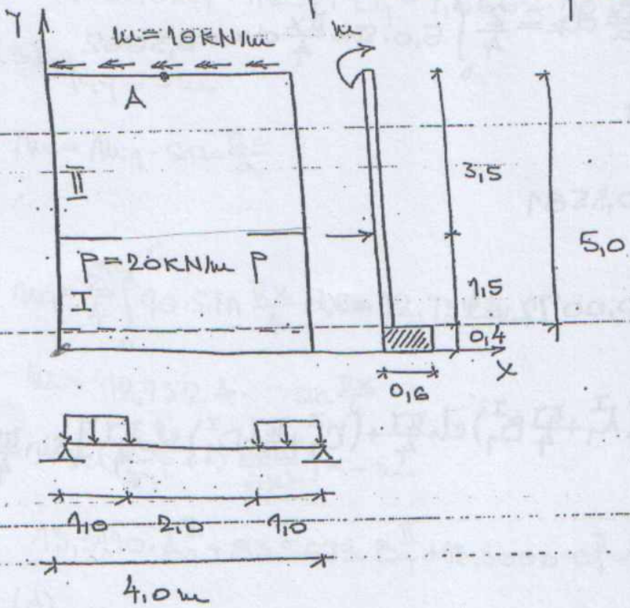
MORIS - LEVY

(7) pitati

06.12.1997.

1. Za pravougaonu ploču opterećenu prema slici odrediti ugib u tački A koristeći samo prvi član reda usvojenog rešenja. U obzir uzeti i sopstvenu težinu ploče.



$E = 30\,GPa$

$\nu = 0{,}15 \qquad h = 0{,}20\,m$

$GI_t = 0$

$\gamma = 25\,KN/m^3$

$g = \gamma \cdot h = 25 \cdot 0{,}2 = 5{,}0\,KN/m^2$

granični uslovi:

$$Y=0 \begin{cases} W^P = W^G & (1) \\ \dfrac{\partial W^P}{\partial Y} = \varphi^G & (2) \end{cases} \qquad Y=5{,}0 \begin{cases} M_Y = -m & (3) \\ \overline{T}_Y = 0 & (4) \end{cases}$$

Prelazni uslovi:

$$Y=1{,}5 \begin{cases} W^{I} = W^{II} & (5) \\ \dfrac{\partial W^{I}}{\partial Y} = \dfrac{\partial W^{II}}{\partial Y} & (6) \end{cases} \qquad \begin{matrix} M_Y^{I} = M_Y^{II} & (7) \\ \overline{T}_Y^{I} = \overline{T}_Y^{II} + p(x) & (8) \end{matrix}$$

I ploča:

$$\boxed{W^{I} = W_0^{I} + W_1^{I}}$$

$$W_0^{I} = W_{0,1} \cdot \sin\frac{\pi x}{4}$$

$$W_{0,1} = \frac{Z_1 \cdot a^4}{K \cdot \pi^4}$$

$$Z_1 = \frac{2}{a}\int_0^a Z(x,y)\sin\frac{\pi x}{a}\,dx = \frac{2}{4}\int_0^4 5{,}0 \cdot \sin\frac{\pi x}{4}\,dx = 6{,}3662$$

$$W_{0,1} = 8{,}1772 \cdot 10^{-4}$$

$$K = \frac{E \cdot h^3}{12(1-\nu^2)} = 20460{,}3581$$

$$I_g = \frac{1}{12} \cdot 0{,}4 \cdot 0{,}6^3 = 0{,}0072\,m^4$$

$$W^{I} = \left[8{,}1772 \cdot 10^{-4} + \left(A_1^{I} + \frac{\pi y}{4}B_1^{I}\right)ch\frac{\pi y}{4} + \left(C_1^{I} + \frac{\pi y}{4}D_1^{I}\right)sh\frac{\pi y}{4}\right]\sin\frac{\pi x}{4}$$

II ploča

$$W^{II} = W_0^{II} + W_1^{II}$$

$$Z_1 = \frac{2}{4}\int_0^4 5{,}0 \sin\frac{\pi x}{4}\,dx = 6{,}3662$$

$$W^{II} = \left[8{,}1772 \cdot 10^{-4} + \left(A_1^{II} + \frac{\pi y}{4}B_1^{II}\right)ch\frac{\pi y}{4} + \left(C_1^{II} + \frac{\pi y}{4}D_1^{II}\right)sh\frac{\pi y}{4}\right] \cdot \sin\frac{\pi x}{4}$$

(1):

$$W_P = W_G$$

$$\frac{\partial^4 W_P}{\partial x^4} = \frac{d^4 W_G}{dx^4} = \frac{T_y}{EI_G}$$

$$\frac{\partial^4 W}{\partial x^4} \cdot EI = -K\left(\frac{\partial^3 W}{\partial y^3} + (2-\nu)\frac{\partial^2 W}{\partial x^2 \partial y}\right)$$

$$82188{,}9206 \cdot A_1^{I} + 11399{,}3664 \cdot B_1^{I} - 8425{,}6187 \cdot C_1^{I} + 67{,}2079 = 0 \; (1)$$

(2) $\frac{\partial w}{\partial y} = \varphi G$

$G I_t = 0 \Rightarrow M_y = 0$

$$-K\left(\frac{\partial^2 w}{\partial y^2} + \nu \frac{\partial^2 w}{\partial x^2}\right) = 0$$

$$0{,}5243\, A_1^{I} + 1{,}2337 \cdot D_1^{I} - 7{,}5662 \cdot 10^{-5} = 0 \quad (2)$$

(3) $M_y = -m$

$m = m_1 \cdot \sin\frac{\pi x}{a}$

$$m_1 = \frac{2}{4}\int_0^4 10 \cdot \sin\frac{\pi x}{4}\, dx = 12{,}7324$$

$$m = 12{,}7324 \cdot \sin\frac{\pi x}{4}$$

$$-K\left(\frac{\partial^2 w}{\partial y^2} + \nu \frac{\partial^2 w}{\partial x^2}\right) = -m$$

$$13{,}3110 \cdot A_1^{II} + 83{,}5673 \cdot B_1^{II} + 13{,}3006 \cdot C_1^{II} + 83{,}5510 \cdot D_1^{II} - 6{,}9796 \cdot 10^{-4} = 0 \quad (3)$$

(4) $\overline{T}_y = 0$

$$-K\left(\frac{\partial^3 w}{\partial y^3} + (2-\nu)\frac{\partial^3 w}{\partial x^2 \partial y}\right) = 0$$

$$10{,}4462 \cdot A_1^{II} + 26{,}8782 \cdot B_1^{II} + 10{,}4544 \cdot C_1^{II} + 26{,}9210 \cdot D_1^{II} = 0 \quad (4)$$

(5) $w^{I} = w^{II}$

$$1{,}1780 \cdot A_1^{I} + 2{,}0947 \cdot B_1^{I} + 1{,}4702 \cdot C_1^{I} + 1{,}7320 \cdot D_1^{I} - 1{,}1780 \cdot A_1^{II} - 2{,}0947 \cdot B_1^{II} - 1{,}4702 \cdot C_1^{II} - 1{,}7320 \cdot D_1^{II} = 0 \quad (5)$$

(6) $\frac{\partial w^{I}}{\partial y} = \frac{\partial w^{II}}{\partial y}$

$$1{,}1547 \cdot A_1^{I} + 2{,}7568 \cdot B_1^{I} + 1{,}3965 \cdot C_1^{I} + 2{,}7998 \cdot D_1^{I} - 1{,}1547 \cdot A_1^{II} - 2{,}7568 \cdot B_1^{II} - 1{,}3965 \cdot C_1^{II} - 2{,}7998 \cdot D_1^{II} = 0 \quad (6)$$

I ploča:

$$\boxed{W^{I} = W_0^{I} + W_1^{I}}$$

$$W_0^{I} = W_{o,1} \cdot \sin\frac{\pi x}{4}$$

$$W_{o,1} = \frac{Z_1 \cdot a^4}{K \cdot \pi^4}$$

$$Z_1 = \frac{2}{a}\int_0^a z(x,y)\sin\frac{\pi x}{a}dx = \frac{2}{4}\int_0^4 5{,}0 \cdot \sin\frac{\pi x}{4}dx = 6{,}3662$$

$$W_{o,1} = 8{,}1772 \cdot 10^{-4}$$

$$K = \frac{E \cdot h^3}{12(1-\nu^2)} = 20460{,}3581$$

$$I_g = \frac{1}{12} \cdot 0{,}4 \cdot 0{,}6^3 = 0{,}0072\,m^4$$

$$W^{I} = \left[8{,}1772 \cdot 10^{-4} + \left(A_1^{I} + \frac{\pi y}{4}B_1^{I}\right)ch\frac{\pi y}{4} + \left(C_1^{I} + \frac{\pi y}{4}D_1^{I}\right)sh\frac{\pi y}{4}\right]\sin\frac{\pi x}{4}$$

II ploča

$$W^{II} = W_0^{II} + W_1^{II}$$

$$Z_1 = \frac{2}{4}\int_0^4 5{,}0 \sin\frac{\pi x}{4}dx = 6{,}3662$$

$$W^{II} = \left[8{,}1772 \cdot 10^{-4} + \left(A_1^{II} + \frac{\pi y}{4}B_1^{II}\right)ch\frac{\pi y}{4} + \left(C_1^{II} + \frac{\pi y}{4}D_1^{II}\right)sh\frac{\pi y}{4}\right] \cdot \sin\frac{\pi x}{4}$$

(1):

$$W_P = W_G$$

$$\frac{\partial^4 w_P}{\partial x^4} = \frac{d^4 w_G}{dx^4} = \frac{\overline{T}_y}{EI_G}$$

$$\frac{\partial^4 w}{\partial x^4} \cdot EI = -K\left(\frac{\partial^3 w}{\partial y^3} + (2-\nu)\frac{\partial^2 w}{\partial x^2 \partial y}\right)$$

$$82188{,}9206 \cdot A_1^{I} + 11399{,}3664 \cdot B_1^{I} - 8425{,}6187 \cdot C_1^{I} + 67{,}2079 = 0 \; (1)$$

(2) $\frac{\partial w}{\partial y} = \varphi G$

$G I_t = 0 \Rightarrow M_y = 0$

$$-K\left(\frac{\partial^2 w}{\partial y^2} + \nu \frac{\partial^2 w}{\partial x^2}\right) = 0$$

$$0{,}5243\, A_1^{I} + 1{,}2337 \cdot D_1^{I} - 7{,}5662 \cdot 10^{-5} = 0 \quad (2)$$

(3) $M_y = -m$

$$m = m_1 \cdot \sin\frac{\pi x}{a}$$

$$m_1 = \frac{2}{4}\int_0^4 10 \cdot \sin\frac{\pi x}{4}\, dx = 12{,}7324$$

$$m = 12{,}7324 \cdot \sin\frac{\pi x}{4}$$

$$-K\left(\frac{\partial^2 w}{\partial y^2} + \nu \frac{\partial^2 w}{\partial x^2}\right) = -m$$

$$13{,}3110 \cdot A_1^{II} + 83{,}5673 \cdot B_1^{II} + 13{,}3006 \cdot C_1^{II} + 83{,}5510 \cdot D_1^{II} - 6{,}9796 \cdot 10^{-4} = 0 \quad (3)$$

(4) $T_y = 0$

$$-K\left(\frac{\partial^3 w}{\partial y^3} + (2-\nu)\frac{\partial^3 w}{\partial x^2 \partial y}\right) = 0$$

$$10{,}4462 \cdot A_1^{II} + 26{,}8782 \cdot B_1^{II} + 10{,}4544 \cdot C_1^{II} + 26{,}9210 \cdot D_1^{II} = 0 \quad (4)$$

(5) $w^{I} = w^{II}$

$$1{,}7780 \cdot A_1^{I} + 2{,}0947 \cdot B_1^{I} + 1{,}4702 \cdot C_1^{I} + 1{,}7320 \cdot D_1^{I} - 1{,}7780 \cdot A_1^{II} -$$
$$- 2{,}0947 \cdot B_1^{II} - 1{,}4702 \cdot C_1^{II} - 1{,}7320 \cdot D_1^{II} = 0 \quad (5)$$

(6) $\frac{\partial w^{I}}{\partial y} = \frac{\partial w^{II}}{\partial y}$

$$1{,}1547 \cdot A_1^{I} + 2{,}7568 \cdot B_1^{I} + 1{,}3965 \cdot C_1^{I} + 2{,}7998 \cdot D_1^{I} -$$
$$- 1{,}1547 \cdot A_1^{II} - 2{,}7568 \cdot B_1^{II} - 1{,}3965 \cdot C_1^{II} - 2{,}7998 \cdot D_1^{II} = 0 \quad (6)$$

(7) $M_Y^{I} = M_Y^{II}$

$$-K\left(\frac{\partial^2 W^{I}}{\partial Y^2} + \nu \frac{\partial^2 W^{I}}{\partial X^2}\right) = -K\left(\frac{\partial^2 W^{II}}{\partial Y^2} + \nu \frac{\partial^2 W^{II}}{\partial X^2}\right)$$

$$0{,}9323 \cdot A_1^{I} + 2{,}9120 \cdot B_1^{I} + 0{,}7708 \cdot C_1^{I} + 3{,}1017 \cdot D_1^{I} - 0{,}9323 \cdot A_1^{II} - 2{,}9120 \cdot B_1^{II} - 0{,}7708 \cdot C_1^{II} - 3{,}1017 \cdot D_1^{II} = 0 \quad (7)$$

(8)

$$\overline{T}_Y^{I} = \overline{T}_Y^{II} + p(x)$$

$$p(x) = p_1 \cdot \sin\frac{\pi x}{a}$$

$$p_1 = \frac{2}{a}\int_0^a p \cdot \sin\frac{\pi x}{a}\,dx = \frac{2}{4}\left[\int_0^1 20 \sin\frac{\pi x}{4}\,dx + \int_3^4 20 \cdot \sin\frac{\pi x}{4}\,dx\right] = 7{,}4585$$

$$p(x) = 7{,}4585 \cdot \sin\frac{\pi x}{4}$$

$$12387{,}0237 \cdot A_1^{I} - 5675{,}2501 \cdot B_1^{I} + 14980{,}9681 \cdot C_1^{I} + 890{,}1228 \cdot D_1^{I} - 12387{,}0237 \cdot A_1^{II} + 5675{,}2501 \cdot B_1^{II} - 14980{,}9681 \cdot C_1^{II} - 890{,}1228 \cdot D_1^{II} - 7{,}4585 = 0 \quad (8)$$

$A_1^{I} = -6{,}6554 \cdot 10^{-4}$ $\qquad$ $A_1^{II} = -9{,}0049 \cdot 10^{-4}$

$B_1^{I} = -4{,}4579 \cdot 10^{-4}$ $\qquad$ $B_1^{II} = 2{,}2313 \cdot 10^{-4}$

$C_1^{I} = 8{,}8142 \cdot 10^{-4}$ $\qquad$ $C_1^{II} = 8{,}6411 \cdot 10^{-4}$

$D_1^{I} = 3{,}4418 \cdot 10^{-4}$ $\qquad$ $D_1^{II} = -2{,}0892 \cdot 10^{-4}$

$X = 2{,}0$

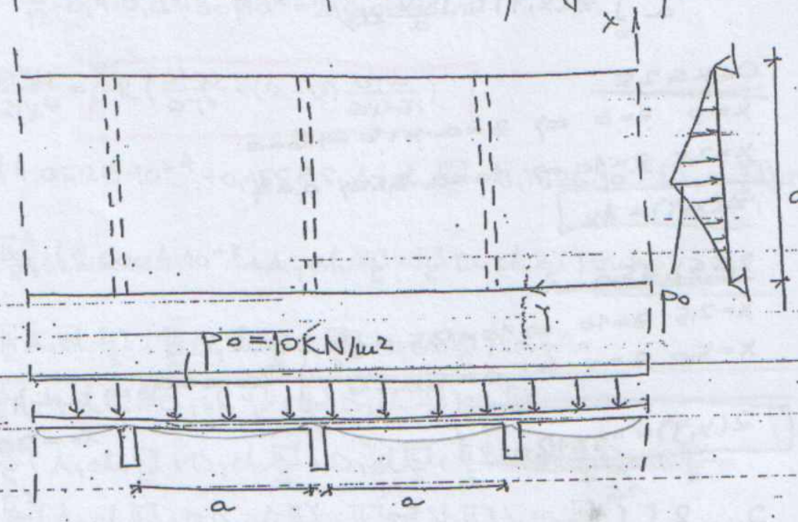
$Y = 5{,}0$

$W_A^{II} = 1{,}3102 \cdot 10^{-3}\,\text{m} = 1{,}3102\,\text{mm}$

MORIS - LEVY

① 26.11.2005.

1. Za kontinualnu ploču opterećenu prema slici odrediti izraze za ugib i presečne sile. Koristiti prvi član usvojenog rešenja.



$E = 30\ GPa$ $\quad h = 0{,}2m$

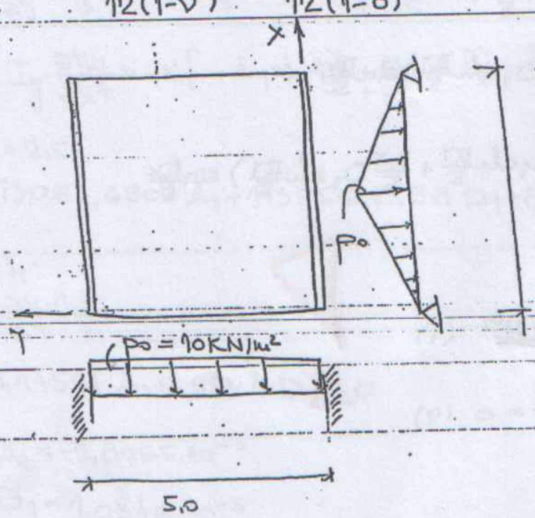
$\nu = 0$

$p = 10 kN/m^2$ $\quad$ grede: $b/h = 0{,}4/0{,}6$

$$K = \frac{E \cdot h^3}{12(1-\nu^2)} = \frac{30 \cdot 10^6 \cdot 0{,}2^3}{12(1-0)} = 20000$$

$z = kx + n$

I $\quad x = 0 \quad z = 0$; $\quad x = 2{,}5 \quad z = 10$ $\quad n = 0$; $\quad 10 = 2{,}5k$; $\quad \boxed{k = 4}$; $\quad \boxed{z = 4x}$



II $\quad x = 2{,}5 \quad z = 10$; $\quad x = 5 \quad z = 0$

$0 = 5k + n$; $\quad 10 = 2{,}5k + n$; $\quad 10 = 2{,}5k$; $\quad \boxed{k = 4}$; $\quad \boxed{n = -20}$

$z = 4x - 20$

$$W_0 = \frac{z_n \cdot a^4}{\pi^4 \cdot n^4 K}$$

$W = W_0 + W_n$

$$z_n = \frac{2}{a}\int_0^a z(x,y) \sin\frac{n\pi x}{a} = \frac{2}{a}\int_0^{a/2} p_0 \cdot$$

$$W_0 = \sum_{n=1}^{\infty} W_n \sin\frac{n\pi x}{a}$$

$$W_n = \frac{4 Z_n \cdot a^4}{K \cdot \pi^4 \cdot n^4}$$

$$Z_n = \frac{2}{a}\int_0^a Z(x,y)\sin\frac{n\pi x}{a}\,dx$$

$0 \le x \le 2{,}5$

$x=0 \quad z=0 \Rightarrow z = a\cdot x + b \Rightarrow b=0$

$x=2{,}5 \quad z=10 \Rightarrow 10 = a\cdot 2{,}5 \Rightarrow a=4$

$\boxed{Z(x,y) = 4x}$

$2{,}5 \le x \le 5{,}0$

$$\left.\begin{array}{l} x=2{,}5 \quad z=10 \Rightarrow 10 = 2{,}5\cdot a + b \\ x=5{,}0 \quad z=0 \Rightarrow 0 = 5a+b \end{array}\right\} 2{,}5a = -10 \Rightarrow a=-4$$

$b = 20$

$\boxed{Z(x,y) = -4x+20}$

$$Z_n = \frac{2}{5}\left[\int_0^{2{,}5} 4x\sin\frac{\pi x}{5}dx + \int_{2{,}5}^{5{,}0}(-4x+20)\sin\frac{\pi x}{5}dx\right] = 8{,}10569$$

$$W_n = \frac{8{,}10569\cdot 5^4}{20000\cdot\pi^4} = 2{,}6004\cdot 10^{-3}$$

$$W_{0,1} = 2{,}6004\cdot 10^{-3}\cdot\sin\frac{\pi x}{5}$$

$$W_{1,1} = \left(A_1 \operatorname{ch}\frac{\pi y}{5} + \frac{\pi y}{5}D_1 \operatorname{sh}\frac{\pi y}{5}\right)\sin\frac{\pi x}{5}$$

$$W = \left(2{,}6004\cdot 10^{-3} + A_1\operatorname{ch}\frac{\pi y}{5} + \frac{\pi y}{5}D_1\operatorname{sh}\frac{\pi y}{5}\right)\sin\frac{\pi x}{5}$$

granični uslovi:

$$y = \pm\frac{b}{2} = \pm 2{,}5 \quad \left\{\begin{array}{ll} W^P = W^G & (1) \\ \dfrac{\partial W^P}{\partial y} = 0 & (2) \end{array}\right.$$

D

(1): $W^P = W^G$

$$\frac{\partial^4 W^P}{\partial x^4} = \frac{\partial^4 W^G}{\partial x^4} = \frac{T_Y}{EI}$$

$$EI_g = \frac{1}{12}\cdot 0{,}4\cdot 0{,}6^3\cdot 30\cdot 10^6 = 216000$$

$$EI_g \frac{\partial^4 W}{\partial x^4} = \boxed{-K\left(\frac{\partial^3 W}{\partial Y^3} + (2-\nu)\frac{\partial^3 W}{\partial x^2 \partial Y}\right)}$$

$$\frac{\partial^4 W}{\partial x^4} = (4{,}0528\cdot 10^{-4} + 0{,}15585\cdot A_1 ch\frac{\pi Y}{5} + 9{,}7926\cdot 10^{-2} D_1 sh\frac{\pi Y}{5}) \sin\frac{\pi x}{5}$$

$$\frac{\partial^4 W}{\partial x^4} = \frac{\pi^4}{5^4}\cdot(2{,}6004\cdot 10^{-3} + A_1 ch\frac{\pi Y}{5} + \frac{\pi Y}{5} D_1 sh\frac{\pi Y}{5}) \sin\frac{\pi x}{5}$$

$$\frac{\partial W}{\partial Y} = \left(\frac{\pi}{5}A_1 sh\frac{\pi Y}{5} + \frac{\pi}{5}D_1 sh\frac{\pi Y}{5} + \frac{\pi^2 Y}{5^2}D_1 ch\frac{\pi Y}{5}\right)\sin\frac{\pi x}{5} =$$
$$= \frac{\pi}{5}\left(A_1 sh\frac{\pi Y}{5} + D_1 sh\frac{\pi Y}{5} + \frac{\pi Y}{5}D_1 ch\frac{\pi Y}{5}\right)\sin\frac{\pi x}{5}$$

$$\frac{\partial^2 W}{\partial Y^2} = \frac{\pi^2}{5}\left(A_1 ch\frac{\pi Y}{5} + D_1 ch\frac{\pi Y}{5} + D_1 ch\frac{\pi Y}{5} + \frac{\pi Y}{5}D_1 sh\frac{\pi Y}{5}\right)\cdot\sin\frac{\pi x}{5} =$$
$$= \frac{\pi^2}{5}\left(A_1 ch\frac{\pi Y}{5} + 2D_1 ch\frac{\pi Y}{5} + \frac{\pi Y}{5}D_1 sh\frac{\pi Y}{5}\right)\cdot\sin\frac{\pi x}{5}$$

$$\frac{\partial^3 W}{\partial Y^3} = \frac{\pi^3}{5^3}\left(A_1 sh\frac{\pi Y}{5} + 2D_1 sh\frac{\pi Y}{5} + D_1 sh\frac{\pi Y}{5} + \frac{\pi Y}{5}D_1 ch\frac{\pi Y}{5}\right)\cdot\sin\frac{\pi x}{5} =$$
$$= \frac{\pi^3}{5^3}\left(A_1 sh\frac{\pi Y}{5} + 3D_1 sh\frac{\pi Y}{5} + \frac{\pi Y}{5}D_1 ch\frac{\pi Y}{5}\right)\cdot\sin\frac{\pi x}{5}$$

$$\frac{\partial^3 W}{\partial x^2 \partial Y} = -\frac{\pi^3}{5^3}\left(A_1 sh\frac{\pi Y}{5} + D_1 sh\frac{\pi Y}{5} + \frac{\pi Y}{5}D_1 ch\frac{\pi Y}{5}\right)\cdot\sin\frac{\pi x}{5}$$

$$EI_g\frac{\partial^4 W}{\partial x^4} = -K\left[-A_1 sh\frac{\pi Y}{5} + D_1 sh\frac{\pi Y}{5} - \frac{\pi Y}{5}D_1 ch\frac{\pi Y}{5}\right]\cdot\sin\frac{\pi x}{5}$$

$Y = 2{,}5$

$$73053{,}6906\, A_1 + 113556{,}5598\cdot D_1 + 87{,}5415 = 0 \qquad (1)$$

(2):

$$\frac{\partial W^P}{\partial Y} = 0$$

$$1{,}44594\cdot A_1 + 3{,}9224\cdot D_1 = 0 \qquad (2)$$

$A_1 = -2{,}8065\cdot 10^{-3}$

$D_1 = 1{,}0346\cdot 10^{-3}$

$$W = \left(2{,}6004\cdot 10^{-3} - 2{,}8065\cdot ch\frac{\pi Y}{5} + \frac{\pi Y}{5}\cdot 1{,}3046\cdot 10^{-3} sh\frac{\pi Y}{5}\right)\cdot\sin\frac{\pi x}{5}$$

Presečne sile:

$$M_x=-K\left(\frac{\partial^2 w}{\partial x^2}+\nu\frac{\partial^2 w}{\partial y^2}\right)=\left(20{,}53196-22{,}15922\cdot ch\frac{\pi y}{5}+5{,}13256\cdot y\, sh\frac{\pi y}{5}\right)\cdot\sin\frac{\pi x}{5}$$

$$M_y=-K\left(\frac{\partial^2 w}{\partial y^2}+\nu\frac{\partial^2 w}{\partial x^2}\right)=-\left(-5{,}82176\cdot ch\frac{\pi y}{5}+5{,}13256\cdot y\, sh\frac{\pi y}{5}\right)\cdot\sin\frac{\pi x}{5}$$

$$M_{xy}=-K(1-\nu)\frac{\partial^2 w}{\partial x\partial y}=\left(13{,}99049\cdot sh\frac{\pi y}{5}-5{,}13256\cdot y\, ch\frac{\pi y}{5}\right)\cdot\sin\frac{\pi x}{5}$$

$$T_x=-K\left(\frac{\partial^3 w}{\partial x^3}+\frac{\partial^3 w}{\partial x\partial y^2}\right)=-\left(10{,}2651\cdot ch\frac{\pi y}{5}-12{,}9006\right)\cdot\cos\frac{\pi x}{5}$$

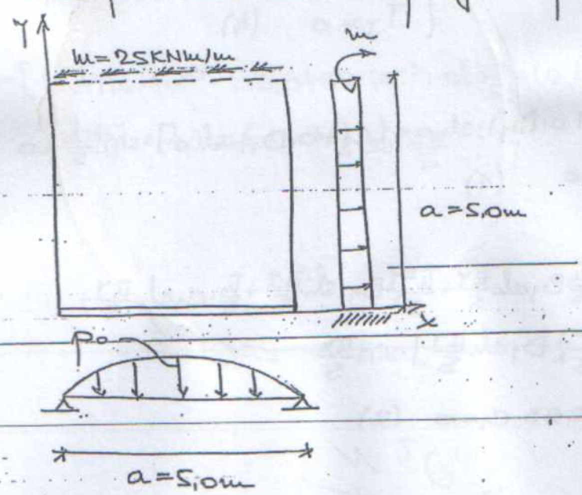
$$T_y=-K\left(\frac{\partial^3 w}{\partial y^3}+\frac{\partial^3 w}{\partial x^2\partial y}\right)=-10{,}2651\cdot sh\frac{\pi y}{5}\cdot\sin\frac{\pi x}{5}$$

MORIS-LEVY

②

24.10.1999.

1. Rešiti problem savijanja ploče opterećene prema slici koristeći samo prvi član reda usvojenog rešenja.



$p_0 = 25\,KN/m^2$

$h = 0,25\,m$

$E = 30\,GPa$

$\nu = 0$

$Z(x,y) = p_0 \cdot \sin\frac{\pi x}{a}$

$$K = \frac{E \cdot h^3}{12(1-\nu^2)} = \frac{30 \cdot 10^6 \cdot 0,25^3}{12(1-0)} = 39062,5$$

$$W = W_0 + W_1$$

$$W_0 = \sum_{n=1}^{\infty} W_n \cdot \sin\frac{n\pi x}{a} = W_{01} \cdot \sin\frac{\pi x}{a}$$

$$W_{0,1} = \frac{Z_n \cdot a^4}{K \cdot \pi^4}$$

$$Z_n = \frac{2}{a}\int_0^a Z(x,y) \cdot \sin\frac{\pi x}{a}\,dx = \frac{2}{5}\int_0^5 p_0 \cdot \sin^2\frac{\pi x}{5}\,dx = 10\int_0^5 \sin^2\frac{\pi x}{5}\,dx = 25$$

$$W_{0,1} = 4,1064 \cdot 10^{-3}$$

$$W_1 = \left[\left(A_1 + \frac{\pi y}{5} B_1\right) \cdot ch\frac{\pi y}{5} + \left(C_1 + \frac{\pi y}{5} \cdot D_1\right) sh\frac{\pi y}{5}\right] \cdot \sin\frac{\pi x}{5}$$

$$W=\left[4{,}1064\cdot 10^{-3}+\left(A_1+\frac{\pi Y}{5}B_1\right)ch\frac{\pi Y}{5}+\left(C_1+\frac{\pi Y}{5}D_1\right)sh\frac{\pi Y}{5}\right]\cdot \sin\frac{\pi X}{5}$$

granični uslovi:

$$Y=0\begin{cases} W=0 & (1)\\ \dfrac{\partial W}{\partial Y}=0 & (2)\end{cases}\qquad Y=5{,}0\begin{cases} M_Y=-m & (3)\\ \overline{T}_Y=0 & (4)\end{cases}$$

(1):

$$\left[4{,}1064\cdot 10^{-3}+(A_1+0\cdot B_1)\cdot ch\,0+(C_1+0\cdot D_1)\cdot sh\,0\right]\cdot \sin\frac{\pi x}{5}=0$$

$$4{,}1064\cdot 10^{-3}+A_1=0 \qquad (1)$$

(2)

$$\frac{\partial W}{\partial Y}=\left[\frac{\pi}{5}\cdot A_1\cdot sh\frac{\pi Y}{5}+\frac{\pi}{5}B_1 ch\frac{\pi Y}{5}+\frac{\pi^2 Y}{5^2}B_1\cdot sh\frac{\pi Y}{5}+\frac{\pi}{5}\cdot C_1 ch\frac{\pi Y}{5}+\right.$$
$$\left.+\frac{\pi}{5}D_1\cdot sh\frac{\pi Y}{5}+\frac{\pi^2 Y}{5^2}D_1 ch\frac{\pi Y}{5}\right]\cdot \sin\frac{\pi X}{5}=0$$

$$0{,}6283\cdot B_1+0{,}6283\cdot C_1=0 \quad (2)$$

(3)

$$M_Y=-m$$

$$M_Y=-K\left(\frac{\partial^3 W}{\partial Y^3}+\cancel{\nu}^{\,0}\frac{\partial^2 W}{\partial X^2}\right)=-m \qquad \frac{\partial^2 W}{\partial\ldots^2}$$

$$\frac{\partial^3 W}{\partial Y^3}=\left[0{,}24805\cdot A_1 sh\frac{\pi Y}{5}+B_1\left(0{,}15585\cdot Y\cdot sh\frac{\pi Y}{5}+0{,}74415\cdot ch\frac{\pi Y}{5}\right)+\right.$$
$$\left.+0{,}24805\, C_1 ch\frac{\pi Y}{5}+D_1\left(0{,}74415\cdot sh\frac{\pi Y}{5}+0{,}15585\cdot Y\cdot ch\frac{\pi Y}{5}\right)\right]\cdot \sin\frac{\pi X}{5}$$

Razvijanje momenta u red:

$$m=\sum_{n=1}^{\infty} m_n\cdot \sin\frac{n\pi x}{a}\,,\; n=1 \Rightarrow m=m_1\cdot \sin\frac{\pi x}{5}$$

$$m_1=\frac{2}{a}\int_0^a m\cdot \sin\frac{\pi x}{a}dx=\frac{2}{5}\cdot 25\cdot\left(-\cos\frac{\pi x}{5}\right)\Big|_0^5\cdot\frac{5}{\pi}=\frac{100}{\pi}$$

$$m=\frac{100}{\pi}\cdot \sin\frac{\pi x}{5}$$

$$-111901{,}0650\cdot A_1-688506{,}9172\cdot B_1-112319{,}7845\cdot C_1-$$
$$-688566{,}2048\cdot D_1+31{,}8310=0 \quad (3)$$

(4)

$$\overline{T}_Y=-K\left[\frac{\partial^3 W}{\partial Y^3}+(2-\nu)\frac{\partial^3 W}{\partial X^2\partial Y}\right]=0$$

$$2{,}8647\cdot A_1+6{,}1242\cdot B_1+2{,}8754\cdot C_1+6{,}1686\cdot D_1=0 \quad (4)$$

$A_1 = -4{,}1064 \cdot 10^{-3}$

$B_1 = -3{,}8482 \cdot 10^{-3}$

$C_1 = 3{,}8482 \cdot 10^{-3}$

$D_1 = 3{,}9337 \cdot 10^{-3}$

$$W = \Big[(2{,}4776 \cdot 10^{-3} \cdot y + 3{,}8482 \cdot 10^{-3}) \cdot sh\frac{\pi y}{5} - (2{,}4179 \cdot 10^{-3} y + 4{,}1064 \cdot 10^{-3}) \cdot$$

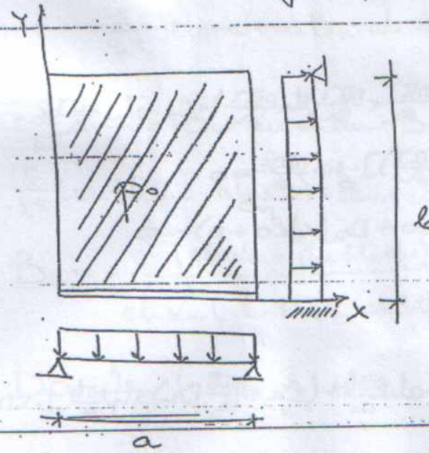
$$\cdot ch\frac{\pi y}{5} + 4{,}1064 \cdot 10^{-3} \Big] \cdot \sin\frac{\pi x}{5}$$

MORIS – LEVY

04.05.2011

(4)

1. Napisati izraz za ugib u proizvoljnoj tački ploče.



$$Z(x,y)=p_0$$

$$W=W_0+W_1$$

$$W_0=\sum_{n=1}^{\infty} W_n\cdot \sin\frac{n\pi x}{a}$$

$$W_n=\frac{Z_n\cdot a^4}{K\cdot\pi^4\cdot n^4}$$

$$Z_n=\frac{2}{a}\int_0^a Z(x,y)\sin\frac{n\pi x}{a}\,dx=\frac{2}{a}\cdot p_0\left(-\cos\frac{n\pi x}{a}\right)\Big|_0^a\frac{a}{n\pi}$$

$$Z_n=\frac{2p_0}{n\pi}(\cos 0-\cos n\pi)=\frac{2p_0}{n\pi}\left(1-(-1)^n\right)=\begin{cases}\frac{4p_0}{n\pi}, & \text{za } n=1,3,5,\ldots\\ 0, & \text{za } n=2,4,6,\ldots\end{cases}$$

$$W_n=\frac{4p_0\cdot a^4}{K\cdot\pi^5\cdot n^5}$$

$$W_1=\sum\left[\left(A_n+\frac{n\pi y}{a}B_n\right)ch\frac{n\pi y}{a}+\left(C_n+\frac{n\pi y}{a}D_n\right)sh\frac{n\pi y}{a}\right]\sin\frac{n\pi x}{a}$$

$$W=\sum_{n=1}^{\infty}\left[\frac{4p_0a^4}{K\pi^5n^5}+\left(A_n+\frac{n\pi y}{a}B_n\right)ch\frac{n\pi y}{a}+\left(C_n+\frac{n\pi y}{a}D_n\right)sh\frac{n\pi y}{a}\right]\cdot\sin\frac{n\pi x}{a}$$

Granični uslovi:

$$Y=0\begin{cases}W=0 & (1)\\ \frac{\partial W}{\partial Y}=0 & (2)\end{cases}\qquad Y=b\begin{cases}W=0 & (3)\\ M_Y=0 \Leftrightarrow \frac{\partial^2 W}{\partial Y^2}=0 & (4)\end{cases}$$

Jexa

(1):

$$W=\sum_{n=1}^{\infty}\left[\frac{4p_0\cdot a^4}{K\cdot\pi^5\cdot n^5}+A_n ch\,0+C_n\cancel{sh\,0}^{\,0}\right]\cdot\sin\frac{n\pi x}{a}=0$$

$$\Rightarrow\ \frac{4p_0\cdot a^4}{K\cdot\pi^5\cdot n^5}+A_n=0 \qquad (1)$$

(2)

$$\frac{\partial W}{\partial y}=\sum_{n=1}^{\infty}\frac{n\pi}{a}\Big[A_n sh\frac{n\pi y}{a}+B_n\Big(ch\frac{n\pi y}{a}+\frac{n\pi y}{a}sh\frac{n\pi y}{a}\Big)+C_n\cdot ch\frac{n\pi y}{a}+$$

$$+D_n\Big(sh\frac{n\pi y}{a}+\frac{n\pi y}{a}\cdot ch\frac{n\pi y}{a}\Big)\Big]\cdot\sin\frac{n\pi x}{a}=0$$

$$\cancel{A_n sh\,0}^{\,0}+B_n(ch\,0+0)+C_n ch\,0+D_n(\cancel{sh\,0}^{\,0}+0)=0$$

$$B_n+C_n=0 \qquad (2)$$

(3)

$$W=\sum_{n=1}^{\infty}\left[\frac{4p_0a^4}{K\pi^5n^5}+\Big(A_n+\frac{n\pi b}{a}B_n\Big)ch\frac{n\pi b}{a}+\Big(C_n+\frac{n\pi b}{a}D_n\Big)sh\frac{n\pi b}{a}\right]\cdot\sin\frac{n\pi x}{a}=0$$

$$\alpha_n=\frac{n\pi b}{a}\qquad S=\frac{4p_0a^4}{K\pi^5}$$

$$(A_n+\alpha_nB_n)ch\,\alpha_n+(C_n+\alpha_nD_n)sh\,\alpha_n+\frac{S}{n^5}=0 \qquad (3)$$

(4):

$$\frac{\partial^2W}{\partial y^2}=\sum_{n=1}^{\infty}\frac{n^2\pi^2}{a^2}\Big[A_n ch\frac{n\pi y}{a}+B_n\Big(2sh\frac{n\pi y}{a}+\frac{n\pi y}{a}ch\frac{n\pi y}{a}\Big)+$$

$$+C_n sh\frac{n\pi y}{a}+D_n\Big(2ch\frac{n\pi y}{a}+\frac{n\pi y}{a}sh\frac{n\pi y}{a}\Big)\Big]\cdot\sin\frac{n\pi x}{a}=0$$

$$A_n ch\,\alpha_n+B_n(2sh\,\alpha_n+\alpha_n ch\,\alpha_n)+C_n\cdot sh\,\alpha_n+$$

$$+D_n(2ch\,\alpha_n+\alpha_n sh\,\alpha_n)=0 \qquad (4)$$

$$(1)\to\ A_n=-\frac{4p_0a^4}{K\pi^5n^5}=-\frac{S}{n^5}$$

$$(2)\to\ C_n=-B_n$$

$$(4)-(3)\to B_n(2sh\,\alpha_n+\alpha_n ch\,\alpha_n-\alpha_n ch\,\alpha_n)+D_n(2ch\,\alpha_n+\alpha_n sh\,\alpha_n-$$

$$-\alpha_n sh\,\alpha_n)=\frac{S}{n^5}$$

$$B_n\cdot sh\,\alpha_n+D_n ch\,\alpha_n=\frac{S}{n^5}$$

$$D_n = \frac{1}{\operatorname{ch}\alpha_n} \cdot \left(\frac{S}{k^5} - B_n \cdot \operatorname{sh}\alpha_n \right)$$

$$(3) \rightarrow -\frac{S}{k^5} \cdot \operatorname{ch}\alpha_n + \alpha_n B_n \cdot \operatorname{ch}\alpha_n - B_n \operatorname{sh}\alpha_n + \frac{\alpha_n \operatorname{sh}\alpha_n}{\operatorname{ch}\alpha_n} \left(\frac{S}{k^5} - B_n \operatorname{sh}\alpha_n \right) = 0$$

$$-\frac{S}{k^5} \operatorname{ch}\alpha_n + B_n \cdot \alpha_n \operatorname{ch}\alpha_n - B_n \operatorname{sh}\alpha_n - B_n \alpha_n \operatorname{sh}\alpha_n \cdot \operatorname{th}\alpha_n + \frac{S}{k^5} \alpha_n \operatorname{th}\alpha_n = 0$$

$$\frac{S}{k^5} (\alpha_n \operatorname{th}\alpha_n - \operatorname{ch}\alpha_n) + B_n \cdot \operatorname{ch}\alpha_n \, (\alpha_n - \operatorname{th}\alpha_n - \alpha_n \operatorname{th}^2\alpha_n) = 0$$

$$B_n = + \frac{\frac{S}{k^5} (\operatorname{ch}\alpha_n - \alpha_n \operatorname{th}\alpha_n)}{\operatorname{ch}\alpha_n (\alpha_n - \operatorname{th}\alpha_n - \alpha_n \operatorname{th}^2\alpha_n)}$$

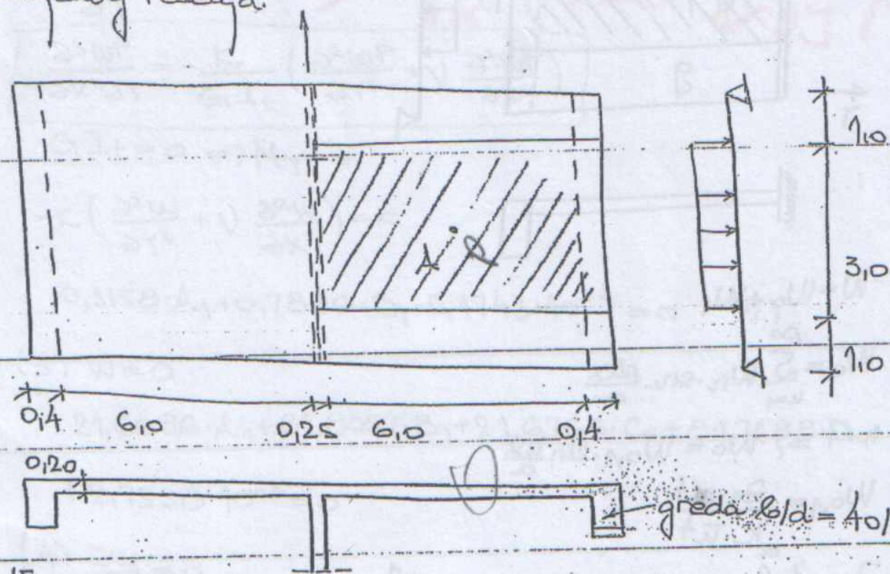
$$D_n = \frac{1}{\operatorname{ch}\alpha_n} \left(\frac{S}{k^5} - \frac{S}{k^5} \cdot \operatorname{th}\alpha_n \, \frac{\operatorname{ch}\alpha_n - \alpha_n \operatorname{th}\alpha_n}{\alpha_n - \operatorname{th}\alpha_n - \alpha_n \operatorname{th}^2\alpha_n} \right)$$

80096563

MORIS-LEVY

(6)
09.02.1998.

1. Usled sopstvene težine konstrukcije i korisnog opterećenja $p=5{,}0\ kN/m^2$, odrediti ugib u tački A koristeći samo prvi član usvojenog rešenja.



$\gamma=25\ kN/m^3$

$\nu=0{,}20$

$E=30\ GPa$

$GI_t=0$

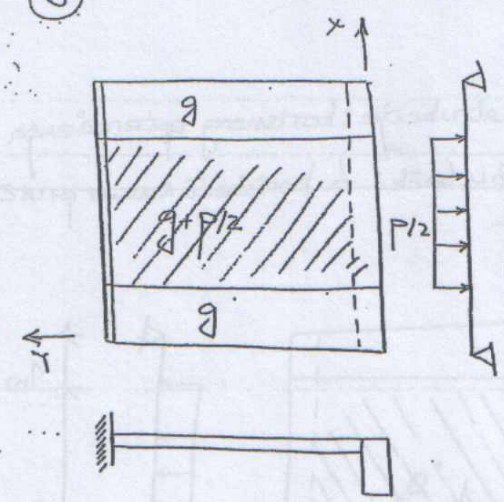
$W_A=W_A^S+W_A^A$

$\boxed{g=0{,}2\cdot 25=5{,}0\ kN/m^2}$

$$K=\frac{E\cdot h^3}{12(1-\nu^2)}=\frac{30\cdot 10^6\cdot 0{,}2^3}{12(1-0{,}2^2)}=20833{,}3$$

$$I_g=\frac{1}{12}\cdot 0{,}4\cdot 0{,}6^3=0{,}0072\ m^4$$

(S)



$$W = W_0 + W_1$$

$$W_0 = \sum_{n=1}^{\infty} W_n \cdot \sin\frac{n\pi x}{a}$$

$$n=1 \Rightarrow W_0 = W_{0,1} \cdot \sin\frac{\pi x}{a}$$

$$W_{0,1} = \frac{Z_1 \cdot a^4}{K \cdot \pi^4}$$

$$Z_1 = \frac{2}{a}\int_0^a Z(x,Y)\sin\frac{\pi x}{a}\,dx = \frac{2}{5}\left[\int_0^1 5{,}0 \cdot \sin\frac{\pi x}{5}\,dx + \int_1^4 7{,}5 \cdot \sin\frac{\pi x}{5}\,dx + \int_4^5 5{,}0 \sin\frac{\pi x}{5}\,dx\right] = 8{,}9414$$

$$W_{0,1} = 2{,}7537 \cdot 10^{-3}$$

$$W^s = \left[2{,}7537 \cdot 10^{-3} + \left(A_1 + \frac{\pi Y}{5} B_1\right) ch\frac{\pi Y}{5} + \left(C_1 + \frac{\pi Y}{5} D_1\right) sh\frac{\pi Y}{5}\right]\sin\frac{\pi x}{5}$$

granični uslovi:

$$Y=0 \begin{cases} W^P = W^G & (1) \\ \dfrac{\partial W^P}{\partial Y} = \varphi^G & (2) \end{cases} \qquad Y = 6{,}0 \begin{cases} W = 0 & (3) \\ \dfrac{\partial W}{\partial Y} = 0 & (4) \end{cases}$$

(1): $W^P = W^G$

$$\boxed{\frac{\partial^4 W^P}{\partial x^4} = \frac{d^4 W^G}{dx^4} = \frac{T_Y}{EI}}$$

$$\frac{\partial^4 W}{\partial x^4}\cdot EI_g = -K\left[\frac{\partial^3 W}{\partial y^3} + (2-\nu)\frac{\partial^3 W}{\partial x^2 \partial y}\right]$$

$$33664{,}5819\cdot A_1 + 6201{,}2553\cdot B_1 - 4134{,}1702\cdot C_1 + 92{,}7042 = 0 \quad (1)$$

(2)
$$\frac{\partial W^P}{\partial y} = \varphi^G \rightarrow \frac{\partial}{\partial x}\left(GI_t \frac{\partial^2 W^P}{\partial x \partial y}\right) = M_y$$

$$\boxed{\frac{\partial^3 W^P}{\partial x^2 \partial y} = -\frac{K}{GI_t}\left(\frac{\partial^2 W^P}{\partial y^2} + \nu\frac{\partial^2 W^P}{\partial x^2}\right)}$$

$$GI_t = 0 \Rightarrow M_y = 0$$

$$-K\left(\frac{\partial^2 W}{\partial y^2} + \nu\frac{\partial^2 W}{\partial x^2}\right) = 0$$

$$0{,}3158\cdot A_1 + 0{,}7896\cdot D_1 - 2{,}1743\cdot 10^{-4} = 0 \quad (2)$$

(3) $W = 0$

$$21{,}6696\cdot A_1 + 81{,}8057\cdot B_1 + 21{,}6766\cdot C_1 + 81{,}7188\cdot D_1 +$$
$$+ 2{,}7538\cdot 10^{-3} = 0$$

(4) $\frac{\partial W}{\partial y} = 0$

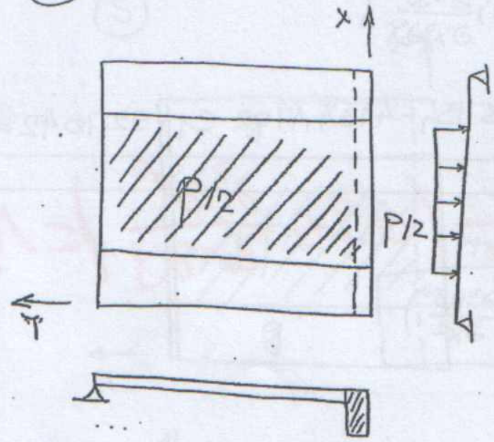
$$13{,}6198\cdot A_1 + 64{,}9797\cdot B_1 + 13{,}6343\cdot C_1 + 65{,}0198\cdot D_1 = 0$$

$$A_1 = -2{,}3335\cdot 10^{-3}$$
$$B_1 = -1{,}0942\cdot 10^{-3}$$
$$C_1 = 1{,}7813\cdot 10^{-3}$$
$$D_1 = 1{,}2088\cdot 10^{-3}$$

$$W^S = \left[2{,}7537\cdot 10^{-3} - (6{,}8748\cdot 10^{-4}\cdot y + 2{,}3335)\cdot ch\frac{\pi y}{5} + (7{,}5949\cdot 10^{-4}\cdot y + 1{,}7813)\cdot sh\frac{\pi y}{5}\right]\cdot \sin\frac{\pi x}{5}$$

$$W^S_A = 1{,}0048\cdot 10^{-3}$$

(A)



$$W = W_0 + W_1$$

$$W_0 = W_{0,1} \cdot \sin\frac{\pi x}{a}$$

$$Z_1 = \frac{2}{a}\int_0^a Z(x,y)\sin\frac{\pi x}{a}dx = \frac{2}{5}\int_1^4 2{,}5\cdot\sin\frac{\pi x}{5}dx = 2{,}5752$$

$$W_{0,1} = \frac{2{,}5752\cdot 5^4}{K\cdot\pi^4} = 7{,}9310\cdot 10^{-4}$$

$$W = \left[7{,}9310\cdot 10^{-4} + \left(A_1 + \frac{\pi y}{5}B_1\right)ch\frac{\pi y}{5} + \left(C_1 + \frac{\pi y}{5}D_1\right)sh\frac{\pi y}{5}\right]\cdot\sin\frac{\pi x}{5}$$

granični uslovi:

$$y=0\ \begin{cases} W^P = W^G & (1) \\ \dfrac{\partial W^P}{\partial y} = \varphi^G & (2)\end{cases} \qquad y=6\ \begin{cases} W^P = 0 \\ M_y = 0\end{cases}$$

(1): $W^P = W^G$

$$\frac{\partial^4 W^P}{\partial x^4} = \frac{d^4 W^G}{dx^4} = \frac{\overline{T}_y}{EI}$$

$$\frac{\partial^4 W}{\partial x^4}EI = -K\left[\frac{\partial^3 W}{\partial y^3} + (2-\nu)\frac{\partial^3 W}{\partial x^2 \partial y}\right]$$

$$33664{,}5819\cdot A_1 + 6201{,}2553\cdot B_1 - 4134{,}1702\cdot C_1 + 26{,}6995 = 0 \quad (1)$$

(2) $\frac{\partial w^P}{\partial y} = \varphi^G$

$GI_t = 0 \Rightarrow M_y = 0$

$-K\left(\frac{\partial^2 w}{\partial y^2} + \nu \frac{\partial^2 w}{\partial x^2}\right) = 0$

$0{,}3158 \cdot A_1 + 0{,}7896 \cdot D_1 - 6{,}2621 \cdot 10^{-5} = 0$ (2)

(3) $w = 0$

$21{,}6696 \cdot A_1 + 81{,}8057 \cdot B_1 + 21{,}6766 \cdot C_1 + 81{,}7188 \cdot D_1 +$
$+7{,}9310 \cdot 10^{-4} = 0$ (3)

(4) $M_y = 0$

$M_y = -K\left(\frac{\partial^2 w}{\partial y^2} + \nu \frac{\partial^2 w}{\partial x^2}\right) = 0$

$6{,}8533 \cdot A_1 + 42{,}9516 \cdot B_1 + 6{,}8461 \cdot C_1 + 42{,}9424 \cdot D_1 -$
$-6{,}2621 \cdot 10^{-5} = 0$ (4)

$A_1 = -6{,}6389 \cdot 10^{-4}$

$B_1 = -3{,}2694 \cdot 10^{-4}$

$C_1 = 5{,}6174 \cdot 10^{-4}$

$D_1 = 3{,}4487 \cdot 10^{-4}$

$$w_1^A = \left[7{,}9310 \cdot 10^{-4} - (2{,}0542 \cdot 10^{-4} \cdot y + 6{,}6390) \cdot ch\frac{\pi y}{5} + (2{,}1669 \cdot 10^{-4} \cdot y + 5{,}6174 \cdot 10^{-4})\, sh\frac{\pi y}{5}\right] \cdot \sin\frac{\pi x}{5}$$

$w_A^A = 3{,}7880 \cdot 10^{-4}$

$w_A = w_A^S + w_A^A = 1{,}3836 \cdot 10^{-3}\ m = 1{,}3836\ mm$