quad p=3$$

за $p<3$ конвергира $p \in (-\infty, 3)$
за $p>3$ дивергира $p \in (3, +\infty)$

16. $$\sum_{n=1}^{\infty} \frac{2^n p^n}{n^2+1} \quad \frac{2^{n+1} p^{n+1}}{(n+1)^2+1} \cdot \frac{n^2+1}{\cancel{2^n p^n}} =$$

$$= \frac{2p(n^2+1)}{n^2+2n+2} = \frac{2pn^2+2p}{n^2+2n+2}$$

$2p<1/:2$ конвергира за $p \in (-\infty, 1/2)$
$p<1/2$ дивергира за $p \in (1/2, +\infty)$

17. $$\sum_{n=1}^{\infty} \frac{(n+2)(p+2)^n}{n!}$$

$$\frac{(n+3)\cdot \cancel{(p+2)^n} \cdot (p+2)}{(n+1) \cdot \cancel{n!}} \cdot \frac{\cancel{n!}}{(n+2) \cdot \cancel{(p+2)^n}} =$$

$$= \frac{(p+2)n+3(p+2)}{n^2+3n+2}$$

$p+2<1$
$p<1-2$
$p<-1$

конвергира за $p \in (-\infty, -1)$
дивергира за $p \in (-1, +\infty)$

18. $$\sum_{n=1}^{\infty} \frac{5^n+3^n}{n} \cdot p^n$$

$$\frac{5^{n+1}+3^{n+1}}{n+1} \cdot \cancel{p^n} \cdot p \cdot \frac{n}{5^n+3^n} \cdot \frac{1}{\cancel{p^n}} =$$

$$= \frac{5^{n+1}\left(1+\left(\frac{3}{5}\right)^{n+1}\right) \cdot p \cdot n}{(n+1)\cancel{5^n}\left(1+\left(\frac{3}{5}\right)^n\right)} = \frac{5pn}{n+1}$$

$5p<1/:5$ конвергира за $p \in (-\infty, \frac{1}{5})$
$p<1/5$ дивергира за $p \in (\frac{1}{5}, +\infty)$

19. $$\sum_{n=1}^{\infty} \frac{2^n+n^3}{n^2} \cdot p^n$$

$$\frac{2^{n+1}+(n+1)^3}{(n+1)^2} \cdot \frac{p^n \cdot p}{2^n+n^3} \cdot \frac{n^2}{p^n} =$$

$$= \frac{(2^n \cdot 2+n^3+3n^2+3n+1)pn^2}{(2^n+n^3)\cdot(n^2+2n+1)} =$$

$$= \frac{\cancel{2^n}(2+0)pn^2}{\cancel{2^n}(1+0)(n^2+2n+1)} = \frac{2pn^2}{n^2+2n+1}$$

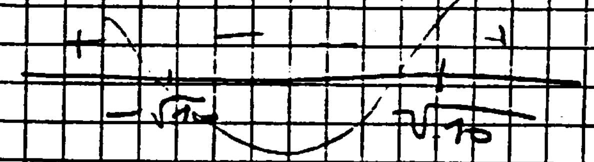
$2p<1/:2$ конвергира за $p \in (-\infty, \frac{1}{2})$
$p<1/2$ дивергира за $p \in (\frac{1}{2}, +\infty)$

20. $$\sum_{n=1}^{\infty} \frac{(p^2-9)^n}{2n+3}$$

$$\frac{(p^2-9)^{n+1}}{2n+5} \cdot \frac{2n+3}{\cancel{(p^2-9)^n}} = \frac{2(p^2-9)n+3(p^2-9)}{2n+5}$$

$$\frac{2(p^2-9)}{2} = p^2-9<1$$

$p^2-9-1<0$
$p^2-10<0$
$(p+\sqrt{10})(p-\sqrt{10})<0$



$p \in (-\sqrt{10}, \sqrt{10})$

конвергира $p \in (-\sqrt{10}, \sqrt{10})$
дивергира $p \in (-\infty, -\sqrt{10}) \cup (\sqrt{10}, +\infty)$

$\sum_{n=1} (3n^2-2)^n (p^2-1)^n$

$\frac{n+1}{(3n^2+6n+3-2)^{n+1}(p^2-1)^{n+1}} \cdot \frac{(3n^2-2)^n (p^2-1)^n}{n} = \frac{(n+1)\cancel{3^n}\cancel{n^{2n}}}{n+1}$

$= \cancel{3^n} \cdot 3 \cancel{n^{2n}} \cdot n^2 (p^2-1) n = 3p^2 n^3 - 3n^3 =$

$3p^2$ увек

$3p^2-3=0$

$3p^2=3 /:3$

$p^2=1$

$p=\pm 1$

$p\in(-1,1)$

За $p_1=1 \vee p_2=-1$ дивергира

За $p_1\neq 1$ и $p_2\neq -1$ конвергир.

22. $\sum_{n=1}^{\infty} (-1)^n \frac{\sqrt[3]{n^3+1}}{n^p}$

Модуларни ред $\sum_{n=1}^{\infty} \frac{1}{n^{p-1}}$ $\quad p-1\le 1$, $p\le 1+1$, $p\le 2$

За $p\in(-\infty,2]$ дивергира => ред условно конвергира.

За $p\in(2,+\infty)$ конвергира => ред апсолутно конвергира

23. $\sum_{n=1}^{\infty} (-1)^n n^p \sqrt{n^2+2n}$

Модуларни ред $\sum_{n=1}^{\infty} \frac{1}{n^{-p-1}}$ $\quad -p-1>1$, $-p>1+1$, $-p>2 /:(-1)$, $p<-2$

1 За $p\in(-\infty,-2)$ конвергира => ред апсолутно конвергира

2 За $p\in[-2,+\infty)$ дивергира => ред условно конвергира

24. $\sum_{n=1} (-1)^{n-1} \frac{(n+4n)^p}{\sqrt[3]{n+1}}$

Модуларни ред $\sum_{n=1}^{\infty} \frac{1}{n^{1/3-2p}}$ $\quad 1/3-2p>1$, $-2p>1-1/3$, $-2p>2/3 /:(-2)$, $p<-1/3$

(1) За $p\in(-\infty,-\frac{1}{3})$ конвергира

$\Downarrow$

Ред апсолутно конвергира

(2) За $p\in[-\frac{1}{3},+\infty)$ дивергира

$\Downarrow$

Ред условно конвергира

25. $\sum_{n=1}^{\infty} (-1)^n \frac{\sqrt[3]{n^3-2}}{n^{2p}}$ $\quad$ Модуларни ред: $\sum_{n=1}^{\infty} \frac{1}{n^{2p-1}}$

$2p-1\le 1$, $2p\le 1+1$, $2p\le 2 /:2$, $p\le 1$

(1) За $p\in(-\infty,1]$ дивергира => условно конвергира

(2) За $p\in(1,\infty)$ конвергира => ред апсолутно конвергира

26. $\sum_{n=1}^{\infty} (-1)^{n-1} n^2 \sin\frac{\pi}{2^n}$

Модуларни ред $\sum_{n=1}^{\infty} n^2 \sin\frac{\pi}{2^n} \sim \sum_{n=1} n^2 \frac{\pi}{2^n}$

$\frac{(n+1)^2\cancel{\pi}}{2^{n+1}} \cdot \frac{\cancel{2^n}}{n^2\cancel{\pi}} = \frac{n^2+2n+1}{2n^2} = \frac{1}{2}$

Модуларни ред конвергира

Ред апсолутно конвергира