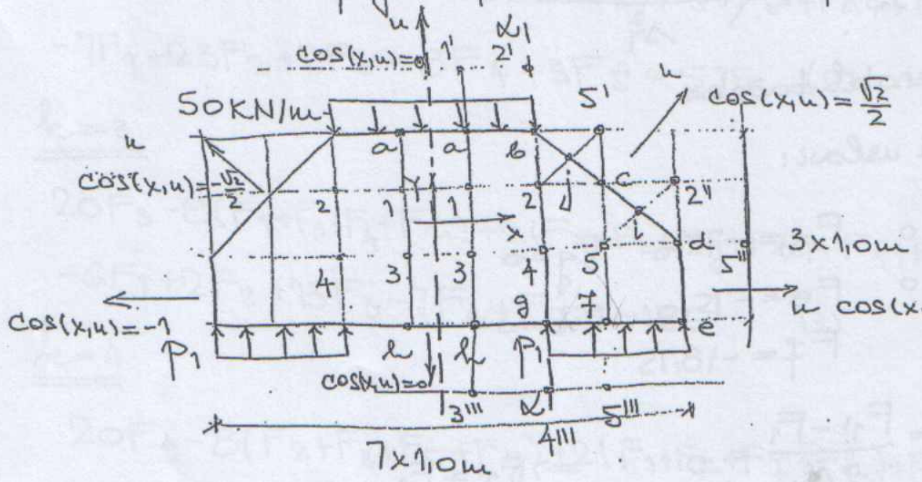
NP

(1)

2. Za ploču napregnutu u svojoj ravni primenom diferencnog postupka:

a) Odrediti vrednost naponske funkcije i njenog izvoda $\frac{\partial F}{\partial n}$ na konturi.

b) Nacrtati dijagram promene sile N_x u preseku α-α

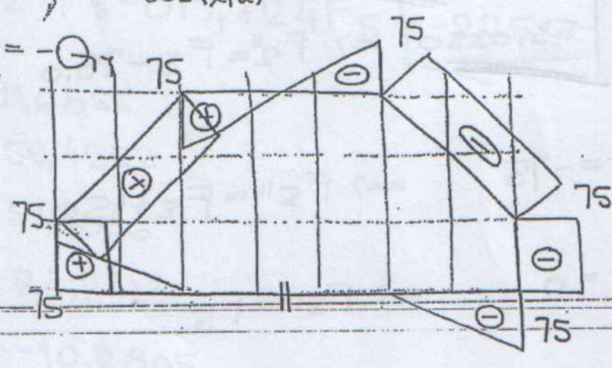


$2 \cdot p_1 \cdot 2{,}0 = 50 \cdot 3 \cdot 1{,}0 \Rightarrow p_1 = 37{,}5 \text{ KN/m}$

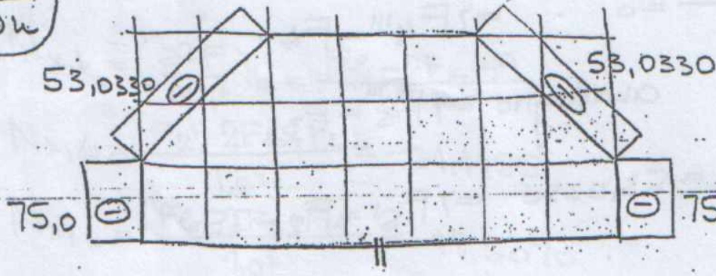
$$\frac{\partial F}{\partial n} = \frac{\partial F}{\partial x} \cdot \frac{\partial x}{\partial n} + \frac{\partial F}{\partial y} \cdot \frac{\partial y}{\partial n}$$

$-Q_y$; $\cos(x,n)$; Q_x ; $\cos(y,n)$

$$\frac{\partial F}{\partial x} = -Q_y$$

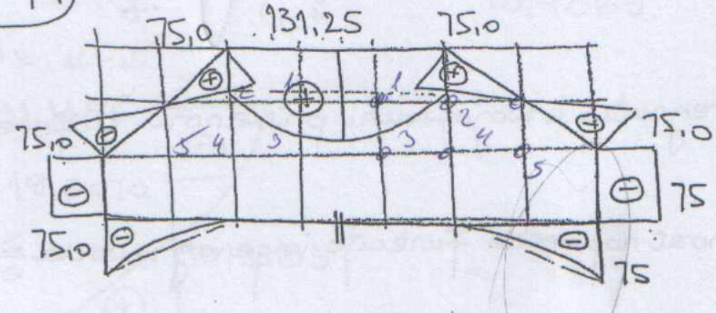


$\frac{\partial F}{\partial n}$



$$\frac{\partial F}{\partial n} = \frac{\partial F}{\partial x} \cdot \frac{\partial x}{\partial n} \quad , Q_x = 0$$

F=M



$$N_{x,k} = \left(\frac{\partial^2 F}{\partial y^2}\right)_k = \frac{F_l - 2F_k + F_i}{\Delta_y^2}$$

5 nepoznatih tačaka

Konturni uslovi:

$F_a = 125{,}0 \quad F_d = -75{,}0 \quad F_g = 0$

$F_b = 75{,}0 \quad F_e = -75{,}0 \quad F_h = 0$

$F_c = 0 \quad F_f = -18{,}75$

$$\left(\frac{\partial F}{\partial u}\right)_a = \frac{F_{1'} - F_1}{2\Delta_y} = 0 \quad \Rightarrow F_{1'} = F_1$$

analogno $\Rightarrow F_{2'} = F_2$

$F_{2''}$ nemože da se odredi preko F_2, mora da se ide preko normale na konturu!

$$\left(\frac{\partial F}{\partial u}\right)_i = \frac{F_{2''} - F_5}{\frac{\sqrt{2}}{2} \cdot 2} = \boxed{\overset{dF/dn}{-53{,}0330}} \Rightarrow F_{2''} = F_5 - 75{,}0$$

$$\left(\frac{\partial F}{\partial u}\right)_d = \frac{F_{5''} - F_5}{2\Delta_x} = -75 \quad \Rightarrow F_{5''} = F_5 + 150$$

$$\left(\frac{\partial F}{\partial u}\right)_f = \frac{F_{5'''} - F_5}{2\Delta_y} = 0 \quad \Rightarrow F_{5'''} = F_5$$

$$\left(\frac{\partial F}{\partial u}\right)_g = \frac{F_{4'''} - F_4}{2\Delta_y} = 0 \quad \Rightarrow F_{4'''} = F_4$$

analogno $\Rightarrow F_{3'''} = F_3$

$$\left(\frac{\partial F}{\partial u}\right)_j = \frac{F_{5'} - F_2}{\frac{\sqrt{2}}{2} \cdot 2} = -53{,}0330 \quad \Rightarrow F_{5'} = F_2 - 75$$

$\underline{\underline{k=1}}$

$$20F_1-8(F_1+F_2+F_3+F_a)+2(F_3+F_4+F_a+F_b)+F_{1'}+F_2+F_c+F_h=0$$

$$13F_1-7F_2-6F_3+2F_4=600 \qquad (1)$$

$\underline{\underline{k=2}}$

$$20F_2-8(F_1+F_4+F_b+F_c)+2(F_3+F_5+\overset{F_2-75{,}0}{\cancel{F_{5'}}}+F_a)+F_1+\overset{F_2}{\cancel{F_{2'}}}+\overset{F_5-75}{\cancel{F_{2''}}}+F_g=0$$

$$-7F_1+23F_2+2F_3-8F_4+3F_5=575 \qquad (2)$$

$\underline{\underline{k=3}}$

$$20F_3-8(F_1+F_3+F_4+F_h)+2(F_1+F_2+F_g+F_i)+F_4+F_5+F_a+\overset{F_3}{\cancel{F_{3'''}}}=0$$

$$-6F_1+2F_2+13F_3-7F_4+F_5=-125 \qquad (3)$$

$\underline{\underline{k=4}}$

$$20F_4-8(F_2+F_3+F_5+F_g)+2(F_1+F_c+F_f+F_h)+F_3+\overset{F_4}{\cancel{F_{4'''}}}+F_b+F_d=0$$

$$2F_1-8F_2-7F_3+21F_4-8F_5=37{,}5 \qquad (4)$$

$\underline{\underline{k=5}}$

$$20F_5-8(F_4+F_c+F_d+F_f)+2(F_2+\overset{F_5-75}{\cancel{F_{2''}}}+F_e+F_g)+F_3+\overset{F_2-75}{\cancel{F_{5'}}}+\overset{F_5+150}{\cancel{F_{5''}}}+\overset{F_5}{\cancel{F_{5'''}}}=$$

$$3F_2+F_3-8F_4+24F_5=-225 \qquad (5)$$

$F_1=91{,}6855$

$F_2=59{,}4252$

$F_3=37{,}4036$

$F_4=24{,}2434$

$F_5=-10{,}2805$

$$N_{x,k}=\left(\frac{\partial^2 F}{\partial y^2}\right)_k=\frac{F_l-2F_k+F_i}{\Delta_y^2}$$

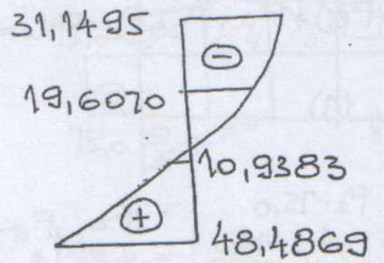
$$N_{x,b}=\frac{F_{2'}-2F_b+F_2}{1{,}0^2}=-31{,}1495$$

$$N_{x,2}=\frac{F_b-2F_2+F_4}{1{,}0^2}=-19{,}6070$$

$$N_{x,4}=\frac{F_2-2F_4+F_g}{1{,}0^2}=10{,}9383$$

$$N_{x,9} = \frac{F_4 - 2F_9 + F_4'''}{1{,}0^2} = 48{,}4869$$

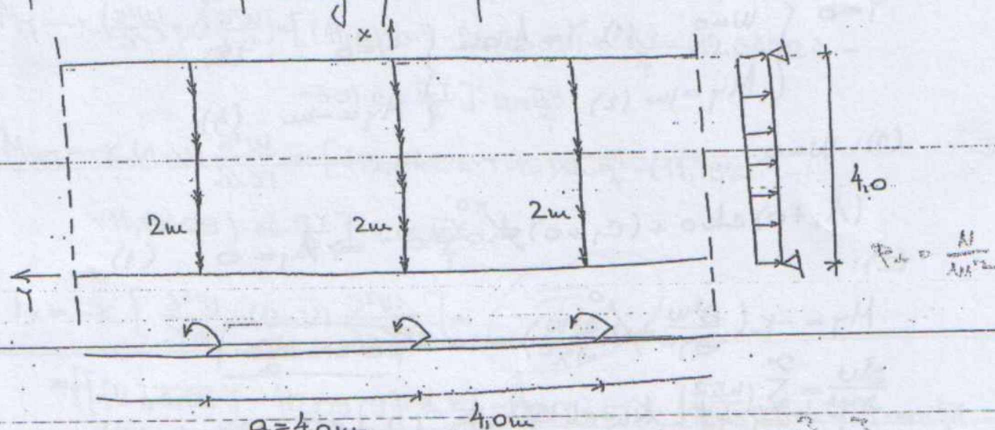
$N_{x,\alpha-\alpha}$



⑤

22.05.2004.

1. Za kontinualnu ploču opterećenu prema slici odrediti rešenje za ugib i presečne sile u proizvoljnoj tački ploče.



$E = 30\ GPa \quad h = 0{,}15m$

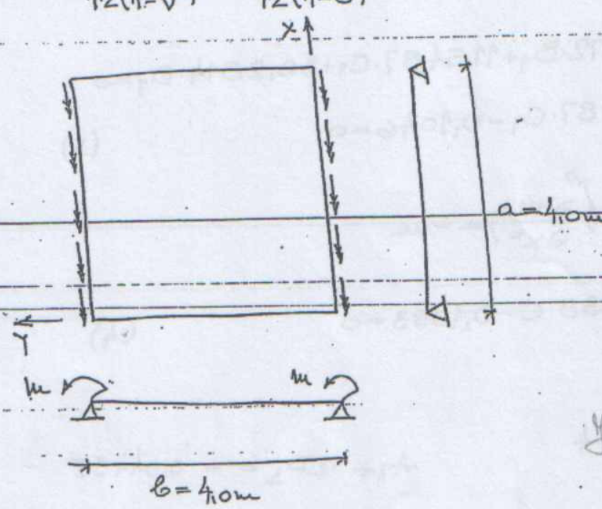
$\nu = 0$

$m_0 = 30 KNm/m$

$m = m_0 \cdot \sin\frac{\pi x}{a}$

$K = \frac{E \cdot h^3}{12(1-\nu^2)} = \frac{30 \cdot 10^6 \cdot 0{,}15^3}{12(1-0)} = 8437{,}50$

$\frac{30\ GPa \cdot 0{,}15^3\ m^3}{m^2} = \frac{30 \cdot 10^9 \frac{N}{m^2} \cdot 0{,}15^3\ m^3}{m^2} =$



$y=0: \begin{cases} M_y = m \\ w = 0 \end{cases}$

$y=b: \begin{cases} M_y = -m \\ w = 0 \end{cases}$

$W = W_o + W_1$

$\boxed{W_o = 0}$

$$W = \sum_{n=1}^{\infty} \left[\left(A_1 + \frac{n\pi y}{a} B_1\right) \cdot ch \frac{n\pi y}{a} + \left(C_1 + \frac{n\pi y}{a} D_1\right) \cdot sh \frac{n\pi y}{a} \right] \cdot \sin \frac{n\pi x}{a}$$

granični uslovi:

$$Y = 0 \begin{cases} W = 0 & (1) \\ M_Y = m & (2) \end{cases} \qquad Y = 4{,}0m \begin{cases} W = 0 & (3) \\ M_Y = -m & (4) \end{cases}$$

(1): $W = 0$

$$(A_1 + 0)\, ch\, 0 + (C_1 + 0)\, \cancel{sh}\, 0 = 0 \Rightarrow A_1 = 0 \qquad (1)$$

(2):

$$M_Y = -K\left(\frac{\partial^2 W}{\partial Y^2} + \cancel{\nu} \frac{\partial^2 W}{\partial X^2} \right) = m = \boxed{30 \cdot \sin \frac{n\pi x}{a}}$$

$$\frac{\partial^2 W}{\partial Y^2} = \sum_{n=1}^{\infty} \left(\frac{n\pi}{4}\right)^2 \left[A_n \cdot ch \frac{n\pi y}{4} + B_n \left(2\, sh \frac{n\pi y}{a} + \frac{n\pi y}{a} ch \frac{n\pi y}{a} \right) + \right.$$
$$\left. + C_n \cdot sh \frac{n\pi y}{4} + D_n \left(2\, ch \frac{n\pi y}{4} + \frac{n\pi y}{4} ch \frac{n\pi y}{4} \right) \right] \cdot \sin \frac{n\pi x}{4}$$

$$\frac{\pi^2}{16} \left[A_1 ch\, 0 + B_1 (2 \cdot \cancel{sh}\, 0 + 0) + C_1 \cancel{sh}\, 0 + D_1 (2\, ch\, 0 + 0) \right] = -\frac{30}{K}$$

$$A_1 + 2D_1 = -\frac{30}{K} \cdot \frac{16}{\pi^2} \Rightarrow D_1 = -2{,}8820 \cdot 10^{-3} \qquad (2)$$

(3): $W = 0$

$$11{,}5915 \cdot A_1 + 36{,}4172 \cdot B_1 + 11{,}5487 \cdot C_1 + 36{,}2814 \cdot D_1 = 0$$

$$36{,}4172 \cdot B_1 + 11{,}5487 \cdot C_1 - 0{,}1046 = 0 \qquad (3)$$

(4):

$$M_Y = -K\left(\frac{\partial^2 W}{\partial Y^2} + \cancel{\nu} \frac{\partial^2 W}{\partial X^2} \right) = -m$$

$$36{,}7116 \cdot B_1 + 7{,}1238 \cdot C - 0{,}1093 = 0 \qquad (4)$$

$B_1 = 3{,}1424 \cdot 10^{-3}$

$C_1 = -8{,}5481 \cdot 10^{-4}$

$A_1 + 2D_1 = -0{,}04495$

$-0{,}0252 + 2D_1 = -0{,}04495$

$2D_1 = -0{,}05475$

$D_1 = -0{,}007875$

$$W = \left[2{,}4680 \cdot ch\frac{\pi y}{4} - (2{,}2635 \cdot 10^{-3} \cdot y + 8{,}5481 \cdot 10^{-4}) \cdot sh\frac{\pi y}{4}\right] \cdot \sin\frac{\pi x}{4}$$

$$M_x = -K\left(\frac{\partial^2 w}{\partial x^2} + \overset{0}{\nu}\frac{\partial^2 w}{\partial y^2}\right) = -\left[12{,}8452 \cdot y \cdot ch\frac{\pi y}{4} - (11{,}7810 \cdot y + 4{,}4490) \cdot sh\frac{\pi y}{4}\right] \sin\frac{\pi x}{4}$$

$$M_y = -K\left(\frac{\partial^2 w}{\partial y^2} + \overset{0}{\nu}\frac{\partial^2 w}{\partial x^2}\right) = \left[(11{,}7810 \cdot y - 28{,}2609) \cdot ch\frac{\pi y}{4} - (12{,}8452 \cdot y - 30) \cdot ch\frac{\pi y}{5}\right] \cdot \sin\frac{\pi x}{4}$$

$$M_{xy} = -K(1-\nu)\frac{\partial^2 w}{\partial x \partial y} = -\left[(12{,}8452 \cdot y - 15) \cdot sh\frac{\pi y}{4} - (11{,}7810 \cdot y - 11{,}9059) \cdot ch\frac{\pi y}{4}\right] \cdot \cos\frac{\pi x}{4}$$

$$T_x = -K\left[\frac{\partial^3 w}{\partial x^3} + (2-\nu)\frac{\partial^3 w}{\partial x \partial y^2}\right] =$$

$$= \left[(27{,}7583 \cdot y - 40{,}8979) \cdot sh\frac{\pi y}{4} + (30{,}2657 \cdot y - 47{,}1239) \cdot ch\frac{\pi y}{4}\right] \cdot \cos\frac{\pi x}{4}$$

$$T_y = \left[(10{,}0886 \cdot y + 11{,}7810) \cdot sh\frac{\pi y}{4} - (9{,}2527 \cdot y + 16{,}3394) \cdot ch\frac{\pi y}{4}\right] \cdot \sin\frac{\pi x}{4}$$

(18)

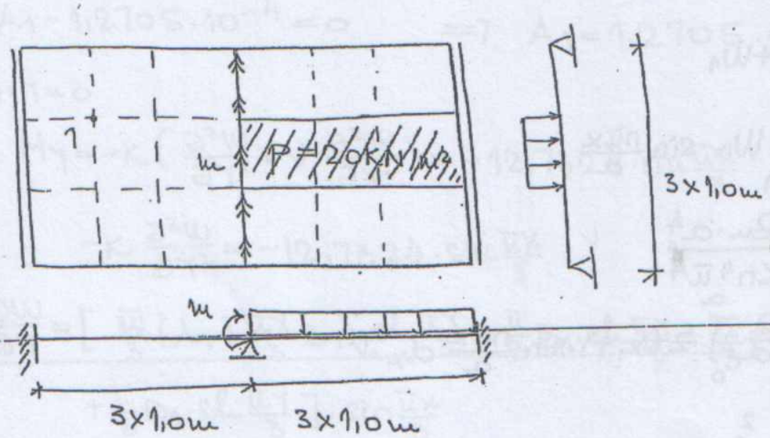
22.04.2008.

1. Za konstrukciju prikazanu na slici:

- Rastaviti opterećenje na simetričan i antimetričan deo.
- Odrediti ugib u tački 1:

a) Usled antimetričnog dela opterećenja analitički. Koristiti prvi član reda usvojenog rešenja.

b) Usled simetričnog dela opterećenja primenom diferencnog postupka



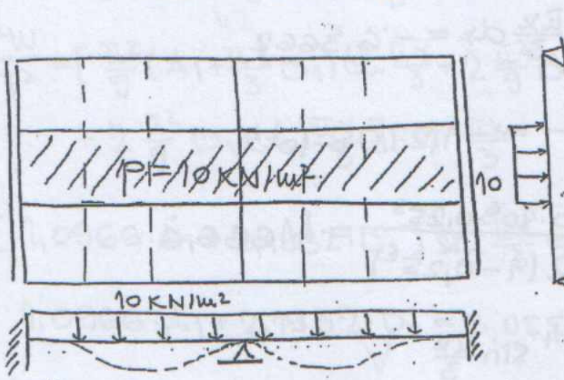
$E = 30 GPa$

$\nu = 0{,}25$

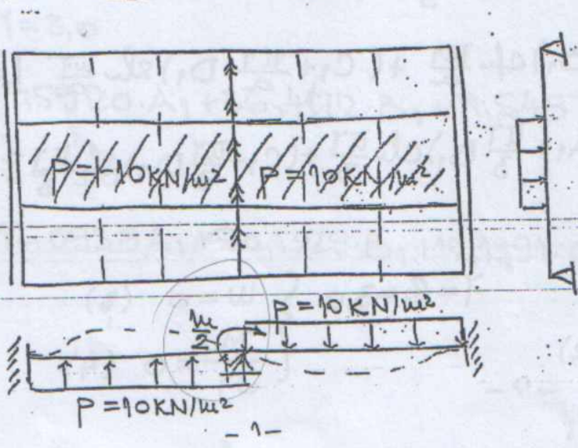
$h = 25 cm$

$m = 20 KNm/m'$

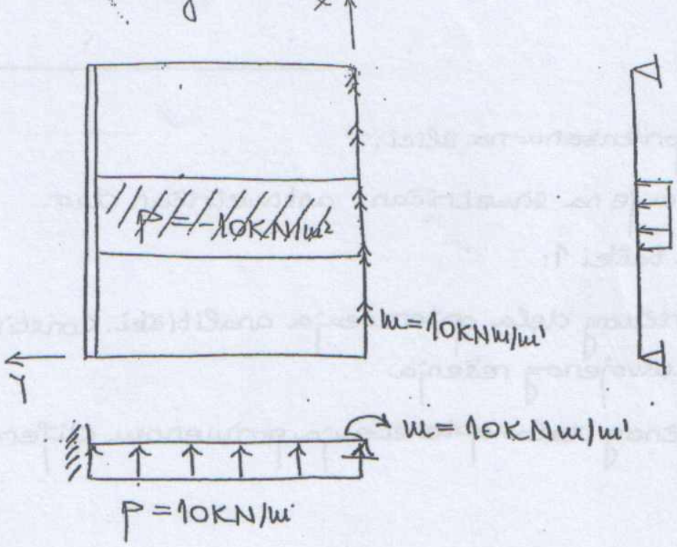
(S)



(A)



a) M. Levy



$$W = W_0 + W_1$$

$$W_0 = \sum_{n=1}^{\infty} W_n \cdot \sin\frac{n\pi x}{a}$$

$$W_n = \frac{Z_n \cdot a^4}{K n^4 \pi^4}$$

$$Z_n = \frac{2}{a}\int_0^a z(x,y) \cdot \sin\frac{n\pi x}{a}\, dx$$

$n = 1$

$$Z_1 = \frac{2}{3}\int_1^2 -10 \cdot \sin\frac{\pi x}{3}\, dx = -6{,}3662$$

$$W_1 = \frac{-6{,}3662 \cdot 3^4}{K \cdot \pi^4} = -1{,}2705 \cdot 10^{-4}$$

$$K = \frac{E \cdot h^3}{12(1-\nu^2)} = \frac{30 \cdot 10^6 \cdot 0{,}25^3}{12(1-0{,}25^2)} = 41666{,}\dot{6}$$

$$W_0 = -1{,}2705 \cdot 10^{-4} \cdot \sin\frac{\pi x}{3}$$

$$W_1 = \left[\left(A_1 + \frac{\pi y}{a} B_1\right) ch\frac{\pi y}{a} + \left(C_1 + \frac{\pi y}{a} D_1\right) sh\frac{\pi y}{a}\right] \sin\frac{\pi x}{a}$$

$$W = \left[-1{,}2705 \cdot 10^{-4} + \left(A_1 + \frac{\pi y}{3} B_1\right) ch\frac{\pi y}{3} + \left(C_1 + \frac{\pi y}{3} D_1\right) sh\frac{\pi y}{3}\right] \sin\frac{\pi x}{3}$$

granični uslovi:

$$y = 0 \begin{cases} W = 0 & (1) \\ M_y = -m & (2) \end{cases} \qquad y = b = 3{,}0 \begin{cases} W = 0 & (3) \\ \dfrac{\partial W}{\partial y} = 0 & (4) \end{cases}$$

Razvoj momenta u red:

$$m = \sum_{n=1}^{\infty} m_n \cdot \sin\frac{n\pi x}{a}$$

$n=1$ $$m_1 = \frac{2}{a}\int_0^a m\cdot\sin\frac{\pi x}{a}\,dx = \frac{2}{3}\int_0^3 10\cdot\sin\frac{\pi x}{3}\,dx = 12{,}7324$$

$$m = 12{,}7324\cdot\sin\frac{\pi x}{3}$$

(1): $Y=0$

$$[-1{,}2705\cdot10^{-4} + (A_1+0\cdot B_1)\,\overset{1}{ch(0)} + (C_1+0\cdot D_1)\,\overset{0}{sh(0)}]\sin\frac{\pi x}{3} = 0$$

$$A_1 - 1{,}2705\cdot10^{-4} = 0 \quad \Rightarrow \quad A_1 = 1{,}2705\cdot10^{-4}$$

(2): $Y=0$

$$M_Y = -K\left(\frac{\partial^2 w}{\partial Y^2} + \nu\frac{\partial^2 w}{\partial x^2}\right) = -12{,}7324\cdot\sin\frac{\pi x}{3}$$

$$-K\,\frac{\partial^2 w}{\partial Y^2} = -12{,}7324\cdot\sin\frac{\pi x}{3}$$

$$\frac{\partial w}{\partial Y} = \left[\frac{\pi}{3}\left(A_1+\frac{\pi Y}{3}B_1\right)\cdot sh\frac{\pi Y}{3} + \frac{\pi}{3}B_1\cdot ch\frac{\pi Y}{3} + \frac{\pi}{3}\left(C_1+\frac{\pi Y}{3}D_1\right)ch\frac{\pi Y}{3} + \right.$$
$$\left. + \frac{\pi}{3}D_1\cdot sh\frac{\pi Y}{3}\right]\cdot\sin\frac{\pi x}{3}$$

$$\frac{\partial^2 w}{\partial Y^2} = \left[\frac{\pi^2}{9}\left(A_1+\frac{\pi Y}{3}B_1\right)ch\frac{\pi Y}{3} + 2\frac{\pi^2}{9}B_1\,sh\frac{\pi Y}{3} + \frac{\pi^2}{9}\left(C_1+\frac{\pi Y}{3}D_1\right)sh\frac{\pi Y}{3} + \right.$$
$$\left. + 2\frac{\pi^2}{9}D_1\cdot ch\frac{\pi Y}{3}\right]\cdot\sin\frac{\pi x}{3}$$

$$[1{,}0966\cdot A_1 + 2{,}1932\cdot D_1]\sin\frac{\pi x}{3} = \frac{1}{K}\cdot12{,}7324\cdot\sin\frac{\pi x}{3}$$

$$1{,}0966\cdot A_1 + 2{,}1932\cdot D_1 = 3{,}0558\cdot10^{-4} \qquad (2)$$

(3) $Y=3{,}0$

$$11{,}5920\cdot A_1 + 36{,}4172\cdot B_1 + 11{,}5487\cdot C_1 + 36{,}2814\,D_1 = 1{,}2705\cdot10^{-4} \qquad (3)$$

(4) $Y=3{,}0$

$$12{,}0938\,A_1 + 50{,}1329\cdot B_1 + 12{,}1391\cdot C_1 + 50{,}2298\cdot D_1 = 0 \qquad (4)$$

$$A_1 = 1{,}2705 \cdot 10^{-4}$$

$$B_1 = -8{,}7627 \cdot 10^{-5}$$

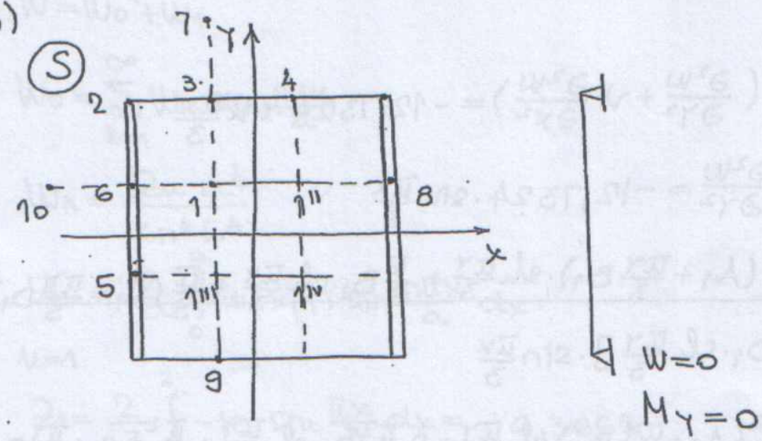
$$C_1 = -7{,}8342 \cdot 10^{-5}$$

$$D_1 = 7{,}5801 \cdot 10^{-5}$$

$$W = \left[-1{,}2705 \cdot 10^{-4} + (1{,}2705 \cdot 10^{-4} - 9{,}1763 \cdot 10^{-5} \cdot Y)\, ch\, \frac{\pi Y}{3} + (-7{,}8342 \cdot 10^{-5} + 7{,}9379 \cdot 10^{-5} \cdot Y) \cdot sh\, \frac{\pi Y}{3}\right] \sin \frac{\pi x}{3}$$

$$W(x=2, Y=2) = -3{,}3148 \cdot 10^{-5}\ m$$

b)



$$W_1 = W_{1''} = W_{1'''} = W_{1IV}$$

$$W_2 = W_3 = W_4 = W_8 = W_9 = W_5 = W_6 = 0$$

$$\left(\frac{\partial W}{\partial x}\right)_6 = \frac{W_1 - W_{10}}{2\Delta x} = 0 \implies W_{10} = W_1$$

$$(M_Y)_3 = 0 \iff \left(\frac{\partial^2 W}{\partial Y^2}\right)_3 = \frac{W_7 - 2W_3 + W_1}{\Delta Y^2} = 0 \implies W_7 = -W_1$$

<u>**k=1**</u>

$$20 \cdot W_1 - 8(\overset{W_1}{\cancel{W_{1''}}} + \overset{W_1}{\cancel{W_{1'''}}} + \overset{0}{\cancel{W_3}} + \overset{0}{\cancel{W_6}}) + 2(\overset{0}{\cancel{W_2}} + \overset{0}{\cancel{W_4}} + \overset{W_1}{\cancel{W_{1IV}}} + \overset{0}{\cancel{W_5}}) +$$

$$+ \overset{-W_1}{\cancel{W_7}} + \overset{0}{\cancel{W_8}} + \overset{0}{\cancel{W_9}} + \overset{W_1}{\cancel{W_{10}}} = \frac{Z_k \cdot \Delta^4}{K}$$

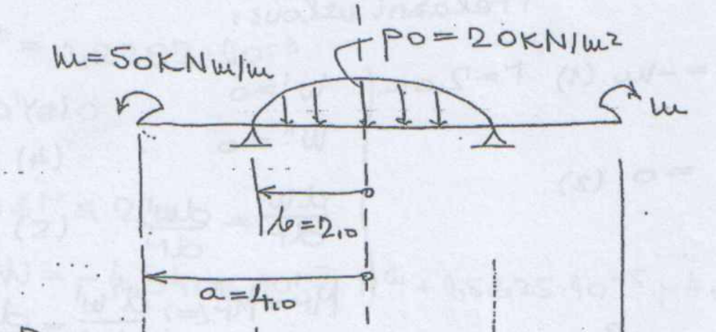
$$20W_1 - 16W_1 + 2W_1 = \frac{10 \cdot 1{,}0^4}{41666{,}\dot{6}}$$

$$W_1 = 4{,}0 \cdot 10^{-5} \ m$$

X (15)

22.06.1998.

1. Za kružnu ploču prikazanu na slici odrediti ugib u tački A

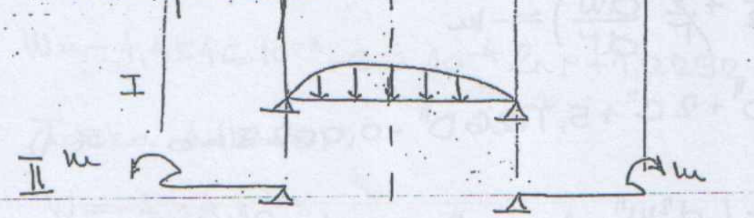


$E = 30\,GPa$

$\nu = 0$

$h = 0{,}20\,m$

Dekompozicija:



$W^{I} = W_0^{I} + W_1^{I}$

$W^{II} = \cancel{W_0^{II}} + W_1^{II}$ — $\int^{+\mu} Ax^2 + Bx + C$ iz upoređne

$M = -\int \frac{dr}{r} \int r\, Z(r)\, dr$

$p(0) = p_0 = 20 \qquad a_1 \cdot 0^2 + a_2 \cdot 0 + a_3 = 20 \Rightarrow a_3 = 20$

$p(2) = 0 \qquad 4a_1 + 4a_2 + 20 = 0$

opšti uslov: $\boxed{p'(0) = 0} \qquad 2a_1 \cdot 0 + a_2 = 0 \quad a_2 = 0$

$a_1 = -5$

$p(r) = -5r^2 + 20$

$M = -\int \frac{dr}{r} \int (-5r^3 + 20r)\, dr = -\int \frac{dr}{r} \left(-\frac{5}{4} r^4 + 10 r^2\right)$

$M = +\frac{5}{16} r^4 - 5r^2$

$W_0 = -\frac{1}{K} \int \frac{dr}{r} \int r M\, dr = +\int \frac{dr}{r} \int \left(5r^3 - \frac{5}{16} r^5\right) dr$

$W_0 = -4{,}3403 \cdot 10^{-7} \cdot r^6 + 0{,}15625 \cdot 10^{-4} \cdot r^4$

$$K = \frac{E \cdot h^3}{12(1-\nu^2)} = 20000$$

$$W^I = -4{,}3403 \cdot 10^{-7} \cdot r^6 + 0{,}15625 \cdot 10^{-4} \cdot r^4 + A' + \cancel{B'}^{0} \cdot \ln r + C' r^2 + \cancel{D'}^{0} r^2 \ln r$$

$$W'' = A'' + B'' \ln r + C'' r^2 + D'' r^2 \ln r$$

granični uslovi: Prelazni uslovi:

$$r = 4{,}0m \begin{cases} M_r'' = -m & (1) \\ T_r'' = 0 & (2) \end{cases} \qquad r = 2{,}0m \begin{cases} w' = 0 & (3) \\ w'' = 0 & (4) \\ \frac{dw'}{dr} = \frac{dw''}{dr} & (5) \\ M_r' = M_r'' \Leftrightarrow \frac{d^2w'}{dr^2} = \frac{d^2w''}{dr^2} & (6) \end{cases}$$

(1):

$$-K\left(\frac{d^2w''}{dr^2} + \cancel{\frac{\nu}{r}}^{0}\frac{dw}{dr}\right) = -m$$

$$-0{,}0625\, B'' + 2C'' + 5{,}7726\, D'' - 0{,}0025 = 0 \qquad (1)$$

(2):

$$T_r = -K \frac{d}{dr}\left(\frac{d^2w''}{dr^2} + \frac{1}{r}\frac{dw''}{dr}\right)$$

$$T_r = -K\left(\frac{d^3w''}{dr^3} + \frac{1}{r}\frac{d^2w''}{dr^2} - \frac{1}{r^2}\frac{dw}{dr}\right)$$

$$T_r'' = -K(D'') = 0$$

$$D'' = 0 \qquad (2)$$

(3): $w' = 0$

$$A' + 4C' + 2{,}2 \cdot 10^{-4} = 0 \qquad (3)$$

(4): $w'' = 0$

$$A'' + 0{,}6932 \cdot B'' + 4C'' + 2{,}7726 \cdot D'' = 0 \qquad (4)$$

(5):

$$4 \cdot C' - 0{,}5 \cdot B'' - 4C'' - 4{,}7726 \cdot D'' + 4{,}16 \cdot 10^{-4} = 0 \qquad (5)$$

(6):

$$2C' + 0{,}25 B'' - 2C'' - 4{,}3863 \cdot D'' + 5{,}416 \cdot 10^{-4} = 0 \qquad (6)$$

$A' = -4{,}38\dot{}\cdot 10^{-3}$

$C' = 1{,}041\dot{6}\cdot 10^{-3}$

$A'' = -4{,}4546\cdot 10^{-3}$

$B'' = -6{,}\dot{6}\cdot 10^{-4}$

$C'' = 1{,}2292\cdot 10^{-3}$

$D'' = 0$

$0 \le r \le 2{,}0$

$W = -4{,}3403\cdot 10^{-7}\cdot r^6 + 1{,}5625\cdot 10^{-5}\cdot r^4 + 1{,}041\dot{6}\cdot 10^{-3} r^2 - 4{,}38\dot{}\cdot 10^{-3}$

$2{,}0 \le r \le 4{,}0$

$W = -4{,}4546\cdot 10^{-3} - 6{,}\dot{6}\cdot 10^{-4}\ln r + 1{,}2292\cdot 10^{-3}\cdot r^2$

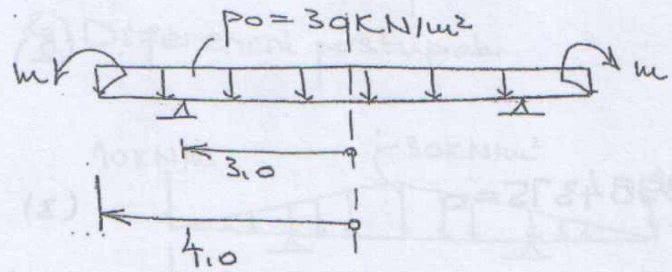
Tačka $A(r=0)$

$W = -4{,}38\dot{}\cdot 10^{-3}\ \text{m}$

17

15.04.2003.

1. Za kružnu ploču prikazanu na slici odrediti moment savijanja m, tako da ugib u centru ploče iznosi 2cm.

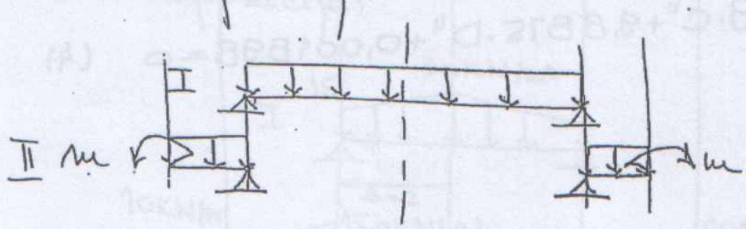


$E = 30\,GPa$

$\nu = 0$

$h = 0,20\,m$

Dekompozicija



$$M = -\int \frac{dr}{r}\int r\,Z(r)\,dr = -30\int \frac{1}{r}\cdot\frac{r^2}{2}\,dr = -30\,\frac{r^2}{4} = -7,5\,r^2$$

$$W_0 = -\frac{1}{K}\int \frac{dr}{r}\int r\,M\,dr = \frac{7,5}{K}\int \frac{dr}{r}\cdot\frac{r^4}{4} = \frac{7,5}{16K}\cdot r^4$$

$$K = \frac{30\cdot 10^6\cdot 0,2^3}{12(1-0^2)} = 20000$$

$$W_0 = 2,34375\cdot 10^{-5}\cdot r^4$$

$$W' = 2,34375\cdot 10^{-5}\cdot r^4 + A' + \cancel{B'}^{\,0}\ln r + C' r^2 + \cancel{D'}^{\,0} r^2 \ln r$$

$$W'' = 2,34375\cdot 10^{-5}\cdot r^4 + A'' + B''\ln r + C'' r^2 + D'' r^2 \ln r$$

Granični uslovi:

$$r = 4,0 \begin{cases} M_r'' = -m & (1) \\ T_r'' = 0 & (2) \end{cases} \qquad r = 3,0 \begin{cases} W' = 0 & (3) \\ W'' = 0 & (4) \\ \dfrac{dw'}{dr} = \dfrac{dw''}{dr} & (5) \\ M_r' = M_r'' \Leftrightarrow \dfrac{d^2w'}{dr^2} = \dfrac{d^2w''}{dr^2} & (6) \end{cases}$$

(1):

$$M_r'' = -K\left(\frac{d^2w}{dr^2} + \cancel{\frac{2}{r}}\frac{dw}{dr}\right) = -m$$

$$-0{,}0625\,B'' + 2C'' + 5{,}7726\cdot D'' + 0{,}0045 - \frac{m}{K} = 0 \qquad (1)$$

(2):

$$T_r = -K\frac{d}{dr}\left(\frac{d^2w}{dr^2} + \frac{1}{r}\frac{dw}{dr}\right) = -K\left(\frac{d^3w}{dr^3} + \frac{1}{r}\frac{d^2w}{dr^2} - \frac{1}{r^2}\frac{dw}{dr}\right) = 0$$

$$D'' + 0{,}003 = 0 \qquad (2)$$

(3):

$$W' = 0$$

$$A' + 9C' + 0{,}001898\,4375 = 0 \qquad (3)$$

(4):

$$W'' = 0$$

$$A'' + 1{,}0986\cdot B'' + 9\cdot C'' + 9{,}8875\cdot D'' + 0{,}001898 = 0 \qquad (4)$$

(5):

$$\frac{dw'}{dr} = \frac{dw''}{dr}$$

$$6C' - 0{,}\dot{3}\cdot B'' - 6C'' - 9{,}5917\cdot D'' = 0 \qquad (5)$$

(6):

$$\frac{d^2w'}{dr^2} = \frac{d^2w''}{dr^2}$$

$$2\cdot C' + 0{,}\dot{1}\cdot B'' - 2C'' - 5{,}1972\cdot D'' = 0 \qquad (6)$$

$$A' = -(0{,}000225\cdot m - 4{,}6779\cdot 10^{-3})$$

$$C' = 0{,}000025\cdot m - 7{,}3070\cdot 10^{-4}$$

$$A'' = -(0{,}000225\cdot m - 0{,}00734)$$

$$B'' = -0{,}027$$

$$C'' = 0{,}000025\,m + 5{,}5651\cdot 10^{-3}$$

$$D'' = -0{,}003$$

$$W(r=0) = -0{,}000225\cdot m + 4{,}6779\cdot 10^{-3} = 0{,}02$$

$$m = -68{,}0982\ \mathrm{KNm}$$

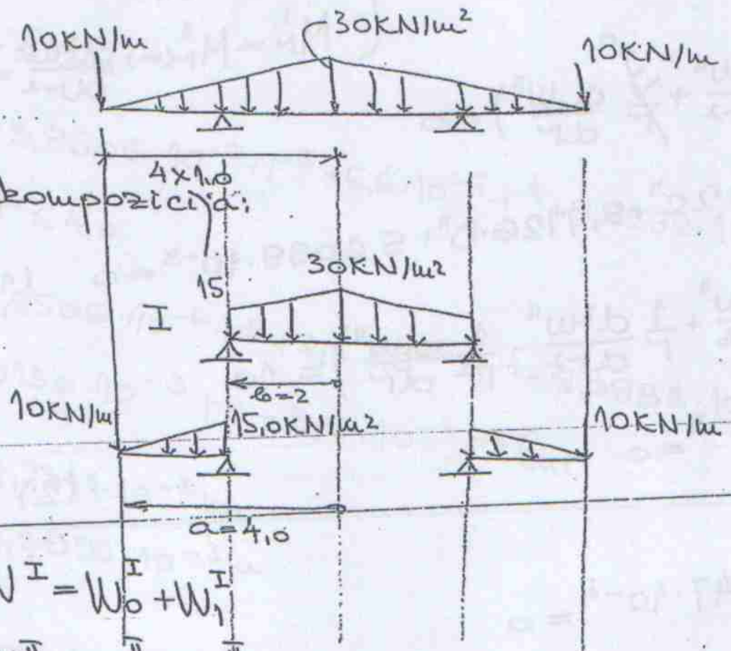
(16)

23.11.2002.

1. Za kružnu ploču prikazanu na slici odrediti ugibe i momente u označenim tačkama primenjujući:

a) Analitičko rešenje

b) Diferencni postupak



$E = 30GPa$

$\nu = 0$

$h = 0{,}15m$

Dekompozicija:

$W^{I} = W_0^{I} + W_1^{I}$

$W^{II} = W_0^{II} + W_1^{II}$

$W_0 = -\frac{1}{K}\int \frac{dr}{r}\int r M\, dr$

$M = -\int \frac{dr}{r}\int r\, z(r)\, dr$

I ploča

$z(r) = 30 - 7{,}5 \cdot r$

$M = -\int \frac{dr}{r}\int r(30-7{,}5r)\,dr = -\int \frac{dr}{r}\left(15r^2 - 2{,}5\cdot r^3\right) = -\left(\frac{15}{2}r^2 - \frac{5}{6}r^3\right)$

$W_0 = +\frac{1}{K}\int \frac{dr}{r}\left(\frac{15}{8}r^4 - \frac{5}{30}r^5\right) = \frac{1}{K}\left(\frac{15}{32}r^4 - \frac{5}{150}r^5\right)$

$K = 8437{,}5$

$W_0^{I} = -3{,}9506\cdot 10^{-6}\cdot r^5 + 5{,}5\cdot 10^{-5}\cdot r^4$

$$W^{I} = -3{,}9506\cdot 10^{-6}\cdot r^5 + 5{,}\dot{5}\cdot 10^{-5}\cdot r^4 + A' + C'\cdot r^2$$

$$W'' = -3{,}9506\cdot 10^{-6}\cdot r^5 + 5{,}\dot{5}\cdot 10^{-5}\, r^4 + A'' + B''\ln r + C'' r^2 + D'' r^2 \ln r$$

granični uslovi:

Prelazni uslovi:

$$r=4 \begin{cases} M_r'' = 0 & (1) \\ T_r'' = 10\,kN/m & (2) \end{cases} \qquad r = 2{,}0 \begin{cases} W' = 0 & (3) \\ W'' = 0 & (4) \\ \dfrac{dW'}{dr} = \dfrac{dW''}{dr} & (5) \\ M_r' = M_r'' \Leftrightarrow \dfrac{d^2W'}{dr^2} = \dfrac{dW''}{dr^2} & (6) \end{cases}$$

(1):
$$M_r'' = -K\left(\frac{d^2W''}{dr^2} + \frac{\cancel{\nu}^{\,0}}{r}\frac{dW''}{dr}\right) = 0$$

$$-0{,}0625\cdot B'' + 2C'' + 5{,}7726\cdot D'' + 5{,}6099\cdot 10^{-3} = 0 \qquad (1)$$

(2):
$$T_r'' = -K\left(\frac{d^3W''}{dr^3} + \frac{1}{r}\frac{d^2W''}{dr^2} - \frac{1}{r^2}\frac{dW''}{dr}\right) = 10$$

$$D'' + 3{,}\dot{5}\cdot 10^{-3} = 0 \qquad (2)$$

(3): $W' = 0$

$$A' + 4C' + 7{,}6247\cdot 10^{-4} = 0 \qquad (3)$$

(4):

$W'' = 0$

$$A'' + 0{,}6932\cdot B'' + 4C'' + 2{,}7726\cdot D'' + 7{,}6247\cdot 10^{-4} = 0 \qquad (4)$$

(5)
$$\frac{dW'}{dr} = \frac{dW''}{dr}$$

$$4C' - 0{,}5B'' - 4C'' - 4{,}7726\cdot D'' = 0 \qquad (5)$$

(6)
$$\frac{d^2W'}{dr^2} = \frac{d^2W''}{dr^2}$$

$$2C' + 0{,}25B'' - 2C'' - 4{,}3863\cdot D'' = 0 \qquad (6)$$

$A' = -4{,}7341 \cdot 10^{-3}$

$C' = 9{,}9292 \cdot 10^{-4}$

$A'' = -9{,}0983 \cdot 10^{-3}$

$B'' = -1{,}4\dot{2} \cdot 10^{-2}$

$C'' = 7{,}0130 \cdot 10^{-3}$

$D'' = -3{,}\dot{5} \cdot 10^{-3}$

$0 \le r \le 2{,}0$

$W = -3{,}9506 \cdot 10^{-6} \cdot r^5 + 5{,}\dot{5} \cdot 10^{-5} \cdot r^4 + 9{,}9292 \cdot 10^{-4} r^2 - 4{,}7341 \cdot 10^{-3}$

$2{,}0 \le r \le 4{,}0$

$W = -3{,}9506 \cdot 10^{-6} \cdot r^5 + 5{,}\dot{5} \cdot 10^{-5} \cdot r^4 + 9{,}0983 \cdot 10^{-3} - 1{,}4\dot{2} \cdot 10^{-2} \cdot \ln r +$
$+ 7{,}0130 \cdot 10^{-3} \cdot r^2 - 3{,}\dot{5} \cdot 10^{-3} r^2 \ln r$

$W_0 = -4{,}7341 \cdot 10^{-3}\,m$

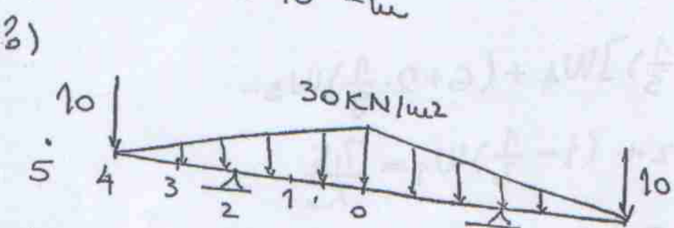
$W_1 = -3{,}6896 \cdot 10^{-3}\,m$

$W_2 = 0$

$W_3 = 6{,}7784 \cdot 10^{-3}\,m$

$W_4 = 1{,}4705 \cdot 10^{-2}\,m$

3)



$\Delta = 1{,}0m$, $C = 0 \rightarrow$ reakcija podloge

$\underline{\underline{=4}}$ $\lambda_m = \frac{\Delta m}{r_m} = \frac{1}{4}$ $\frac{5}{6} r^3)$

$\nu = 0$ (1)

$= 10$ (2)

$M_r = -\frac{K}{\Delta^2}\left[W_{m+1}\left(1 + \frac{\nu \cdot \lambda_m}{2}\right) - 2W_m + W_{m-1}\left(1 - \frac{\nu \lambda_m}{2}\right)\right]$

$1r = -K\left[W_5(1+0) - 2W_4 \ldots\right]$

$$W_5 - 2W_4 + W_3 = 0 \qquad (1)$$

(2):
$$T_r = 10{,}0$$

$$T_{r,m} = -\frac{K}{2\Delta^3}\left[W_{m+2} - W_{m+1}(2 - 2\lambda_m + \lambda_m^2) - 4\lambda_m W_m + \right.$$
$$\left. + W_{m-1}(2 + 2\lambda_m + \lambda_m^2) - W_{m-2}\right]$$

$$T_{r,4} = -\frac{K}{2}\left[W_6 - W_5\left(2 - 2\cdot\frac{1}{4} + \frac{1}{16}\right) - 4\cdot\frac{1}{4}\cdot W_4 + W_3\left(2 + 2\frac{1}{4} + \frac{1}{16}\right) - \right.$$
$$\left. - W_2\right] = 10$$

$$W_6 - 1{,}5625\cdot W_5 - W_4 + 2{,}5625\cdot W_3 - W_2 = -\frac{20}{K} \qquad (2)$$

(3) $m = 4$

$$\Delta\Delta W = \frac{Z - \cancel{\alpha t}^{\,0}}{K}$$

$$(1+\lambda_m)W_{m+2} - \left[2(2+\lambda_m) + \frac{\lambda_m^2}{2}(2-\lambda_m)\right]W_{m+1} + \left(6 + 2\lambda_m^2 + \frac{C}{K}\Delta^4\right)W_m -$$
$$-\left[2(2-\lambda_m) + \frac{\lambda_m^2}{2}(2+\lambda_m)\right]\cdot W_{m-1} + (1-\lambda_m)W_{m-2} = \frac{Z_m\cdot\Delta^4}{K}$$

$$\left(1+\frac{1}{4}\right)W_6 - \left[4 + \frac{1}{2} + \frac{1}{32}\left(2 - \frac{1}{4}\right)\right]\cdot W_5 + (6 + 2\cdot 0{,}25^2 + 0)W_4 -$$
$$-\left[2\left(2-\frac{1}{4}\right) + \frac{1}{32}\left(2+\frac{1}{4}\right)\right]\cdot W_3 + \left(1 - \frac{1}{4}\right)W_2 = 0$$

$$1{,}25\cdot W_6 - 4{,}5546875\cdot W_5 + 6{,}125W_4 - 3{,}5703125\,W_3 + 0{,}75W_2 = 0 \qquad (3)$$

(4) $m = 3 \quad \lambda_3 = \frac{1}{3}$

$$\left(1+\frac{1}{3}\right)W_5 - \left[2\left(2+\frac{1}{3}\right) + \frac{1}{2\cdot 9}\left(2-\frac{1}{3}\right)\right]W_4 + \left(6 + 2\cdot\frac{1}{9}\right)W_3 -$$
$$-\left[2\left(2-\frac{1}{3}\right) + \frac{1}{18}\left(2+\frac{1}{3}\right)\right]\cdot W_2 + \left(1-\frac{1}{3}\right)W_1 = \frac{7{,}5}{K}$$

$$1{,}\dot{3}W_5 - 4{,}759\,W_4 + 6{,}\dot{2}\,W_3 - 3{,}463\cdot W_2 + 0{,}\dot{6}\,W_1 = \frac{7{,}5}{K} \qquad (4)$$

(5) $m = 2$

$$W_2 = 0$$

(6) $m = 1 \quad \lambda_1 = 1$

$$2W_3 - [2(2+1) + 0{,}5(2-1)]\,W_2 + (8)W_1 - \left[2 + \frac{1}{2}\cdot 3\right]\cdot W_0 +$$
$$+ 0 = \frac{22{,}5}{K}$$

$$2W_3 - 6{,}5W_2 + 8W_1 - 3{,}5W_0 = \frac{22{,}5}{K} \qquad (6)$$

(7) $u = 0$

$$\frac{8}{3}\,\frac{W_2 - 4W_1 + 6W_0 - \overset{W_1}{\cancel{4W_{1'}}} + \overset{W_2}{\cancel{W_{2'}}}}{\Delta^4} = \frac{30}{K}$$

$$\frac{16}{3}W_2 - \frac{64}{3}W_1 + 16W_0 = \frac{30}{K} \qquad (7)$$

$W_0 = -3{,}6074 \cdot 10^{-3}$

$W_1 = -2{,}8722 \cdot 10^{-3}$

$W_2 = 0$

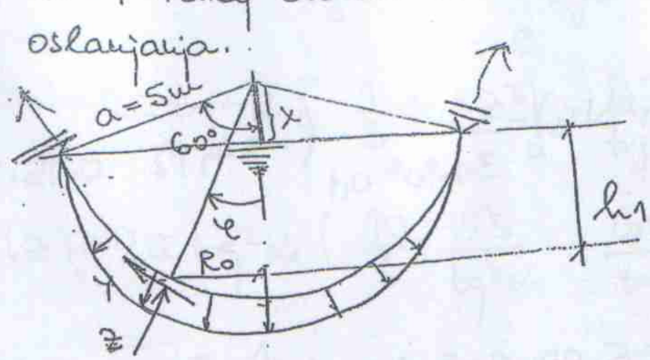
$W_3 = 6{,}5092 \cdot 10^{-3}$

$W_4 = 1{,}3867 \cdot 10^{-2}$

①

28.02.2004.

3. $\alpha = ?$ Tako da su im $\Delta R_o^{sfere} = \Delta R_o^{kupe}$ namestima oslanjanja.



$R_1 = a$

$a = 5m$

$E = 30 GPa \quad \nu = 0,15$

$\delta = 0,1m \quad \gamma = 10 kN/m^3$

$Y = 0$

$Z = -\gamma \cdot h_1$

$h_1 = a \cdot \cos\varphi - x = 5 \cdot \cos\varphi - 5 \cdot \cos 60° = 5\cos\varphi - 2,5$

$R_o = a \cdot \sin\varphi$

$Z = -\gamma(5\cos\varphi - 2,5)$

$$P(\varphi) = 2\pi \int_0^{60} (Y\sin\varphi + Z\cos\varphi) R_o R_1 d\varphi$$

$$P(\varphi) = 2\pi \int_0^{60} -\gamma(5\cos\varphi - 2,5) \cdot \cos\varphi \cdot 25 \cdot \sin\varphi \, d\varphi$$

$P(\varphi) = -20\pi \cdot 25 \cdot 0,52088 = -818,1231$

$$N_\varphi = \frac{-P(\varphi)}{2\pi R_o \cdot \sin\varphi} = \frac{+818,1231}{2\pi \cdot 5 \cdot \sin^2\varphi} = \frac{+818,1231}{2\pi \cdot 5 \cdot \sin^2 60°}$$

$N_\varphi = +34,72$

$$\frac{N_\varphi}{R_1} + \frac{N_\theta}{R_2} + Z = 0$$

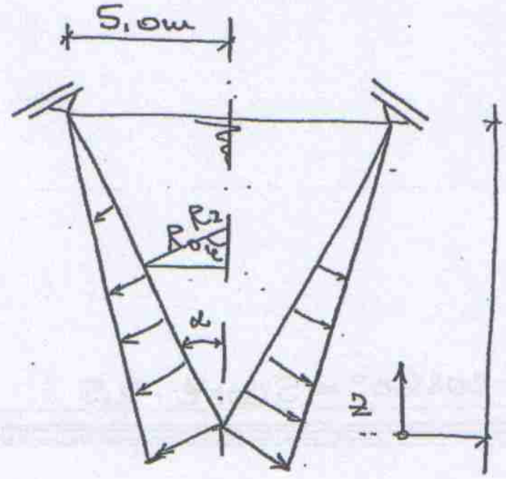
$$N_\Theta = -a\cdot z - N_\varphi = -a\cdot\gamma\cdot(5\cos\overset{\text{u osloncu}}{60°} - 2{,}5) - 34{,}72$$

$$N_\Theta = -34{,}72$$

$$\varepsilon_\Theta = \frac{1}{Eh}(N_\Theta - \nu N_\varphi) = \frac{1}{30\cdot 10^6\cdot 0{,}1}(-34{,}72 - 0{,}15\cdot 34{,}72)$$

$$\varepsilon_\Theta = -1{,}331\cdot 10^{-5}$$

$$\Delta R_0^S = R_0\cdot\varepsilon_\Theta = -5\cdot\sin 60°\cdot 1{,}331\cdot 10^{-5} = -5{,}7635\cdot 10^{-5}$$



$$R_0 = z\cdot tg\alpha$$

$$R_2 = \frac{R_0}{\cos\alpha} = z\cdot\frac{tg\alpha}{\cos\alpha}$$

$$Y = 0$$

$$Z = -\gamma(l - z)$$

$$l = \frac{5{,}0}{tg\alpha}$$

$$Z = -10\left(\frac{5}{tg\alpha} - z\right) = 10z - \frac{50}{tg\alpha}$$

$$P(z) = 2\pi\,\frac{tg\alpha}{\cos\alpha}\int_{0}^{l}(Y\cos\alpha + Z\sin\alpha)\,z\,dz$$

$$P(z) = 2\pi\,\frac{tg\alpha}{\cos\alpha}\int_{0}(10z - \frac{50}{tg\alpha})\sin\alpha\, z\,dz$$

$$P(z) = 2\pi \, tg^2\alpha \int_0^{\ell} \left(10z^2 - \frac{50}{tg\alpha} z\right) dz$$

$$P(z) = 2\pi \, tg^2\alpha \left[\frac{1}{3} \cdot 10 \cdot z^3 \Big|_0^{\ell} - \frac{25}{tg\alpha} z^2 \Big|_0^{\ell} \right]$$

$$P(z) = 2\pi \, tg^2\alpha \left(\frac{10}{3} \cdot \frac{125}{tg^3\alpha} - \frac{25}{tg\alpha} \cdot \frac{25}{tg^2\alpha} \right)$$

$$P(z) = 2\pi \frac{1}{tg\alpha} (-208,\dot{3}) = -\frac{1308,9969}{tg\alpha}$$

$$N_\ell = \frac{-P(z)}{2\pi \cdot \cos\alpha \cdot R_0} = \frac{1308,9969}{2\pi \cdot \cos\alpha \cdot z \cdot tg^2\alpha}$$

$$N_\ell = \frac{208,\dot{3}}{\cos\alpha \cdot \frac{5}{tg\alpha} \cdot tg^2\alpha} = \frac{41,\dot{6}}{\cos\alpha \cdot tg\alpha} = \frac{41,\dot{6}}{\sin\alpha}$$

$$\cancel{N_\theta = -2 R_2 = -\gamma(\ell - \overset{=\ell}{z}) \overset{=\ell}{z} \frac{tg\alpha}{\cos\alpha}} = 0 \quad \text{(uoslonen)}$$

$$\varepsilon_\theta = \frac{1}{Eh}(N_\theta - \nu N_\ell) = \frac{1}{30 \cdot 10^6 \cdot 0,1} \left(0 - 0,15 \frac{41,\dot{6}}{\sin\alpha}\right)$$

$$\varepsilon_\theta = -2,08\dot{3} \cdot 10^{-6} \cdot \frac{1}{\sin\alpha}$$

$$\Delta R_0^k = \varepsilon_\theta \cdot R_0 = -2,08\dot{3} \cdot 10^{-6} \cdot \frac{1}{\sin\alpha} \cdot \cancel{z}^{\,\ell} \cdot tg\alpha$$

$$\Delta R_0 = -2,08\dot{3} \cdot 10^{-6} \cdot \frac{1}{\sin\alpha} \frac{5}{\cancel{tg\alpha}} \cdot \cancel{tg\alpha} = -5,7635 \cdot 10^{-5}$$

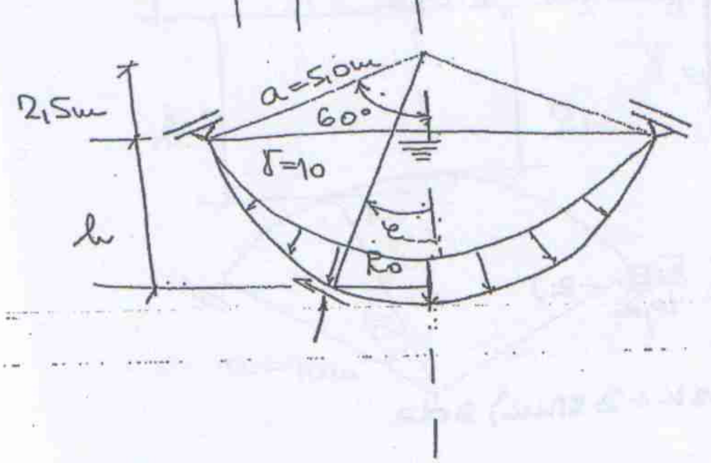
$\sin\alpha = 0,1807357$

$\alpha = 10,4126°$

① 2

28.02.2004.

3. $\alpha = ?$ Tako da su im $\Delta R_0^{sfere} = \Delta R_0^{kupe}$ na mestima oslanjanja



$a = 5{,}0\,m$

$E = 30\,GPa$

$\nu = 0{,}15$

$\delta = 0{,}1\,m$

$\gamma = 10\,kN/m^3$

$Y = 0$

$Z = -\gamma \cdot h = -\gamma\,(a \cdot \cos\varphi - a \cdot \cos 60°) = -50\,(\cos\varphi - 0{,}5)$

$$P(\varphi) = 2\pi \int_0^{60} (Y \sin\varphi + Z \cos\varphi) \cdot R_0 \cdot R_1\, d\varphi$$

$R_1 = R_2 = a = 5{,}0\,m$

$R_0 = a \cdot \sin\varphi$

$$P(\varphi) = 2\pi \int_0^{60} -50 \cdot (\cos\varphi - 0{,}5) \cdot \cos\varphi \cdot 25 \cdot \sin\varphi\, d\varphi = -818{,}1231$$

$$N_\varphi = \frac{-P(\varphi)}{2\pi \cdot R_0 \cdot \sin\varphi} = \frac{818{,}1231}{2\pi \cdot 5 \cdot \sin^2 60°} = 34{,}72$$

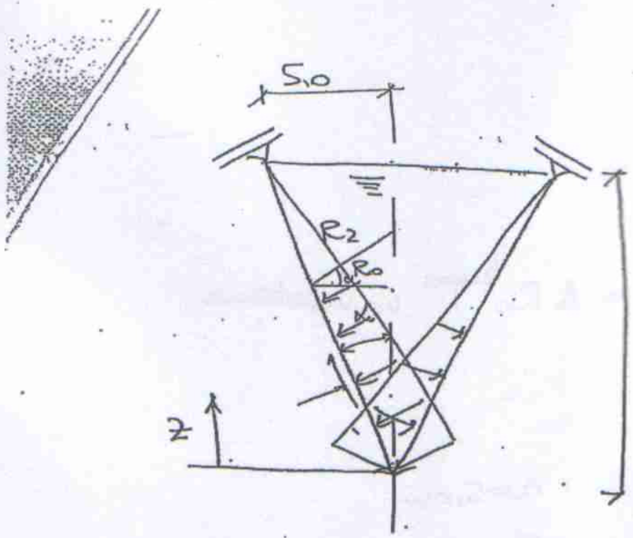
$$\frac{N_\varphi}{R_1} + \frac{N_\theta}{R_2} + Z = 0 \Rightarrow N_\theta = +5 \cdot 50(\cos 60 - 0{,}5) - 34{,}72$$

$N_\theta = -34{,}72$

$$\varepsilon_\theta = \frac{1}{Eh}(N_\theta - \nu N_\varphi) = \frac{1}{30 \cdot 10^6 \cdot 0{,}1}(-34{,}72 - 0{,}15 \cdot 34{,}72)$$

$\varepsilon_\theta = -1{,}3310 \cdot 10^{-5}$

$\Delta R_0^{sfere} = \varepsilon_\theta \cdot R_0 = -1{,}3310 \cdot 10^{-5} \cdot 5 \cdot \sin 60° = -5{,}7635 \cdot 10^{-5}$



$H = \dfrac{5{,}0}{tg\alpha}$

$R_0 = z \cdot tg\alpha$

$Y = 0$

$Z = -\gamma_w \cdot (H - z) = -\gamma_w \left(\dfrac{5{,}0}{tg\alpha} - z \right)$

$P(z) = 2\pi \dfrac{tg\alpha}{\cos\alpha} \int_0^z (Y\cos\alpha + Z\sin\alpha)\, z\, dz$

$P(z) = 2\pi \dfrac{tg\alpha}{\cos\alpha} \int_0^z -10\left(\dfrac{5{,}0}{tg\alpha} - z\right)\sin\alpha \cdot z\, dz$

$P(z) = 2\pi \dfrac{tg\alpha}{\cos\alpha} \left[\int_0^{5/tg\alpha} -50 \cdot \cos\alpha \cdot z\, dz + \int_0^{5/tg\alpha} 10 z^2 \sin\alpha\, dz \right]$

$P(z) = 2\pi \dfrac{tg\alpha}{\cos\alpha} \left[-50\cos\alpha \cdot \dfrac{1}{2} \dfrac{25}{tg^2\alpha} + \sin\alpha \cdot \dfrac{1}{3} \dfrac{1250}{tg^3\alpha} \right]$

$P(z) = -1250\pi \dfrac{1}{tg\alpha} + 833{,}\dot{3} \dfrac{\pi}{tg\alpha} = \dfrac{-1308{,}9969}{tg\alpha}$

$N_\varphi = \dfrac{-P(z)}{2\pi \cdot z \cdot \sin\alpha} = \dfrac{1308{,}9969}{2\pi \cdot \dfrac{5}{tg\alpha} \cdot \sin\alpha \cdot tg\alpha} = \dfrac{41{,}\dot{6}}{\sin\alpha}$

$N_\theta = -Z \cdot R_2 = -10\left(\dfrac{5}{tg\alpha} - \dfrac{5}{tg\alpha}\right) \cdot z \cdot \dfrac{tg\alpha}{\cos\alpha} = 0$

$\varepsilon_\theta = \dfrac{1}{Eh}(N_\theta - \nu N_\varphi) = -2{,}08\dot{3} \cdot 10^{-6} \cdot \dfrac{1}{\sin\alpha}$

$\Delta R_0^{kupe} = \varepsilon_\theta \cdot R_0 = -2{,}08\dot{3} \cdot 10^{-6} \cdot \dfrac{1}{\sin\alpha} \cdot \dfrac{5{,}0}{\cancel{tg\alpha}} \cdot \cancel{tg\alpha} = \Delta R$

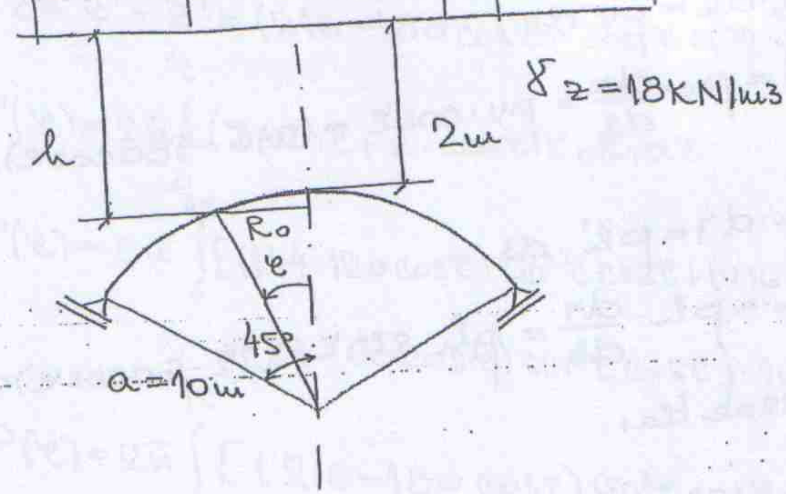
$\sin\alpha = 0{,}1807$

$\alpha = 10{,}4126°$

(2)

02.09.2002.

3. Za sfernu ljusku na slici odrediti reakcije oslonaca i pomeranje u tački oslanjanja usled pritiska tla.



Horizontalni pritisak računati po formuli:

$$p_h = \gamma_z \cdot h \cdot tg^2\left(45^\circ - \frac{\phi}{2}\right)$$

$E = 30\,GPa$

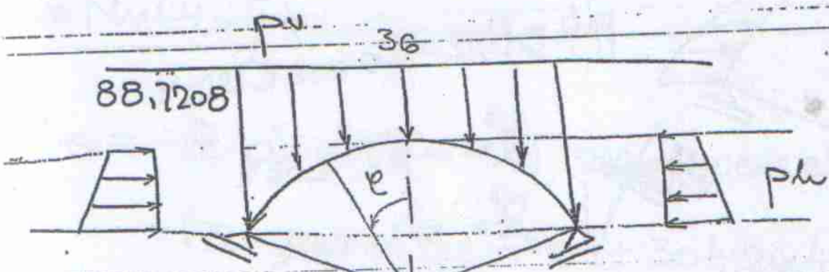
$\nu = 0{,}2$

$\phi = 30^\circ \rightarrow$ ugao unutrašnjeg trenja

$$h = 2 + a - a\cos\varphi = 2 + 10(1 - \cos\varphi)$$

$$R_0 = a \cdot \sin\varphi = 10 \cdot \sin\varphi$$

$$R_1 = R_2 = a = 10\,m$$

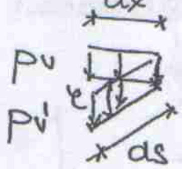


$$p_v = \gamma_z \cdot h = 18 \cdot (2 + 10(1 - \cos\varphi)) = 36 + 180(1 - \cos\varphi)$$

$$p_v = 216 - 180 \cdot \cos\varphi$$

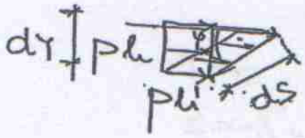
$$p_h = \gamma z \cdot tg\left(45 - \frac{30}{2}\right) \cdot \left(2 + 10(1 - \cos\varphi)\right)$$

$$p_h = 18 \cdot tg\,30^\circ \left(2 + 10 - 10\cos\varphi\right) = 72 - 60 \cdot \cos\varphi$$



$$p_v \cdot dx = p_v' \cdot ds$$

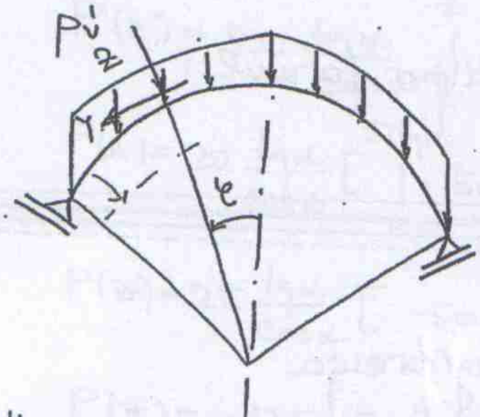
$$p_v' = p_v \cdot \frac{dx}{ds} = p_v \cdot \cos\varphi = (216 - 180\cos\varphi)\cos\varphi$$



$$p_h \cdot dy = p_h' \cdot ds$$

$$p_h' = p_h \cdot \frac{dy}{ds} = p_h \cdot \sin\varphi = (72 - 60\cos\varphi)\sin\varphi$$

I Vertikalni pritisak tla:

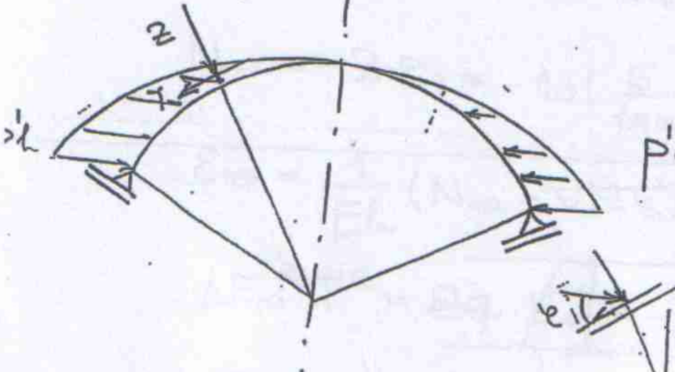


$$p_v' = (216 - 180\cos\varphi)\cos\varphi$$

$$Y' = p_v' \cdot \sin\varphi$$

$$Z' = p_v' \cdot \cos\varphi$$

II Horizontalni pritisak tla:



$$p_h' = (72 - 60\cos\varphi)\sin\varphi$$

$$Y'' = -p_h' \cdot \cos\varphi$$

$$Z'' = p_h' \cdot \sin\varphi$$

$$Y = Y' + Y'' = (216 - 180\cos\varphi)\cos\varphi \cdot \sin\varphi - (72 - 60\cos\varphi)\cdot\sin\varphi\cos\varphi$$

$$Y = (144 - 120\cos\varphi)\sin\varphi\cos\varphi$$

$$Z = Z' + Z'' = (216 - 180\cos\varphi)\cdot\cos^2\varphi + (72 - 60\cos\varphi)\sin^2\varphi$$

$$P(\varphi) = 2\pi\int_0^{\varphi}(Y\cdot\sin\varphi + Z\cdot\cos\varphi)R_0R_1\,d\varphi$$

$$P(\varphi) = 2\pi\int_0^{45}[(144 - 120\cos\varphi)\cdot\sin^2\varphi\cos\varphi + (216 - 180\cos\varphi)\cos^3\varphi + $$
$$+ (72 - 60\cos\varphi)\sin^2\varphi\cos\varphi]\cdot 100\cdot\sin\varphi\,d\varphi$$

$$P(\varphi) = 2\pi\int_0^{45}[(216 - 180\cos\varphi)\sin^2\varphi\cos\varphi +$$
$$+ (216 - 180\cos\varphi)\cos^3\varphi]\cdot 100\sin\varphi\,d\varphi$$

$$P(\varphi) = 200\pi\int_0^{\pi/4}(216 - 180\cos\varphi)\cos\varphi\,\underbrace{(\sin^2\varphi + \cos^2\varphi)}_{1}\sin\varphi\,d\varphi$$

$$P(\varphi) = 200\pi\int_0^{\pi/4}(216 - 180\cos\varphi)\cos\varphi\sin\varphi\,d\varphi$$

$$P(\varphi) = 9558{,}7376$$

$$N_\varphi = \frac{-P(\varphi)}{2\pi R_0\cdot\sin\varphi} = \frac{-9558{,}7376}{2\pi\cdot 10\cdot\sin^2\frac{\pi}{4}} = -304{,}2641$$

$$R = N_\varphi\left(\varphi = \frac{\pi}{4}\right) = -304{,}2641$$

$$N_\theta = -Z\cdot a - N_\varphi = -10[(216 - 180\cos\varphi)\cos^2\varphi +$$
$$+ (72 - 60\cos\varphi)\sin^2\varphi] + 304{,}2641$$

$$N_\theta\left(\varphi = \frac{\pi}{4}\right) = -559{,}3398$$

$$\varepsilon_\varphi = \frac{1}{Eh}(N_\varphi - \nu N_\Theta) = \frac{1}{Eh}(N_\varphi - 0{,}2 N_\Theta) = -\frac{192{,}3961}{Eh}$$

$$\varepsilon_\Theta = \frac{1}{Eh}(N_\Theta - \nu N_\varphi) = \frac{-498{,}4870}{Eh}$$

$$v = \left[\int \frac{R_1 \cdot \varepsilon_\varphi - R_2 \varepsilon_\Theta}{\sin\varphi} d\varphi + c\right] \cdot \sin\varphi$$

$$\varphi = \frac{\pi}{4}$$

$$v = \frac{1}{Eh}$$

$$N_\varphi = \frac{818{,}1231}{2\pi \cdot 5 \cdot \sin^2\varphi} =$$

Ja mislim da je $v=0$?

$$w = \frac{1}{\cos\alpha}(v \cdot \sin\alpha - R_0 \cdot \varepsilon_\Theta) =$$

② 2

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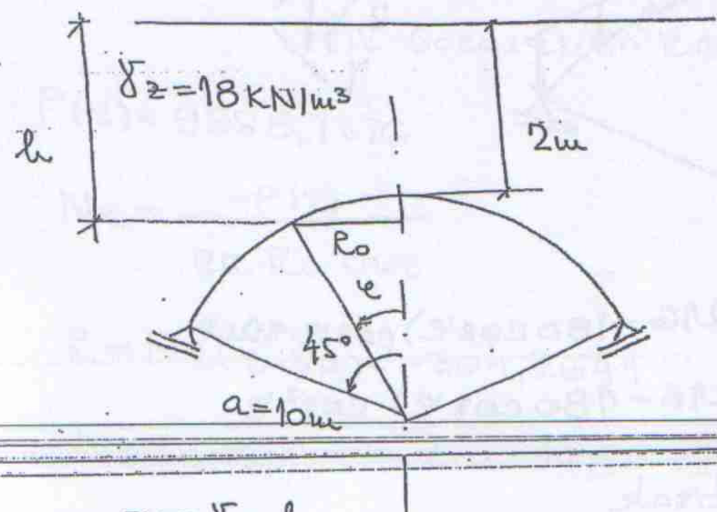
3. Za sfernu ljusku na slici odrediti reakcije oslonaca i pomeranje u tački oslanjanja usled pritiska tla.

Horizontalni pritisak računati po formuli:

$p_h = \gamma_z \cdot h \cdot tg^2(45° - \phi/2)$



$E = 30\ GPa$

$\nu = 0{,}2$

$\phi = 30°$

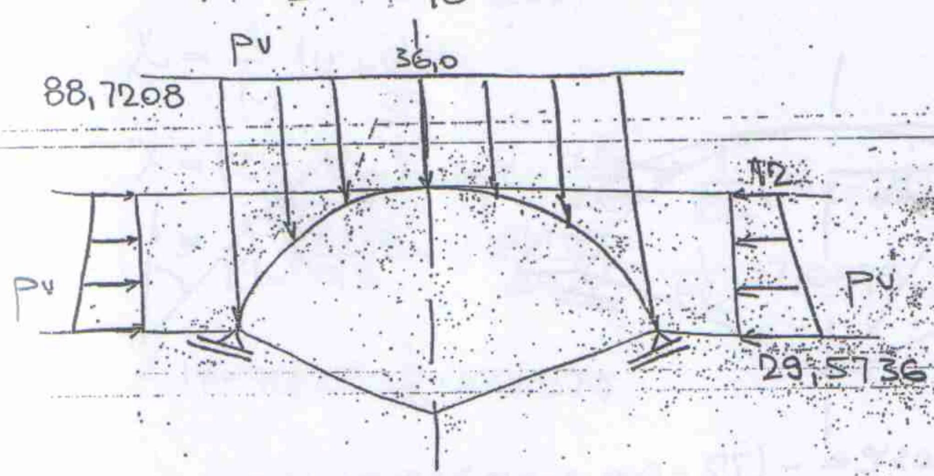
$p_v = \gamma_z \cdot h$

$h = 2 + a - a \cdot \cos\varphi = 2 + a(1 - \cos\varphi) = 2 + 10(1 - \cos\varphi)$

$p_v = 18[2 + 10(1 - \cos\varphi)] = 216 - 180 \cdot \cos\varphi$

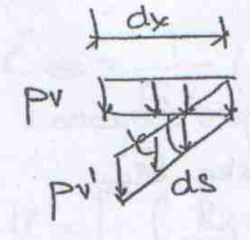
$R_0 = a \cdot \sin\varphi = 10 \sin\varphi$

$R_1 = R_2 = a = 10$



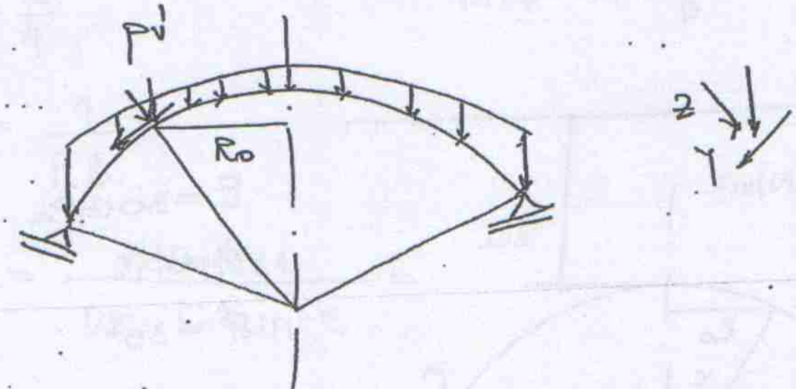
$p_h = 72 - 60 \cos\varphi$

I Vertikalan pritisak tla



$$pv' \cdot ds = pv\,dx \Rightarrow pv' = pv \cdot \frac{dx}{ds} = pv \cdot \cos\varphi$$

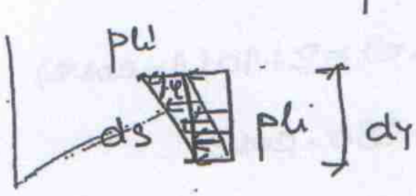
$$pv' = 216\cos\varphi - 180 \cdot \cos^2\varphi$$



$$Y_1 = pv' \cdot \sin\varphi = (216 - 180\cos\varphi)\cos\varphi \cdot \sin\varphi$$

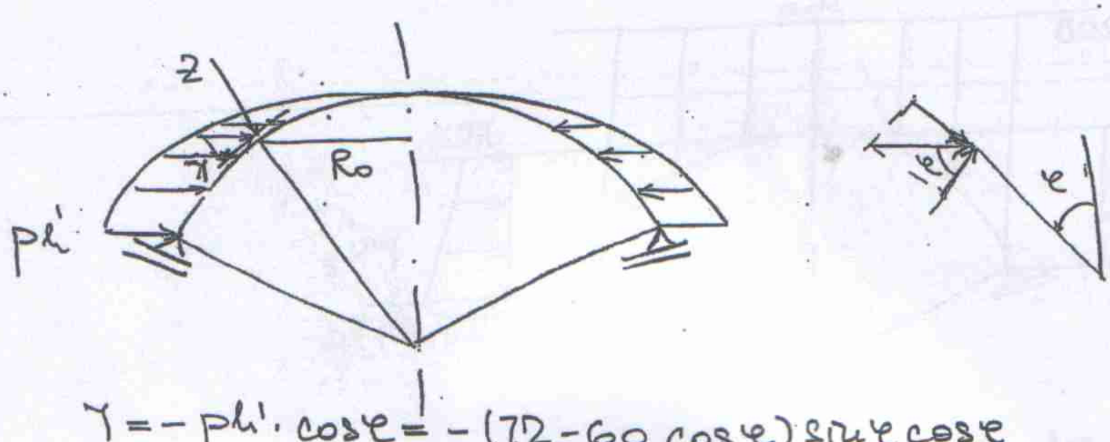
$$Z_1 = pv' \cdot \cos\varphi = (216 - 180\cos\varphi) \cdot \cos^2\varphi$$

II Horizontalni pritisak



$$ph' = ph \cdot \frac{dy}{ds} = ph \cdot \sin\varphi$$

$$ph' = (72 - 60\cos\varphi)\sin\varphi$$



$$Y = -ph' \cdot \cos\varphi = -(72 - 60\cos\varphi)\sin\varphi\cos\varphi$$

$$Z = ph' \cdot \sin\varphi = (72 - 60\cos\varphi)\sin^2\varphi$$

$$Y = Y_1 + Y_2 = (216 - 180\cos\varphi - 72 + 60\cos\varphi)\cos\varphi\sin\varphi$$

$$Y = (144 - 120\cos\varphi)\cos\varphi\sin\varphi$$

$$Z = Z_1 + Z_2 = (216 - 180\cos\varphi)\cos^2\varphi + (72 - 60\cos\varphi)\sin^2\varphi$$

$$P(\varphi) = 2\pi \int^{\varphi} (Y \cdot \sin\varphi + Z\cos\varphi) R_0 R_1 d\varphi$$

$$P(\varphi) = 200\pi \int_0^{45^\circ} \left[(144 - 120\cos\varphi) \cdot \cos\varphi\sin^2\varphi + (216 - 180\cos\varphi)\cos^3\varphi + \right.$$
$$\left. + (72 - 60\cos\varphi) \cdot \sin^2\varphi\cos\varphi \right] \cdot \sin\varphi\, d\varphi$$

$$P(\varphi) = 9558{,}7376$$

$$N_\varphi = \frac{-P(\varphi)}{2\pi \cdot R_0 \cdot \sin\varphi}$$

$$R = N_\varphi(\varphi = \pi/4) = -304{,}2641$$

$$\cancel{N_\theta(\varphi = \pi/4) = -2 \cdot a - N_\varphi = -287{,}2078}$$

$$(\Delta R_0)_{(\varphi = \pi/4)} = \varepsilon_\theta(\varphi = \pi/4) \cdot R_0$$

$$\varepsilon_\theta(\varphi = \pi/4) = \frac{1}{Eh}(N_\theta - \nu N_\varphi) = -\frac{1}{Eh} 226{,}3550$$

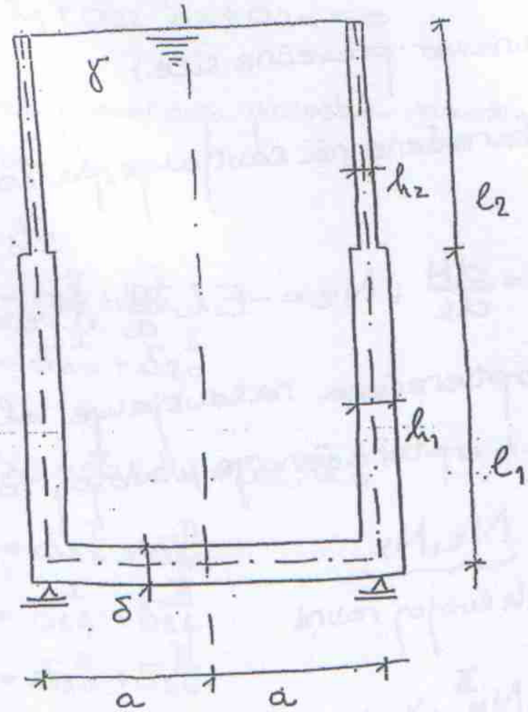
$$\Delta R_0(\varphi = \pi/4) = -\frac{1}{Eh} \cdot 1600{,}5714\ m$$

$$\chi = \frac{1}{R_1}\left(v + \frac{dw}{d\varphi}\right)$$

$$\chi = (\varepsilon_\varphi - \varepsilon_\theta) \cdot ctg\,\varphi - \frac{d\varepsilon_\theta}{d\varphi} = \frac{1}{Eh}(N_\varphi - \nu N_\theta - N_\theta + \nu N_\varphi) ctg\,\varphi -$$
$$- \frac{1}{Eh}\left(\frac{dN_\theta}{d\varphi} - \nu \frac{dN_\varphi}{d\varphi}\right) = -\frac{1}{Eh} \cdot 37{,}6879$$

$$\chi_{(\varphi = \pi/4)} = -\frac{1}{Eh} \cdot 37{,}6879$$

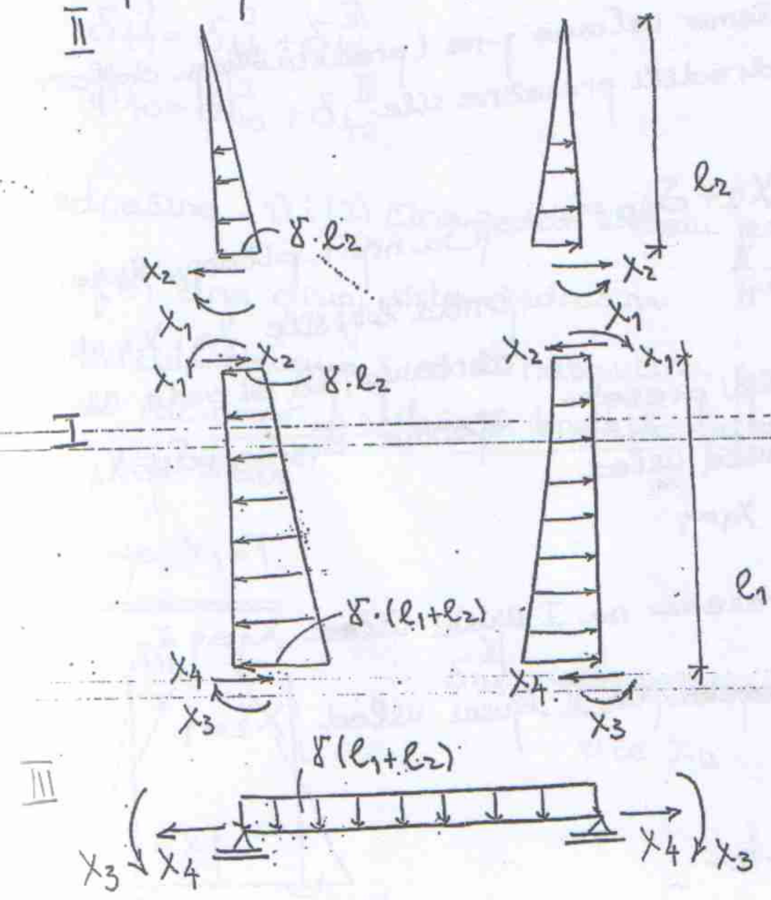
3.



Za obe ljuske važi: $\beta_1 \cdot l_1 > 5$

$\beta_2 \cdot l_2 > 5$

Dekompozicija:



Momenti X_1 i X_3 imaju konstantnu vrednost po obimu, jer je opterećenje rotaciono simetrično.

Sada treba da sračunamo presečne sile:

1. Ljuske I i II su opterećene na savijanje, pa ćemo od presečnih sila imati:

$M_x = -K \frac{d^2w}{dx^2}$, $T_x = \frac{dM}{dx}$, $N_\varphi = -Eh\frac{w}{a}$, $M_\varphi = \nu M_x$

2. Kružna ploča je opterećena na savijanje usled p i momenta X_3, a usled sile X_4 opterećena je u svojoj ravni.

$\underbrace{M_\varphi, M_r, T_\varphi}_{\text{Savijanje}}$ $\underbrace{N_\varphi, N_x}_{\text{u svojoj ravni}}$

$N_\varphi^{II} = N_{\varphi_0}^{II} + N_{\varphi_1}^{II} \cdot X_1 + N_{\varphi_2}^{II} \cdot X_2 + \dots$

Od spoljašnjeg opterećenja — Od nepoznatih jediničnih sila

Treba da napišemo uslovne j-ne (predstavljaju deformacione uslove) da bi odredili presečne sile.

(1) $\delta_{11} X_1 + \delta_{12} X_2 + \delta_{10} = 0$

$\delta_{11} = \delta_{11}^{I} + \delta_{11}^{II}$

Razlika obrtanja preseka na granici I i II ljuske usled jediničnog momenta $X_1 = 1$

Da nije u pitanju duga ljuska onda bi i sile X_3 i X_4 uticale na obrtanje, pa bi ovde imali i članove $\delta_{13} \cdot X_3$ i $\delta_{14} \cdot X_4$.

δ_{11}^{I} - obrtanje preseka na I ljusci usled $X_1 = 1$

δ_{11}^{II} - obrtanje preseka na II ljusci usled $X_2 = 1$

$\delta_{12} = \delta_{12}^{I} + \delta_{12}^{II}$

δ_{12} - razlika obrtanja preseka između I i II ljuske usled $X_2=1$

$\delta_{10}=\delta_{10}^{I}+\delta_{10}^{II}$

(2) $\delta_{21}\cdot X_1+\delta_{22}\cdot X_2+\delta_{20}=0$

Razlika pomeranja preseka između I i II ljuske usled jediničnog momenta $X_1=1$

$\delta_{21}=\delta_{12}$

$\delta_{22}=\delta_{22}^{I}+\delta_{22}^{II}$

$\delta_{20}=\delta_{20}^{I}+\delta_{20}^{II}$

(3) $\delta_{33}X_3+\delta_{34}\cdot X_4+\delta_{30}=0$

$\delta_{33}=\delta_{33}^{I}+\delta_{33}^{III}$

$\delta_{34}=\delta_{34}^{I}+\delta_{34}^{III}$

$\delta_{30}=\delta_{30}^{I}+\delta_{30}^{III}$

(4) $\delta_{43}\cdot X_3+\delta_{44}\cdot X_4+\delta_{40}=0$

$\delta_{43}=\delta_{34}$

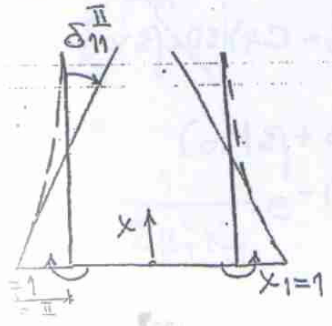
$\delta_{44}=\delta_{44}^{I}+\delta_{44}^{III}$

$\delta_{40}=\delta_{40}^{I}+\delta_{40}^{III}$

Jednačine (1) i (2) čine jedan sistem jednačina, a jednačine (3) i (4) čine drugi sistem jednačina.

Ova dva sistema j-na su nezavisna, pa možemo odvojeno da ih računamo, a u slučaju kratkih ljuski imali bismo 4 j-ne sa 4 nepoznate.

Stanje $X_1=1$



δ_{21}^{II} — Usvojeno je da je ⊕ jer je u pravcu sile X_2

Momenti X_1 i X_3 imaju konstantnu vrednost po obimu, jer je opterećenje rotaciono simetrično.

Sada treba da sračunamo presečne sile:

1. Ljuske I i II su opterećene na savijanje, pa ćemo od presečnih sila imati:

$M_x = -K \frac{d^2w}{dx^2}$, $T_x = \frac{dM}{dx}$, $N_\varphi = -Eh\frac{w}{a}$, $M_\varphi = \nu M_x$

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$\underbrace{M_\varphi, M_r, T_\varphi}_{\text{Savijanje}}$ $\qquad$ $\underbrace{N_\varphi, N_x}_{\text{u svojoj ravni}}$

$N_\varphi^{II} = N_{\varphi_0}^{II} + \underbrace{N_{\varphi_1}^{II} \cdot X_1 + N_{\varphi_2}^{II} \cdot X_2 + \ldots}$

Od spoljašnjeg opterećenja $\rightarrow$ Od nepoznatih jediničnih sila

Treba da napišemo uslovne j-ne (predstavljaju deformacione uslove) da bi odredili presečne sile.

(1) $\delta_{11}X_1 + \delta_{12}X_2 + \delta_{10} = 0$

$\delta_{11} = \delta_{11}^{I} + \delta_{11}^{II}$

Razlika obrtanja preseka na granici I i II ljuske usled jediničnog momenta $X_1 = 1$

Da nije u pitanju duga ljuska onda bi i sile X_3 i X_4 uticale na obrtanje, pa bi ovde imali i članove $\delta_{13} \cdot X_3$ i $\delta_{14} \cdot X_4$.

δ_{11}^{I} - obrtanje preseka na I ljusci usled $X_1 = 1$

δ_{11}^{II} - obrtanje preseka na II ljusci usled $X_2 = 1$

$\delta_{12} = \delta_{12}^{I} + \delta_{12}^{II}$

δ_{12} - razlika obrtanja preseka između I i II ljuske usled $X_2=1$

$\delta_{10}=\delta_{10}^{I}+\delta_{10}^{II}$

(2) $\delta_{21}\cdot X_1+\delta_{22}\cdot X_2+\delta_{20}=0$

$\downarrow$

Razlika pomeranja preseka između I i II ljuske usled jediničnog momenta $X_1=1$

$\delta_{21}=\delta_{12}$

$\delta_{22}=\delta_{22}^{I}+\delta_{22}^{II}$

$\delta_{20}=\delta_{20}^{I}+\delta_{20}^{II}$

(3)

$\delta_{33}X_3+\delta_{34}\cdot X_4+\delta_{30}=0$

$\delta_{33}=\delta_{33}^{I}+\delta_{33}^{III}$

$\delta_{34}=\delta_{34}^{I}+\delta_{34}^{III}$

$\delta_{30}=\delta_{30}^{I}+\delta_{30}^{III}$

(4) $\delta_{43}\cdot X_3+\delta_{44}\cdot X_4+\delta_{40}=0$

$\delta_{43}=\delta_{34}$

$\delta_{44}=\delta_{44}^{I}+\delta_{44}^{III}$

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Jednačine (1) i (2) čine jedan sistem jednačina, a jednačine (3) i (4) čine drugi sistem jednačina.

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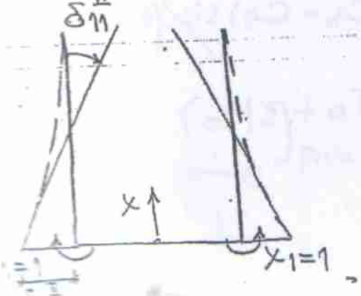
Stanje $X_1=1$



δ_{21}^{II} → Usvojeno je da je ⊕ jer je u pravcu sile X_2

$$W = e^{\beta x}(C_1 \cos\beta x + C_2 \sin\beta x) + e^{-\beta x}(C_3 \cos\beta x + C_4 \sin\beta x)$$

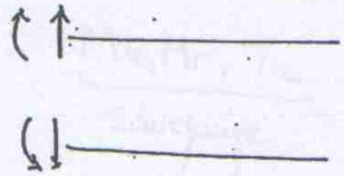
Duga ljuska $\rightarrow$ uticaji opadaju udaljavajući se od opterećenog kraja ljuske, pa je za $x \rightarrow \infty$

$$W = 0$$

$$\Rightarrow C_1 = C_2 = 0$$

$$W = e^{-\beta x}(C_3 \cos\beta x + C_4 \sin\beta x)$$

$$x = 0 \left\{ \begin{array}{l} M_x = M_0 \\ T_x = T_0 \end{array} \right.$$



$$M_x = -K \frac{d^2 w}{dx^2}$$

$$\frac{dw}{dx} = -\beta e^{-\beta x}(C_3 \cos\beta x + C_4 \sin\beta x) + \beta e^{-\beta x}(-C_3 \sin\beta x + C_4 \cos\beta x)$$

$$\frac{d^2 w}{dx^2} = +\beta^2 e^{-\beta x}\cancel{(C_3 \cos\beta x + C_4 \sin\beta x)} - \beta^2 e^{-\beta x}(-C_3 \sin\beta x + C_4 \cos\beta x) -$$
$$- \beta^2 e^{-\beta x}(-C_3 \sin\beta x + C_4 \cos\beta x) + \beta^2 e^{-\beta x}\cancel{(-C_3 \cos\beta x - C_4 \sin\beta x)}$$

$$\frac{d^2 w}{dx^2} = 2\beta^2 e^{-\beta x}(C_3 \sin\beta x - C_4 \cos\beta x)$$

$$\frac{d^3 w}{dx^3} = 2\left[-\beta^3 e^{-\beta x}(C_3 \sin\beta x - C_4 \cos\beta x) + \beta^3 e^{-\beta x}(C_3 \cos\beta x + C_4 \sin\beta x)\right]$$

(1): $-K \cdot 2\beta^2 \cancelto{1}{e^{-\beta x}}(C_3 \cancelto{0}{\sin\beta x} - C_4 \cancelto{1}{\cos\beta x}) = M_0$

$$C_4 = \frac{M_0}{2\beta^2 K}$$

(2):

$$-K \cdot 2 \cdot \beta^3 \cancelto{1}{e^{-\beta x}}\left[(C_3 + C_4)\cancelto{1}{\cos\beta x} + (C_3 - C_4)\cancelto{0}{\sin\beta x}\right] = T_0$$

$$C_3 = -\frac{T_0}{2\beta^3 K} - \frac{M_0}{2\beta^2 K} = -\frac{1}{2\beta^3 K}(T_0 + \beta M_0)$$

$$w = -e^{-\beta x} \cdot \frac{1}{2\beta^3 K}\left[(T_0+\beta M_0)\cos\beta x - \beta M_0 \sin\beta x\right]$$

↳ Ovu j-nu koristim

$M_0 = 1$

$T_0 = 0$

$$W = -\frac{1}{2\beta^2 K_2} e^{-\beta x}\left[\beta\cos\beta x - \beta \sin\beta x\right]$$

$$W^{II} = -\frac{1}{2\beta_2^2 K_2} e^{-\beta_2 x}\left(\cos\beta_2 x - \sin\beta_2 x\right)$$

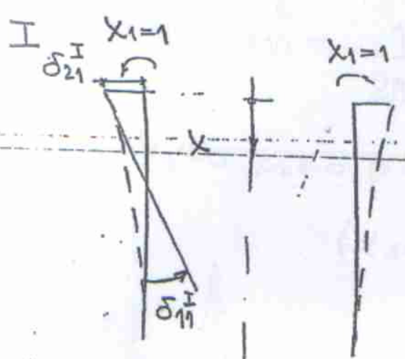
$$\frac{dW^{II}}{dx} = -\frac{1}{2\beta_2^2 K_2}\left[-\beta_2 e^{-\beta_2 x}(\cos\beta_2 x - \sin\beta_2 x) + \beta_2 e^{-\beta_2 x}(-\sin\beta_2 x - \cos\beta_2 x)\right]$$

$$\frac{dW^{II}}{dx} = \frac{1}{2\beta_2 K_2} e^{-\beta_2 x}\, 2\cos\beta_2 x$$

$$\beta_2 = \sqrt[4]{\frac{3(1-\nu^2)}{a^2 \cdot h_2^2}} \qquad K_2 = \frac{E \cdot h_2^3}{12(1-\nu^2)}$$

$$\delta_{11}^{II} = \left(\frac{dW^{II}}{dx}\right)_{x=0} = \frac{1}{\beta_2 \cdot K_2}$$

$$\delta_{21}^{II} = \left(W^{II}\right)_{x=0} = \frac{1}{2\beta_2^2 K_2}$$



$M_0 = 1$

$T_0 = 0$

$$W^{I} = -\frac{1}{2\beta_1^2 K_1} e^{-\beta_1 x}\left(\cos\beta_1 x - \sin\beta_1 x\right)$$

$$\frac{dW^{I}}{dx} = \frac{1}{2\beta_1 K_1} e^{-\beta_1 x} \cdot 2\cos\beta_1 x$$

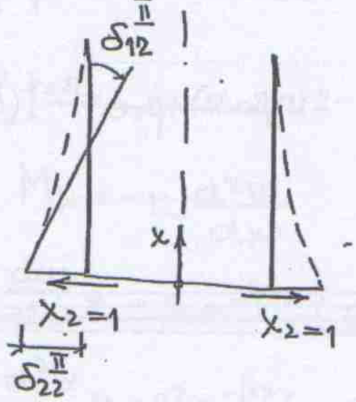
$$\delta_{11}^{I} = \left(\frac{dW^{I}}{dx}\right)_{x=0} = \frac{1}{\beta_1 K_1}$$

$$\delta_{21}^{I} = -(W^{I})_{x=0} = -\frac{1}{2\beta_1^2 K_1}$$

$\ominus$ jer X_2 na ljusci I deluje → i a pomeranje δ_{21}^{I} je u suprotnom smeru

Stanje $X_2 = 1$

II



$$W^{II} = -\frac{1}{2\beta_2^3 K_2} e^{-\beta_2 x}\left[(T_0 + \beta_2 M_0)\cos\beta_2 x - \beta_2 M_0 \sin\beta_2 x\right]$$

$M_0 = 0$

$T_0 = 1$

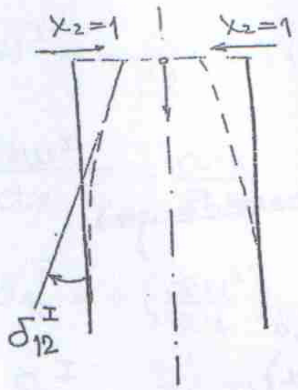
$$W^{II} = -\frac{1}{2\beta_2^3 K_2} e^{-\beta_2 x} \cos\beta_2 x$$

$$\frac{dW^{II}}{dx} = +\frac{1}{2\beta_2^2 K_2} e^{-\beta_2 x}(\cos\beta_2 x + \sin\beta_2 x)$$

$$\delta_{12}^{II} = \left(\frac{dW^{II}}{dx}\right)_{x=0} = \frac{1}{2\beta_2^2 K_2}$$

$$\delta_{22}^{II} = (W^{II})_{x=0} = \mp\frac{1}{2\beta_2^3 K_2}$$

I



$M_0=0$

$T_0=-1$

$$W^{I}=\frac{1}{2\beta_1^3 K_1}e^{-\beta_1 x}\cos\beta_1 x$$

$$\frac{dW^{I}}{dx}=-\frac{1}{2\beta_1^2 K_1}e^{-\beta_1 x}(\cos\beta_1 x+\sin\beta_1 x)$$

$$\delta_{12}^{I}=-\left(\frac{dW^{I}}{dx}\right)_{x=0}=-\frac{1}{2\beta_1^2 K_1^2}$$

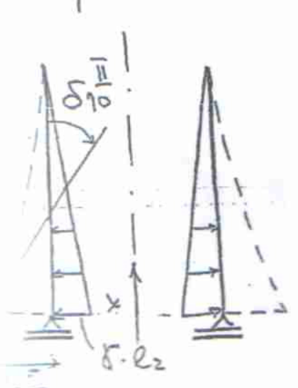
$$\delta_{22}^{I}=(W^{I})_{x=0}=\frac{1}{2\beta_1^3 K_1}$$

Znak koeficijenta δ_{ij} gledamo tako da su ⊕ ako su u pravcu delovanja sile X_i, a ne po formuli!

$$\delta_{11}=\delta_{11}^{I}+\delta_{11}^{II}=\frac{1}{\beta_2 K_2}+\frac{1}{\beta_1 K_1}$$

$$\delta_{12}=\delta_{12}^{I}+\delta_{12}^{II}=-\frac{1}{2\beta_1^2 K_1}+\frac{1}{2\beta_2^2 K_2}$$

Stanje $X_1=0, X_2=0, p\neq 0$



$$Z = -\gamma(l_2 - x)$$

$$X = Y \equiv 0$$

$$N_\varphi = -Z \cdot R = +a\gamma(l_2 - x)$$

$N_{\varphi x} = 0$ (rotaciono simetrično opterećenje)

$$N_x = 0$$

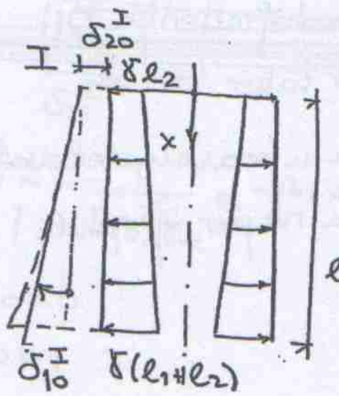
$$W = \Delta R = R \cdot \varepsilon_\varphi = a\frac{1}{Eh_2}(N_\varphi - \nu \cancel{N_x}^{0})$$

$$W^{II} = -\frac{a}{Eh_2} \cdot a\gamma(l_2 - x) = -\frac{a^2\gamma(l_2 - x)}{Eh_2}$$

$$\frac{dW^{II}}{dx} = \frac{a^2\gamma}{Eh_2}$$

$$\delta_{10}^{II} = \left(\frac{dW^{II}}{dx}\right)_{x=0} = \frac{a^2\gamma}{Eh_2}$$

$$\delta_{20}^{II} = +(W^{II})_{x=0} = \frac{a^2\gamma(l_2 - x)}{Eh_2}$$



$$Z = -\gamma(l_2 + x)$$

$$X = Y \equiv 0$$

$$N_\varphi = -Z \cdot R = a \cdot \gamma(l_2 + x)$$

$$N_{\varphi x} = 0$$

$$N_x = 0$$

$$W = \Delta R = -a \cdot \varepsilon_\varphi = -a\frac{1}{Eh_1}(N_\varphi - \nu \cancel{N_x}^{0})$$

$$W^{I} = -\frac{a}{E h_1} \cdot a\gamma(l_2 + x) = -\frac{a^2\gamma}{E h_1}(l_2 + x)$$

$$\frac{dW^{I}}{dx} = -\frac{a^2\gamma}{E h_1}$$

$$\delta_{10}^{I} = -\left(\frac{dW^{I}}{dx}\right)_{x=0} = -\frac{a^2\gamma}{E h_1}$$

$$\delta_{20}^{I} = -(W^{I})_{x=0} = -\frac{a^2\gamma}{E h_1} \cdot l_2$$

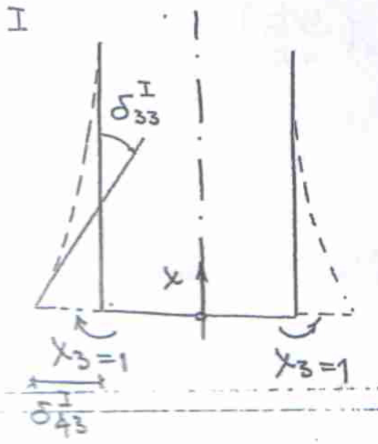
$$\delta_{10} = \delta_{10}^{I} + \delta_{10}^{II} = \frac{a^2\gamma}{E}\left(\frac{1}{h_2} - \frac{1}{h_1}\right)$$

$$\delta_{20} = \delta_{20}^{I} + \delta_{20}^{II} = \frac{a^2\gamma}{E}\left[\frac{l_2 - x}{h_2} - \frac{l_2}{h_1}\right]$$

$\Longrightarrow X_1, X_2$

Stanje $X_3 = 1$

I



$M_0 = 1$

$T_0 = 0$

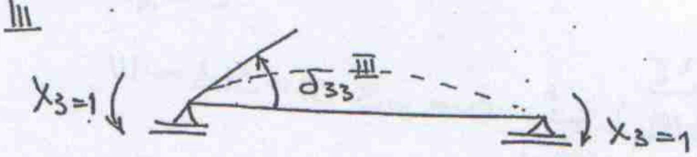
$$W^{I} = -\frac{1}{2\beta_1^2 k_1} e^{-\beta_1 x}(\cos\beta_1 x - \sin\beta_1 x)$$

$$\frac{dW^{I}}{dx} = \frac{1}{2\beta_1 k_1} e^{-\beta_1 x}\, 2\cos\beta_1 x$$

$$\delta_{33}^{I} = +\left(\frac{dw^{I}}{dx}\right)_{x=0} = \frac{1}{\beta_1 k_1}$$

$$\delta_{43}^{I} = -(w^{I})_{x=0} = -\frac{1}{2\beta_1^2 k_1}$$

III



$$w^{III} = \cancel{w_0^{III}}^{\to 0} + w_1^{III} = C_1 + C_2 r^2 + C_3 \ln r + C_4 r^2 \ln r$$

$$r \Rightarrow 0 \quad w \to \infty \quad \Rightarrow C_3 = C_4 \equiv 0$$

$$w^{III} = C_1 + C_2 r^2$$

granični uslovi:

$$r = a \begin{cases} w = 0 \\ M_r = -1 \end{cases}$$

$$M_r = -K\left(\frac{\partial^2 w}{\partial r^2} + \frac{\nu}{r}\frac{\partial w}{\partial r}\right)$$

$$\frac{\partial w}{\partial r} = 2C_2 r$$

$$\frac{\partial^2 w}{\partial r^2} = 2C_2$$

$$M_r = -K\left(2C_2 + \frac{\nu}{r}\cdot 2C_2 r\right) = -2KC_2(1+\nu) = -1 \Rightarrow C_2 = \frac{1}{2K(1+\nu)}$$

$$C_1 = -\frac{a^2}{2K(1+\nu)}$$

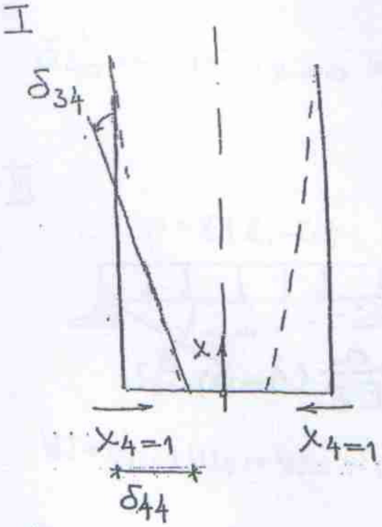
$$w^{III} = -\frac{a^2}{2K(1+\nu)} + \frac{1}{2K(1+\nu)} r^2$$

$$\delta_{33}^{III} = \left(\frac{dw}{dr}\right)_{r=a} = \frac{a}{K_3(1+\nu)}$$

$\delta_{43}^{III} = 0$ → Kod ploča savijanje i naprezanje u ravni nije spregnuto za razliku od ljuski.

Stanje $X_4=1$

I



$M_o=0$

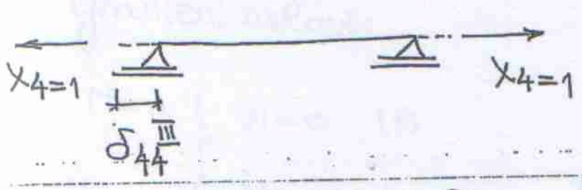
$T_o=-1$

$$W^I=\frac{1}{2\beta_1^3K_1}e^{-\beta_1 x}\cos\beta_1 x$$

$$\delta_{44}^I=(W^I)_{x=0}=\frac{1}{2\beta_1^3K_1}$$

$$\delta_{34}^I=-\left(\frac{dW^I}{dx}\right)_{x=0}=-\frac{1}{2\beta_1^2K_1}$$

III



$$F=\cancel{A}^{0}\ln r+Br^2+\cancel{C}^{0}r^2\ln r+\cancel{D}^{0}$$ → Nema uticaja na naprezanje pa usvajamo $D=0$

za $x=0$ $F\to\infty \Rightarrow A=C=0$

$F=B\cdot r^2$

granični uslovi:

$r=a:\{\ N_r=1$

$$N_r = \frac{1}{r}\frac{\partial F}{\partial r} = \frac{1}{r}\cdot 2Br = 2B = 1 \Rightarrow B = \frac{1}{2}$$

$$F = \frac{1}{2} r^2$$

$$N_\varphi = \frac{\partial^2 F}{\partial r^2} = 1$$

$$N_{\varphi r} = 0$$

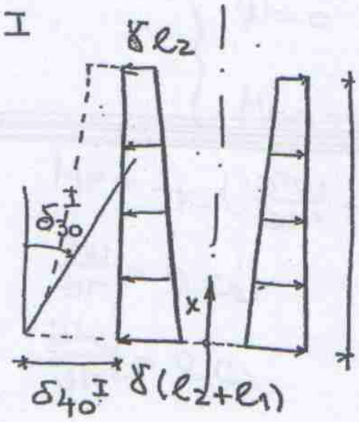
$$u = +r \cdot \varepsilon_\varphi = a \cdot \frac{1}{E\delta}(N_\varphi - \nu N_r) = \frac{a}{E\delta}(1-\nu)$$

$$\delta_{44}^{\text{III}} = (u^{\text{III}})_{r=a} = \frac{a(1-\nu)}{E\delta}$$

$$\delta_{34}^{\text{III}} = 0$$

$$X_3 = 0,\ X_4 = 0,\ p \neq 0$$

I



$$Z = -\gamma(\ell_2 + \ell_1 - x)$$

$$X = Y \equiv 0$$

$$N_\varphi = -Z \cdot R = a\gamma(\ell_2 + \ell_1 - x)$$

$$N_{\varphi x} = 0$$

$$N_x = 0$$

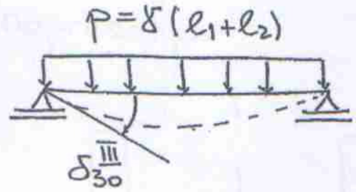
$$W = \Delta R = a \cdot \varepsilon_\varphi = a\frac{1}{Eh_1}(N_\varphi - \nu N_x) = \frac{a^2\gamma}{Eh_1}(\ell_2 + \ell_1 - x)$$

$$\frac{dw}{dx} = -\frac{a^2\gamma}{Eh_1}$$

$$\delta_{30}^{I} = \left(\frac{dw}{dx}\right)_{x=0} = \frac{a^2\gamma}{Eh_1}$$

$$\delta_{40}^{I} = -(w^{I})_{x=0} = -\frac{a^2\gamma}{Eh_1}(l_1+l_2)$$

III



$$W = W_0 + W_1 = W_0 + C_1 + C_2 r^2 + \cancel{C_3} \ln r + \cancel{C_4} r^2 \ln r$$

$$W_0 = -\frac{1}{K}\int\frac{dr}{r}\int r \cdot M dr$$

$$M = -\int\frac{dr}{r}\int r\, z(r) dr = -\int\frac{dr}{r}\int r \cdot p dr = -p\int\frac{1}{r}\frac{r^2}{2}dr = -\frac{pr^2}{4}$$

$$W_0 = \frac{p}{4K_3}\int\frac{dr}{r}\int r^3 dr = \frac{p}{16K_3}\int\frac{dr}{r} \cdot r^4 = \frac{p}{64K} r^4$$

$$W^{III} = \frac{\gamma(l_1+l_2)}{64K_3} \cdot r^4 + C_1 + C_2 r^2$$

$$\frac{dw}{dr} = \frac{\gamma(l_1+l_2)}{16 \cdot K_3} \cdot r^3 + 2C_2 r$$

granični uslovi:

$$r = a \begin{cases} W = 0 & (1) \\ M_r = 0 & (2) \end{cases}$$

$$(2): \quad M_r = -K\left(\frac{\partial^2 w}{\partial r^2} + \frac{\nu}{r}\frac{\partial w}{\partial r}\right)$$

$$M_r = -K\left(\frac{3}{16}\frac{\gamma(l_1+l_2)}{K_3} \cdot a^2 + 2C_2\right) = 0 \Rightarrow C_2 = -\frac{3a^2\gamma(l_1+l_2)}{32K_3}$$

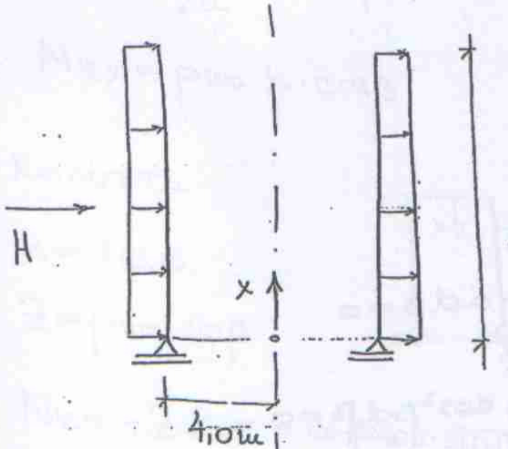
$$(1): \quad C_1 = -\frac{\gamma(l_1+l_2)}{64K_3}a^4 + \frac{3a^4\gamma(l_1+l_2)}{32K_3}$$

$$\Rightarrow X_3, X_4$$

④

25.10.2003.

3. Za cilindričnu ljusku, koja je opterećena opterećenjem $p_w = p_{w0} \cdot \sin\beta$, odrediti sile u preseku koristeći uslove ravnoteže konačnog elementa ljuske. U ljusci vlada membransko stanje naprezanja.

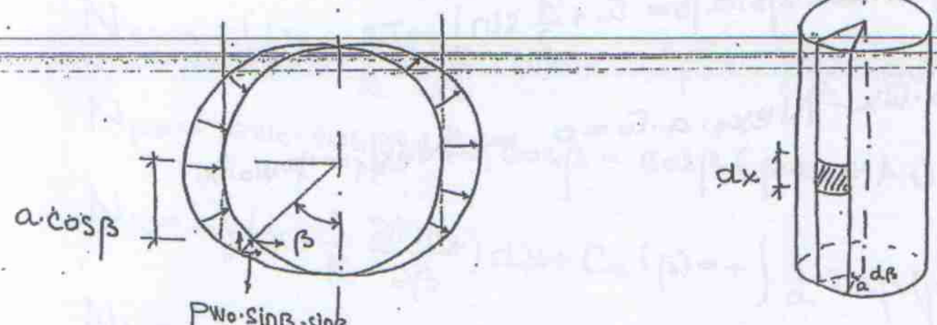


$E = 30\,GPa$

$\nu = 0{,}2$

$h = 0{,}10\,m$

$p_{w0} = 1{,}5\,KN/m^2$



$$H = \int_F p_w\,dF = \int_0^{2\pi}\int_0^{x} p_{w0}\cdot\sin\beta\cdot\sin\beta\cdot \underbrace{dx\cdot a\cdot d\beta}_{dF} = p_{w0}\int_0^{2\pi}\int_0^{x} a\cdot\sin^2\beta\, d\beta\, dx$$

$$H = a\cdot p_{w0}\cdot x\int_0^{2\pi}\sin^2\beta\, d\beta = a\,p_{w0}\cdot x\cdot\int_0^{2\pi}(1-\cos^2\beta)\,d\beta$$

$$\int_0^{2\pi}(1-\cos^2\beta)\,d\beta = \int_0^{2\pi}d\beta - \frac{1}{2}\int_0^{2\pi}(1+\cos2\beta)\,d\beta = 2\pi - \frac{1}{2}\cdot2\pi - \frac{1}{2}\sin2\beta\Big|_0^{2\pi}$$

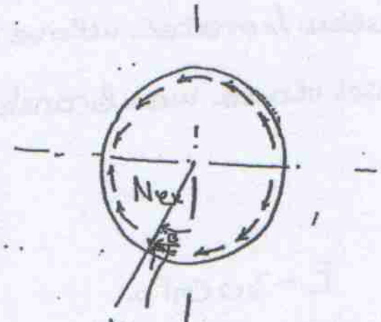
$$J_1 = \pi$$

$$H = a\cdot p_{w0}\cdot\pi\cdot x$$

$$N_\varphi = N_{\varphi 1} \cdot \sin\beta$$

$$N_{\varphi x} = N_{\varphi x 1} \cdot \cos\beta$$

$$N_x = N_{x1} \cdot \sin\beta$$



$N_{\varphi x} \cdot \cos\beta$

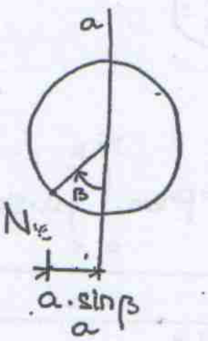
$$\Sigma H = 0 \qquad H - \int_0^{2\pi} N_{\varphi x 1} \cdot \cos\beta \cdot \cos\beta \cdot a\, d\beta = 0$$

$$a \cdot p_{w0} \cdot \pi \cdot x - N_{\varphi x 1} \cdot a \int_0^{2\pi} \cos^2\beta\, d\beta = 0$$

$$J_1 = \frac{1}{2} \int_0^{2\pi} (1 + \cos 2\beta)\, d\beta = \pi + \frac{1}{2} \sin \Big|_0^{2\pi} = \pi$$

$$a \cdot p_{w0} \cdot \pi x - N_{\varphi x 1} \cdot a \cdot \pi = 0 \implies N_{\varphi x 1} = p_{w0} \cdot x$$

$$\Sigma M_{a-a} = 0$$



$$H \cdot \frac{x}{2} - \int_0^{2\pi} N_{x1} \sin\beta \cdot a \sin\beta \cdot a\, d\beta = 0$$

$$a \cdot p_{w0} \cdot \pi \cdot x \cdot \frac{x}{2} - N_{x1} \cdot a^2 \int_0^{2\pi} \sin^2\beta\, d\beta = 0$$

$$a \cdot p_{w0} \cdot \pi \cdot x \frac{x}{2} - N_{x1} \cdot a^2 \cdot \pi = 0 \implies N_{x1} = \frac{p_{w0}}{2a} \cdot x^2$$

$$\frac{N_\varphi}{R} + Z = 0$$

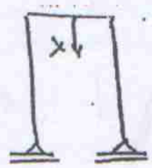
$$Z = p_w = p_{w0} \cdot \sin\beta$$

$$N_\varphi = -a \cdot p_{w0} \cdot \sin\beta$$

$$N_x = \frac{p_{w0}}{2a} x^2 \cdot \sin\beta$$

$$N_{\varphi x} = p_{w0} \cdot x \cdot \cos\beta$$

* Kontrola



$$X = Y \equiv 0$$

$$Z = p_{w0} \cdot \sin\beta$$

$$N_\varphi = -Z \cdot R = -a \cdot p_{w0} \cdot \sin\beta$$

$$N_{\varphi x} = -\int \left(Y + \frac{1}{R}\frac{\partial N_\varphi}{\partial \varphi}\right) dx + C_1(\beta) = +\int \frac{1}{a}\, a\, p_{w0} \cos\beta \, dx + C_1(\beta)$$

$$N_{\beta x} = p_{w0} \cdot \cos\beta x + A_1 \cdot \cos\beta = \cos\beta \left(p_{w0} x + A_1\right)$$

$$N_x = -\int \left(X + \frac{1}{R}\frac{\partial N_{\beta x}}{\partial \beta}\right) dx + C_2(\beta) = +\int \frac{1}{a} \sin\beta \left(p_{w0} x + A_1\right) dx + C_2(\beta)$$

$$N_x = \frac{1}{2a} \sin\beta \, p_{w0} \cdot x^2 + \frac{1}{a} \sin\beta \, A_1 + A_2 \sin\beta$$

granični uslovi:

$$x = 0 \begin{cases} N_x = 0 & (1) \\ N_{\beta x} = 0 & (2) \end{cases}$$

(1): $A_1 = 0$

(2): $A_2 = 0$

$$N_\beta = -a\, p_{w0} \cdot \sin\beta$$

$$N_{\beta x} = p_{w0} \cdot x \cdot \cos\beta$$

$$N_x = \frac{1}{2a} p_{w0} \cdot x^2 \cdot \sin\beta$$

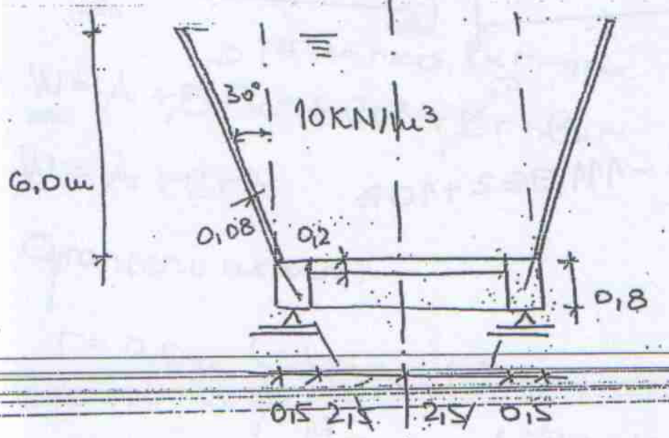
2: (5)..

23.09.2002.

3. Za rezervoar prikazan na slici:

a) Odrediti presečne sile u konusnoj ljusci. Veza između konusne ljuske i kružnog prstena je takva da u ljusci vlada membransko stanje naprezanja.

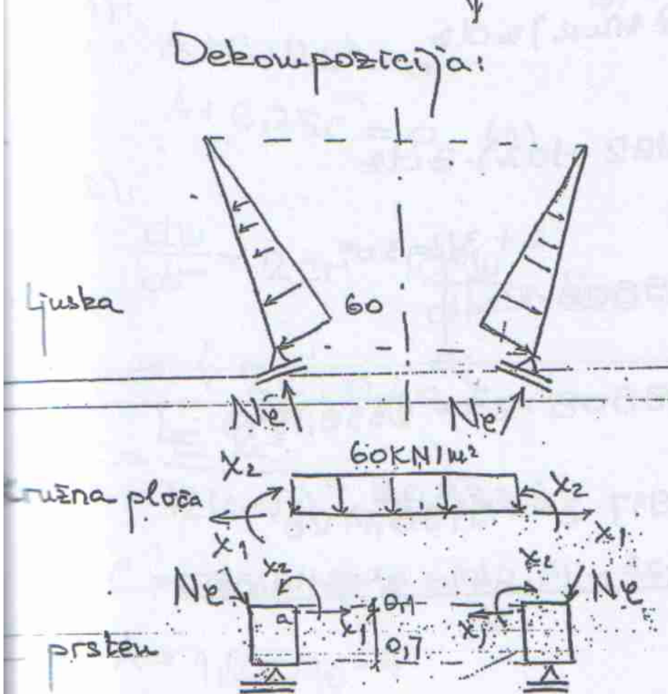
b) Sračunati maksimalni normalni napon u kružnom prstenu.



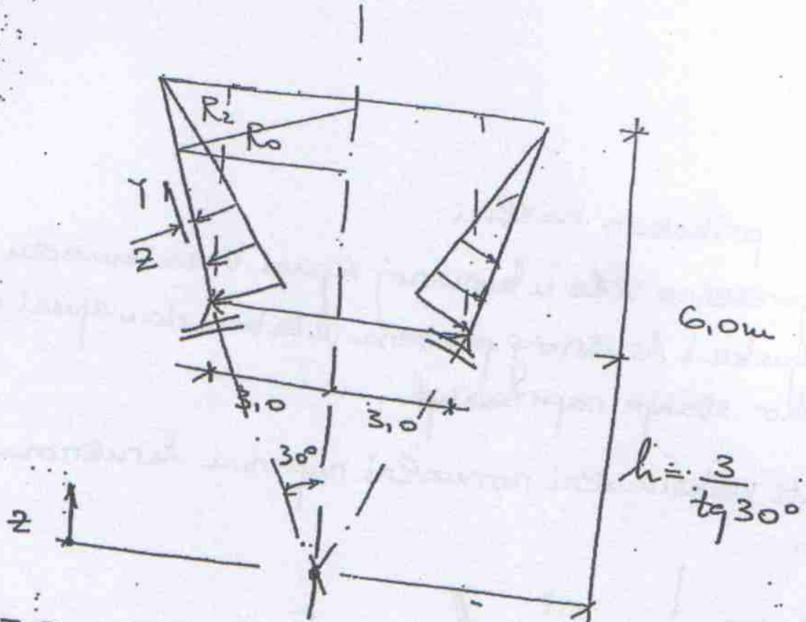
E=30 GPa

ν=0,2

Dekompozicija:



a).



$h = \frac{3}{tg30°} = 5,1962$

$Y \equiv 0$

$Z = -10\cdot(11,1962 - z) = -111,962 + 10z$

$R_1 \to \infty$

$R_0 = z \cdot tg\alpha$

$R_2 = \frac{R_0}{\cos\alpha} = z\,\frac{tg\alpha}{\cos\alpha}$

$P(z) = 2\pi\,\frac{tg\alpha}{\cos\alpha}\int_h^z (Y\cos\alpha + Z\sin\alpha)\,z\,dz$

$P(z) = 2\pi\,\frac{tg\alpha}{\cos\alpha}\int_{3/tg30°}^{z} (-111,962 + 10z)\cdot z\,dz$

$P(z) = 2\pi\,\frac{tg\alpha}{\cos\alpha}\left[\frac{10}{3}x^3 - 55,9808\cdot x^2\right]\Big|_{z}^{6+3/tg30°}$

$P(z) = 2\pi\,\frac{tg\alpha}{\cos\alpha}\left[-\frac{10}{3}z^3 + 55,9808\cdot z^2 - 2339,1343\right]$

$P(z) = -13,9626\cdot z^3 + 234,4917\cdot z^2 - 9798,1428$

$N_\varphi = \frac{-P(z)}{2\pi z\cdot\sin\alpha} = +\frac{4,4\cdot z^3 - 74,6410\cdot z^2 + 3118,8457}{z}$

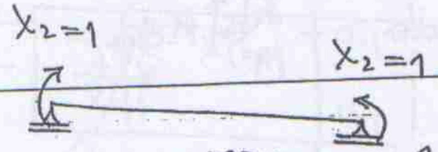
$\cancel{\frac{N_\varphi}{R_1}}^{0} + \frac{N_\theta}{R_2} + Z = 0 \Rightarrow N_\theta = -Z\cdot R_2 = (111,962 - 10z)\cdot z\,\frac{tg\alpha}{\cos\alpha}$

$N_\Theta = -6{,}\dot{6} \cdot z^2 + 74{,}6410 \cdot z$

$N_\Theta (z = 5{,}1962) = 332{,}3760$ KN/m

b)

Stanje $X_2 = 1$



$\underline{W} = A + \cancel{B}\ln r + Cr^2 + \cancel{D} r^2 \ln r$ 0 per za $r=0$, $\ln r \to \infty$

$\underline{W} = A + Cr^2$

Granični uslovi:

$$r = 2{,}5m \begin{cases} w = 0 \quad (1) \\ M_r = -k\left(\dfrac{\partial^2 w}{\partial r^2} + \dfrac{\nu}{r}\dfrac{\partial w}{\partial r}\right) = 1 \end{cases}$$

(1):

$A + C \cdot 2{,}5^2 = 0$

$A + 6{,}25C = 0 \quad (1)$

(2):

$\dfrac{dw}{dr} = 2Cr \qquad \dfrac{d^2w}{dr^2} = 2C$

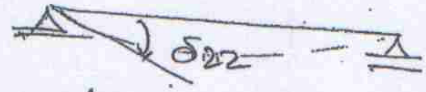
$-k(2C + 0{,}4C) = 1$

$k = \dfrac{E \cdot h^3}{12(1-\nu^2)} = 20833{,}\dot{3}$

$C = -2 \cdot 10^{-5}$

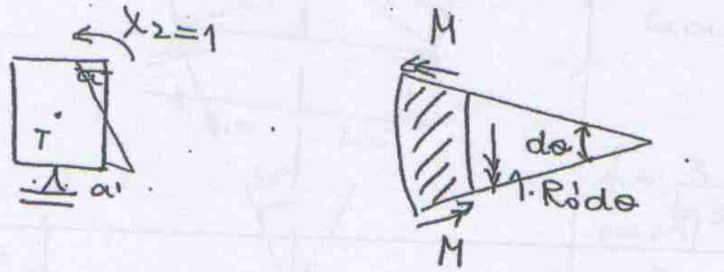
$A = 1{,}25 \cdot 10^{-4}$

$W = 1{,}25 \cdot 10^{-4} - 2 \cdot 10^{-5} \cdot r^2$



$$E\delta_{22}^{pl} = E\cdot\frac{dw}{dr} = E\cdot 2\cdot C = 4\cdot 10^{-5}\cdot 30\cdot 10^{6} = 1200$$

Prsten:



$$M d\theta = 1\cdot R_0' d\theta$$

$$M = R_0'$$

$$E\delta_{22}^{pr} = \frac{Eu_a - Eu_{a'}}{0,7}$$

$$Eu_a = E\varepsilon_a\cdot R_0' = E\frac{\sigma_a}{E}R_0' = \frac{M}{I}\cdot z\cdot R_0'$$

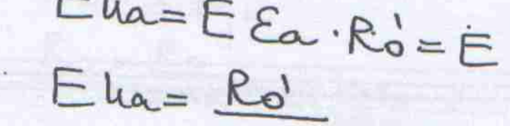
$$Eu_a = \frac{R_0'}{\frac{1}{12}\cdot 0,5\cdot 0,8^3}\cdot 0,3\cdot R_0' = 106,3477 = -E\delta_{12}$$

$$Eu_{a'} = -\frac{R_0'}{\frac{1}{12}\, 0,5\cdot 0,8^3}\cdot 0,4\, R_0' = -141,7969$$

$$E\delta_{22}^{pr} = 354,4922$$

$$E\delta_{22} = E\delta_{22}^{pl} + E\delta_{22}^{pr} = 1554,4922$$

<u>Stange $X_1=1$</u>



$$F = \cancel{D} + \cancel{A\ln r} + Br^2 + \cancel{Cr^2\ln r}$$

$$F = Br^2$$

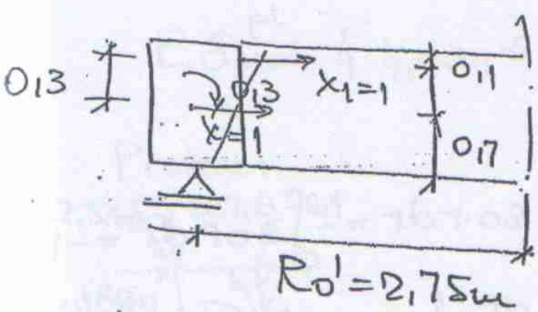
Granicni uslovi: $\left\{ r = 2,5m \Rightarrow N_r = 1,0 \right.$

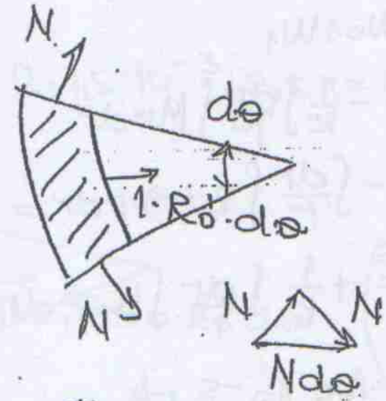
$N_r = N_\varphi = 1{,}0 \quad (B = 0{,}5)$

$E\delta_{11}^{pl} = Eu = E \cdot \varepsilon_\varphi \cdot r = E \dfrac{1}{Eh}(N_\varphi - \nu N_r) r$

$E\delta_{11}^{pl} = \dfrac{1}{0{,}2} \, 0{,}8 \cdot 2{,}5 = 10$

Prsten:

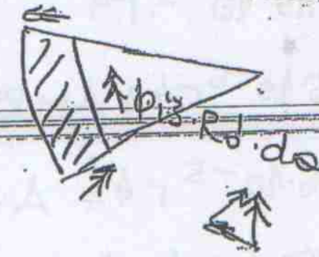




$N d\varphi + 1 \cdot R_0' \cdot d\varphi = 0$

$N = -R_0'$

$\cancel{M d\varphi + 0{,}3 R_0' d\varphi = 0}$



$M = -0{,}3 R_0'$

$E\delta_{11} = E u_a = E \cdot \varepsilon_a \cdot R_0' = \sigma_a \cdot R_0'$

$E\delta_{11}^{pr} = -\left(\dfrac{-R_0'}{b \cdot d} + \dfrac{-0{,}3 R_0'}{\frac{1}{12} b d^3} \cdot 0{,}3 \right) \cdot R_0' = +50{,}8105$

$E\delta_{21}^{pr} = +E \, \dfrac{u_a - u_a'}{0{,}7}$

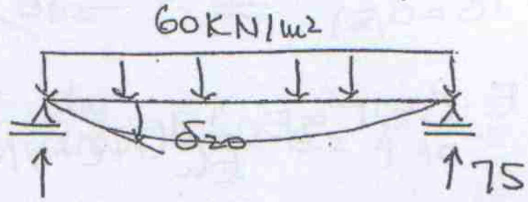
$E u_a' = \left(-\dfrac{R_0}{b \cdot d} - \dfrac{-0{,}3 R_0'}{\frac{1}{12} b \cdot d^3} \cdot 0{,}4 \right) \cdot 2{,}75 = 23{,}6328$

$E\delta_{21}^{pr} = + \dfrac{+50{,}8105 + 23{,}6328}{0{,}7} = -106{,}3477$

$E\delta_{11} = E\delta_{11}^{pl} + E\delta_{11}^{pr} = 60{,}8105$

Stanje $X_i=0$

Ploča:



$$E\delta_{20}=E\left(\frac{dw}{dr}\right)_{r=2,5}$$

$$w=w_0+w_1$$

$$w_0=-\frac{1}{K}\int\frac{dr}{r}\int M_r\,dr$$

$$M=-\int\frac{dr}{r}\int z(r)\cdot r\,dr=-\int\frac{dr}{r}\int 60\cdot r\,dr=-\int 30r\,dr=-15r^2$$

$$w_0=+\frac{1}{K}\int\frac{dr}{r}\int 15r^3\,dr=\frac{15}{K}\int\frac{dr}{r}\,\frac{1}{4}r^4=\frac{15}{16K}r^4$$

$$w_0=4{,}5\cdot10^{-5}\cdot r^4$$

$$w=4{,}5\cdot10^{-5}r^4+A_1+\cancel{B}^{\to 0}\ln r+Cr^2+\cancel{D}^{\to 0}r^2\ln r$$

$$w=4{,}5\cdot10^{-5}\,r^4+A+Cr^2$$

Granični uslovi:

$$r=2{,}5\ \begin{cases} w=0 & (1)\\ M_r=0 & (2)\end{cases}$$

(1):

$$A+C\cdot2{,}5^2=-4{,}5\cdot10^{-5}\cdot2{,}5^3$$

$$A+6{,}25C=-1{,}7578\cdot10^{-3}\ ;(1)$$

(2)

$$M_r=-K\left(\frac{\partial^2w}{\partial r^2}+\frac{\nu}{r}\frac{\partial w}{\partial r}\right)=0$$

$$\frac{\partial w}{\partial r}=2Cr+1{,}8\cdot10^{-4}r^3$$

$$\frac{\partial^2w}{\partial r^2}=2C+5{,}4\cdot10^{-4}r^2$$

$$2\cdot C+5{,}4\cdot 10^{-4}\cdot 2{,}5^2+0{,}4\,C+0{,}2\cdot 1{,}8\cdot 10^{-4}\cdot 2{,}5^2=0$$

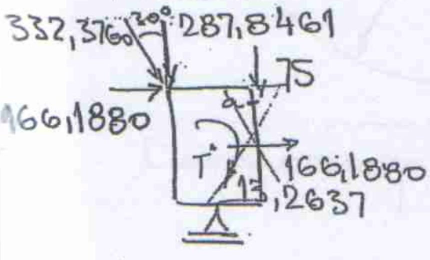
$$C=-1{,}5\cdot 10^{-3}$$

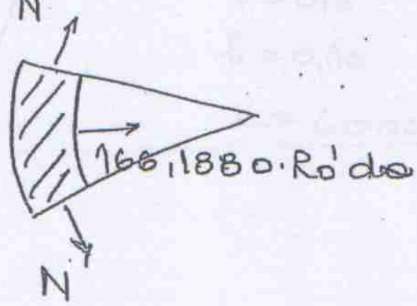
$$A=8{,}671875\cdot 10^{-3}$$

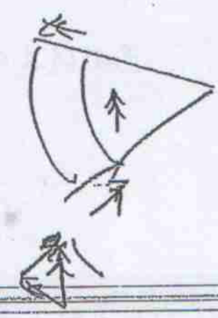
$$W=4{,}5\cdot 10^{-5}\,r^4+8{,}671875\cdot 10^{-3}-1{,}5\cdot 10^{-3}\,r^2=0$$

$$E\delta_{20}^{pl}=4\cdot 4{,}5\cdot 10^{-5}\cdot 2{,}5^3-2\cdot 1{,}5\cdot 10^{-3}\cdot 2{,}5=140625$$

Prsten:







$$\underline{N\,d\varphi=-166{,}1880\cdot R_o'\,d\varphi}$$

$$N=-166{,}1880\cdot R_o'$$

$$M\,d\varphi=-13{,}2637\cdot R_o'\,d\varphi$$

$$M=-13{,}2637\cdot R_o'$$

$$Eu_a=E\cdot\varepsilon_a\cdot R_o'=E\frac{\sigma_a}{E}\cdot R_o'=\left(\frac{N}{bd}+\frac{M}{I}\cdot 0{,}3\right)\cdot R_o'$$

$$Eu_a=\left(-\frac{166{,}188\cdot 2{,}75}{0{,}5\cdot 0{,}8}-\frac{13{,}2637\cdot 2{,}75}{\frac{1}{12}\cdot 0{,}5\cdot 0{,}8^3}\cdot 0{,}3\right)\cdot 2{,}75$$

$$Eu_a=+6647{,}2156=+E\delta_{10}^{pr}$$

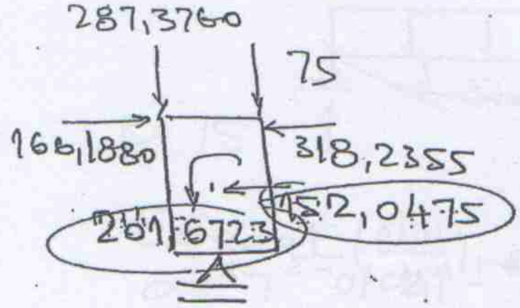
$$E\delta_{20}^{pr}=E\frac{u_a-u_a'}{0{,}7}=\frac{6647{,}2156+1220{,}3290}{0{,}7}=$$

$$E\delta_{20}^{pr}=11239{,}3494$$

$$E\delta_{20}=151864{,}3494$$

$x_1 = -318{,}2355$

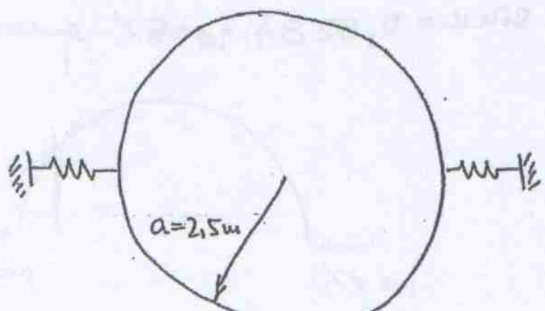
$x_2 = -119{,}4653$



⑥

21.09.2008.

3. Za sfernu ljusku prikazanu na slici odrediti presečne sile i reakciju elastičnog linijskog oslonca usled zagrevanja ljuske za 50°C.



$\alpha_t = 10^{-5}\ 1/°C$

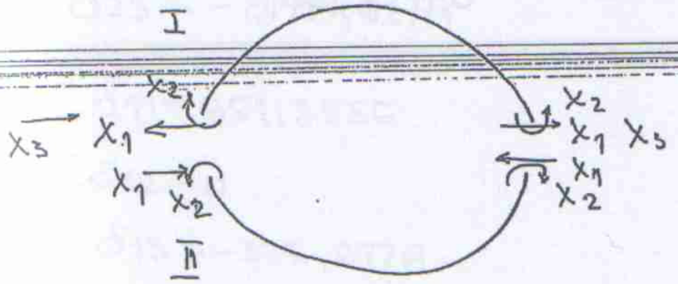
$E = 30\ GPa$

$\nu = 0{,}3$

$h = 0{,}10$

$K = 60000\ kN/m^2$

Dekompozicija:



$\cancel{\delta_{11}X_1 + \delta_{12}X_2 + \delta_{13}X_3 + \delta_{10} = 0}$

$\delta_{21}X_1 + \delta_{22}X_2 + \delta_{23}X_3 + \delta_{20} = 0$

$\delta_{31}X_1 + \delta_{32}X_2 + \delta_{33}X_3 + \delta_{30} = 0$

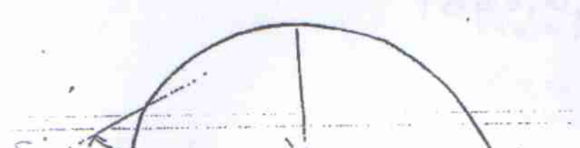
$\delta_{11} = \delta_{11}^{I} + \delta_{11}^{II}$, $\delta_{12} = \delta_{12}^{I} + \delta_{12}^{II}$, $\delta_{13} = \delta_{13}^{I}$

$\delta_{21} = \delta_{12}$, $\delta_{22} = \delta_{22}^{I} + \delta_{22}^{II}$, $\delta_{23} = \delta_{23}^{I}$

$\delta_{31} = \delta_{31}^{I}$, $\delta_{32} = \delta_{32}^{I}$, $\delta_{33} = \delta_{33}^{I} + \delta_{33}^{III} = \delta_{33}^{I} + \frac{1}{K}$

$\delta_{10} = \delta_{10}^{I} + \delta_{10}^{II}$, $\delta_{20} = \delta_{20}^{I} + \delta_{20}^{II}$, $\delta_{30} = \delta_{30}^{I}$

$X_1 = 1$



$$\delta_{11} = \Delta R_0(\varphi=\alpha) = 2\aleph \frac{R_0}{E h} \sin\alpha$$

$$\aleph = \sqrt{\frac{a}{h}\sqrt{3(1-\nu^2)}} = 6{,}5136$$

$$R_0 = a \cdot \sin\varphi = a \cdot \sin 90^\circ = a = 2{,}5\,m$$

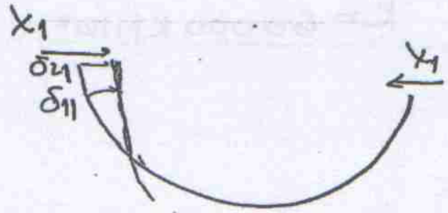
$$\delta_{11}^{I} = 2 \cdot 6{,}5136 \frac{2{,}5}{30 \cdot 10^6 \cdot 0{,}1} \cdot \sin 90 = 1{,}0856 \cdot 10^{-5}$$

$$\delta_{21}^{I} = \chi_{(\varphi=\alpha)} = \frac{2 \cdot \aleph^2}{E \cdot h} \cdot \sin\alpha = 2{,}8284 \cdot 10^{-5}$$

$$E\delta_{11}^{I} = 325{,}6778$$

$$E\delta_{21}^{I} = 848{,}5281$$

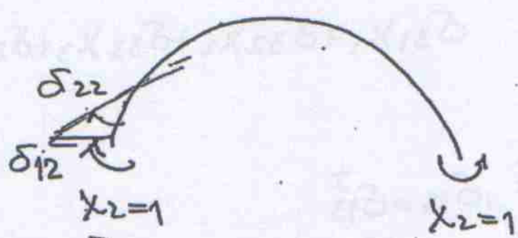
$$E\delta_{31}^{I} = -325{,}6778$$



$$E\delta_{11}^{II} = 325{,}6778$$

$$E\delta_{21}^{II} = -848{,}5281$$

Stange $X_2 = 1$



$$\delta_{22}^{I} = \frac{a}{K \cdot \aleph} \quad , K = \frac{E h^3}{12(1-\nu^2)} = 2747{,}2527$$

$$E\delta_{22}^{I} = \frac{2{,}5}{K \cdot \aleph} = 4191{,}2592$$

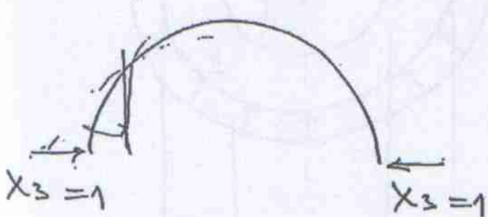
$$E\delta_{12}^{I} = E \cdot \frac{2\aleph^2}{E h} \cdot \sin\alpha = 848{,}5281$$

$$E\delta_{32}^{I} = -848{,}5281$$

$\delta_{22}^{\text{II}} = 4191,2592$

$\delta_{12}^{\text{II}} = -848,5281$

Stanje $X_3=1$



$\delta_{33}^{\text{I}} = 325,6778$

$\delta_{13}^{\text{I}} = -325,6778$

$\delta_{23}^{\text{I}} = -848,5281$

$E\delta_{11} = 651,3556$

$E\delta_{12} = 0$

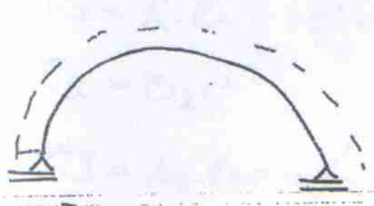
$E\delta_{13} = -325,6778$

$E\delta_{22} = 8382,5184$

$E\delta_{23} = -848,5281$

$E\delta_{33} = 325,6778 + \frac{1}{60000} \cdot E = 825,6778$

Stanje $X_i = 0 \quad t = 50°C$



$Y = Z = 0$

$N_\varphi = N_\theta = 0$

$\varepsilon_\varphi = \varepsilon_\theta = \alpha_t \cdot t$

$E\delta_{10}^{\text{I}} = E \cdot \Delta R_0 = E \cdot \varepsilon_\theta \cdot R_0 = \alpha_t \cdot t \cdot 2,5 \cdot 30 \cdot 10^6 = 37500$

$E\delta_{20}^{\text{I}} = E \cdot \chi$

$$\chi = \frac{1}{R_1}\left(v + \frac{dw}{d\varphi}\right)$$

$$v = \left[\int \frac{R_1 \varepsilon_t - \cancel{R_2 \varepsilon_\varphi}^{\,0}}{\sin\varphi} d\varphi + c\right] \sin\varphi = c \cdot \sin\varphi$$

Granični uslov: $\varphi = 90°$ $v = 0$ $\Rightarrow$ $c = 0$

$v = 0$

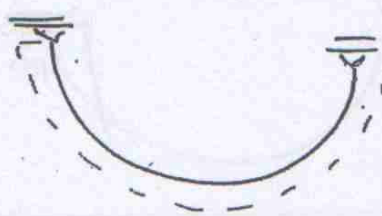
$W = v \cdot ctg\varphi - \varepsilon_\varphi \cdot R_2 = -\alpha t \cdot t \cdot a$

$\frac{dw}{d\varphi} = 0$

$\chi = 0$

$E\delta_{20}^{I} = 0$

$E\delta_{30}^{I} = -37500$



$E\delta_{10}^{II} = -37500$

$E\delta_{20}^{II} = 0$

$E\delta_{10} = E\delta_{10}^{I} + E\delta_{10}^{II} = 0$

$E\delta_{20} = 0$

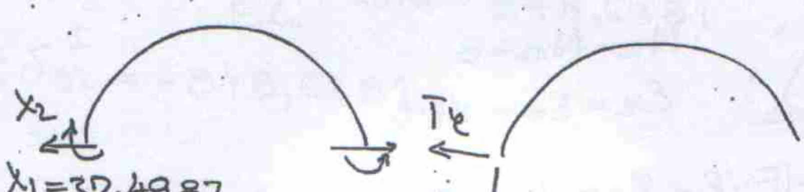
$E\delta_{30} = -37500$

$X_1 = 32,4987$

$X_2 = 6,5794$

$X_3 = 64,9974 = R$

Sile u preseku:



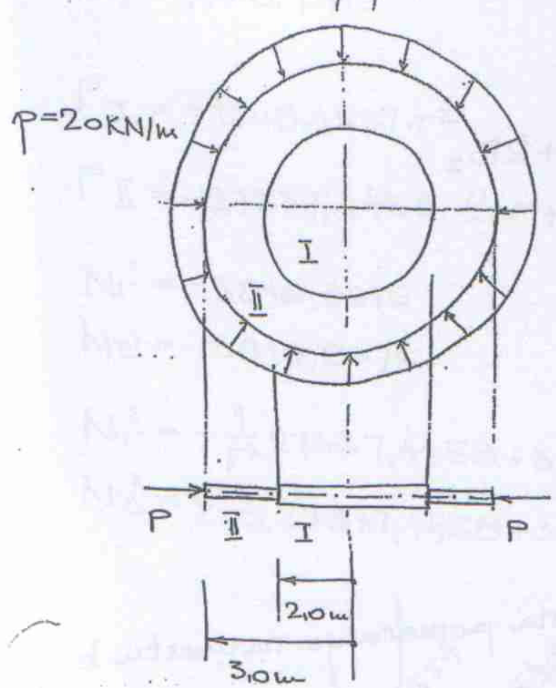
$X_1 = 32,4987$

$T_\varphi = 32,4987$

$M_\varphi = 6,5794$ $N_\varphi = 0$

⑦ 2

2. Usled zadatog opterećenja i zagrevanja ploče 1 za $t=100°C$, sračunati i nacrtati dijagrame sila u preseku.



$E = 210\,GPa$

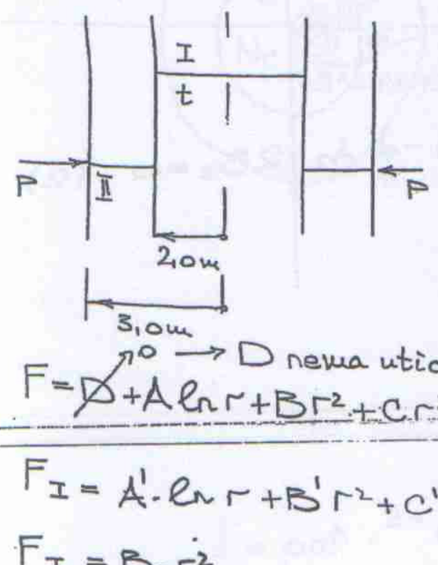
$\nu = 0{,}3$

$\alpha_t = 10^{-5}\ 1/°C$

$h_I = 0{,}1\,m$

$h_{II} = 0{,}06\,m$

Zbog diskontinuiteta debljine, ali i zbog dejstva t samo na jednu ploču, sistem razdvajamo na dve ploče



$$F = \cancel{D} + A\ln r + Br^2 + Cr^2\ln r$$

$D \to 0$ → D nema uticaja na naprezanje, pa možemo da usvojimo da je $D=0$

$$F_I = A'\cdot \ln r + B' r^2 + C' r^2 \ln r \Rightarrow A' = C' = 0 \ (\text{jer za } r=0, \ln r \to \infty$$

$$F_I = B_2 r^2$$

$$F_{II} = A_2 \cdot \ln r + B_2 \cdot r^2 + \cancel{C_2} r^2 \ln r$$

$C_2 = 0$ → Važi za kružne ploče napregnute u svojoj ravni i opterećene rotaciono simetričnim opterećenjem po svojoj konturi

$$F_{II} = A_2 \cdot \ln r + B_2 \cdot r^2$$

$$N_r = \frac{1}{r}\frac{dF}{dr}$$
$$N_\varphi = \frac{d^2F}{dr^2}$$

$$N_r' = \frac{1}{r}(2B_1 r) = 2B_1$$
$$N_\varphi' = 2B_1$$

$$N_r'' = \frac{1}{r}\left(\frac{1}{r}A_2 + 2B_2 r\right) = \frac{1}{r^2}A_2 + 2B_2$$
$$N_\varphi'' = -\frac{1}{r^2}A_2 + 2B_2$$

Granični uslovi:

$$r = 3{,}0m \left\{ N_r'' = -p \quad (1) \right.$$

Prelazni uslovi:

$$r = 2{,}0m \begin{cases} N_r' = N_r'' \\ u_r' = u_r'' \end{cases}$$ $\rightarrow$ radijalna pomeranja na mestu kontakta

(1): $r = 3{,}0m$

$$\frac{1}{9}A_2 + 2B_2 = -20 \quad (1)$$

(2) $r = 2{,}0m$

$$2B_1 = \frac{1}{4}A_2 + 2B_2 \Rightarrow 2B_1 - \frac{1}{4}A_2 - 2B_2 = 0 \quad (2)$$

(3) $r = 2{,}0m$

$$u_r = r \cdot \varepsilon_\varphi$$
$$\varepsilon_\varphi' = \frac{1}{Eh_1}(N_\varphi' - \nu N_r') + \alpha_t \cdot t^o$$
$$\varepsilon_\varphi'' = \frac{1}{Eh_2}(N_\varphi'' - \nu N_r'')$$

$$\frac{1}{210\cdot 10^6\cdot 0{,}1}[2B_1 - 0{,}6B_1] + 10^{-5}\cdot 100 =$$
$$= \frac{1}{210\cdot 10^6\cdot 0{,}06}\left[-\frac{1}{4}A_2 + 2B_2 - \frac{1}{4}\cdot 0{,}3A_2 - 0{,}6B_2\right]$$

$$0{,}084\cdot B_1 + 1260 = -0{,}0325\cdot A_2 + 0{,}14B_2 \quad (3)$$

$B_1 = -1908,4337$

$A_2 = -27337,4458$

$B_2 = 1508,7470$

$F_I = -1908,4337 \cdot r^2$

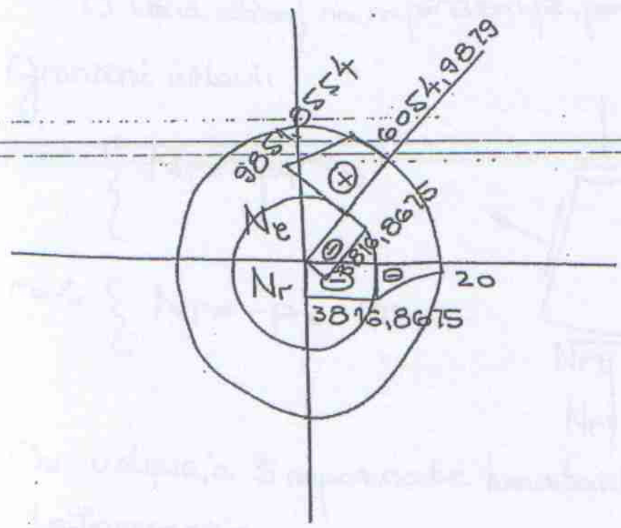
$F_{II} = -27337,4458 \cdot \ln r + 1508,7470 \cdot r^2$

$N_r' = -3816,8675$

$N_\theta' = -3816,8675$

$N_r'' = -\frac{1}{r^2} 27337,4458 + 3017,4940$

$N_\theta'' = +\frac{1}{r^2} 27337,4458 + 3017,4940$



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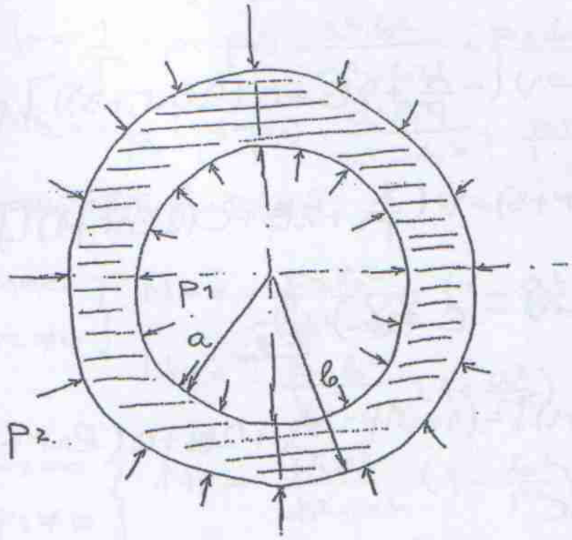
2.



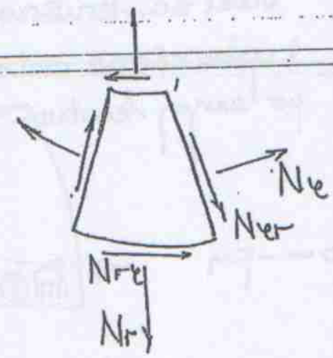
$$F = \cancel{D}^{0} + A\ln r + Br^2 + Cr^2\ln r$$

D nema uticaj na naprezanje, pa možemo da usvojimo da je $D=0$

Granični uslovi:

$r=a \; \{ \; N_r = -p_1 \quad (1)$

$r=b \; \{ \; N_r = -p_2 \quad (2)$



Dva uslova, a 3 nepoznate konstante, treba nam još jedan uslov → deformacija

$$\varepsilon_r = \frac{du}{dr}$$

$$\varepsilon_\varphi = \frac{u}{r} \Rightarrow u = r\cdot\varepsilon_\varphi \Rightarrow \frac{du}{dr} = \varepsilon_\varphi + r\cdot\frac{d\varepsilon_\varphi}{dr}$$

$$\boxed{\varepsilon_r = \varepsilon_\varphi + r\frac{d\varepsilon_\varphi}{dr}} \quad (3)$$

$$\varepsilon_r = \frac{1}{Eh}(N_r - \nu N_\varphi)$$

$$\varepsilon_\varphi = \frac{1}{Eh}(N_\varphi - \nu N_r)$$

$$N_r = \frac{1}{r}\frac{dF}{dr} = \frac{1}{r^2}A + 2B + C(2\ln r + 1)$$

$$N_\varphi = \frac{d^2F}{dr^2} = -\frac{1}{r^2}A + 2B + C(2\ln r + 3).$$

$$\frac{1}{Eh}\left[\frac{A}{r^2} + 2B + C(\ln r + 1) - \nu\left(-\frac{A}{r^2} + 2B + C(2\ln r + 3)\right)\right] =$$

$$= \frac{1}{Eh}\left[-\frac{A}{r^2} + 2B + C(2\ln r + 3) - \nu\left(\frac{A}{r^2} + 2B + C(2\ln r + 1)\right)\right] +$$

$$+ r\left(\frac{2}{r^3}A + \frac{2}{r}C + \frac{2}{r^3}A\cdot\nu - C\frac{2\nu}{r}\right)\cdot\frac{1}{Eh}$$

$$(1+\nu)\left[\frac{A}{r^2} + 2B + C(\ln r + 1)\right] - (1+\nu)\left[-\frac{A}{r^2} + 2B + C(\ln r + 3)\right] =$$

$$= (1+\nu)\frac{2}{r^2}A + (1-\nu)2C$$

$$(1+\nu)\left[\frac{2A}{r^2} - 2C\right] = (1+\nu)\frac{2A}{r^2} + (1-\nu)2C$$

$$2C(1+\nu+1-\nu) = 0 \Rightarrow C = 0$$

$\boxed{C=0}$ ⟶ Važi za kružne ploče napregnute u svojoj ravni i opterećene rotaciono simetričnim opterećenjem po svojoj konturi.

(1): $r = a$

$$\frac{1}{a^2}A + 2B = -p_1 \Rightarrow B = -\frac{p_1}{2} - \frac{A}{2a^2}$$

(2): $r = b$

$$\frac{1}{b^2}A + 2B = -p_2$$

$$\frac{1}{b^2}A - p_1 - \frac{A}{a^2} = -p_2 \Rightarrow A\left(\frac{1}{b^2} - \frac{1}{a^2}\right) = p_1 - p_2$$

$$A = (p_1 - p_2)\frac{a^2b^2}{a^2 - b^2}$$

$$B = -\frac{p_1}{2} - (p_1 - p_2)\frac{b^2}{2(a^2 - b^2)} = \frac{-p_1a^2 + \cancel{p_1b^2} - \cancel{p_1b^2} + p_2b^2}{2(a^2 - b^2)}$$

$$B = \frac{-p_1a^2 + p_2b^2}{2(a^2 - b^2)}$$

$$F = (p_1 - p_2)\frac{a^2 b^2}{a^2 - b^2}\cdot \ln r + \frac{-p_1 a^2 + p_2 b^2}{2(a^2 - b^2)}\cdot r^2$$

$$N_r = \frac{1}{r^2}(p_1 - p_2)\frac{a^2 b^2}{a^2 - b^2} + \frac{p_2 b^2 - p_1 a^2}{a^2 - b^2}$$

$$N_\varphi = -\frac{1}{r^2}(p_1 - p_2)\frac{a^2 b^2}{a^2 - b^2} + \frac{p_2 b^2 - p_1 a^2}{a^2 - b^2}$$

Specijalni slučajevi

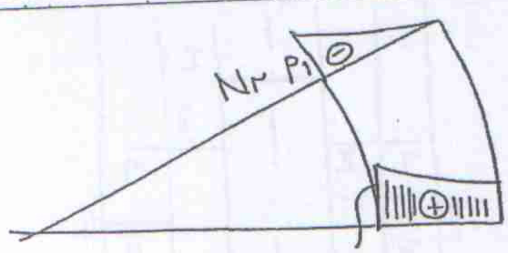
$$\left.\begin{matrix} p_1 = 0 \\ p_2 \neq 0 \end{matrix}\right\} \quad N_r = -\frac{p_2 b^2}{(b^2 - a^2)}\left(1 - \frac{a^2}{r^2}\right)$$

$$N_\varphi = -\frac{p_2 b^2}{b^2 - a^2}\left(1 + \frac{a^2}{r^2}\right)$$

$$\left.\begin{matrix} p_2 = 0 \\ p_1 \neq 0 \end{matrix}\right\} \quad N_r = \frac{p_1 a^2}{b^2 - a^2}\left(1 - \frac{b^2}{r^2}\right)$$

$$N_\varphi = \frac{p_1 a^2}{b^2 - a^2}\left(1 + \frac{b^2}{r^2}\right)$$

Za $p_2 = 0$
$p_1 \neq 0$



$$N_{\varphi,max} = \frac{b^2 + a^2}{b^2 - a^2}\, p_1$$

$$F = (p_1 - p_2)\frac{a^2 b^2}{a^2 - b^2}\cdot \ln r + \frac{-p_1 a^2 + p_2 b^2}{2(a^2 - b^2)}\cdot r^2$$

$$N_r = \frac{1}{r^2}(p_1 - p_2)\frac{a^2 b^2}{a^2 - b^2} + \frac{p_2 b^2 - p_1 a^2}{a^2 - b^2}$$

$$N_\varphi = -\frac{1}{r^2}(p_1 - p_2)\frac{a^2 b^2}{a^2 - b^2} + \frac{p_2 b^2 - p_1 a^2}{a^2 - b^2}$$

Specijalni slučajevi

$$\left.\begin{matrix} p_1 = 0 \\ p_2 \neq 0 \end{matrix}\right\} \quad N_r = -\frac{p_2 b^2}{(b^2 - a^2)}\left(1 - \frac{a^2}{r^2}\right)$$

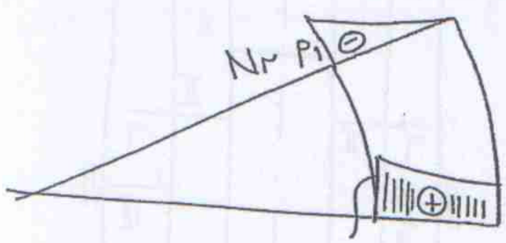
$$N_\varphi = -\frac{p_2 b^2}{b^2 - a^2}\left(1 + \frac{a^2}{r^2}\right)$$

$$\left.\begin{matrix} p_2 = 0 \\ p_1 \neq 0 \end{matrix}\right\} \quad N_r = \frac{p_1 a^2}{b^2 - a^2}\left(1 - \frac{b^2}{r^2}\right)$$

$$N_\varphi = \frac{p_1 a^2}{b^2 - a^2}\left(1 + \frac{b^2}{r^2}\right)$$

Za $p_2 = 0$

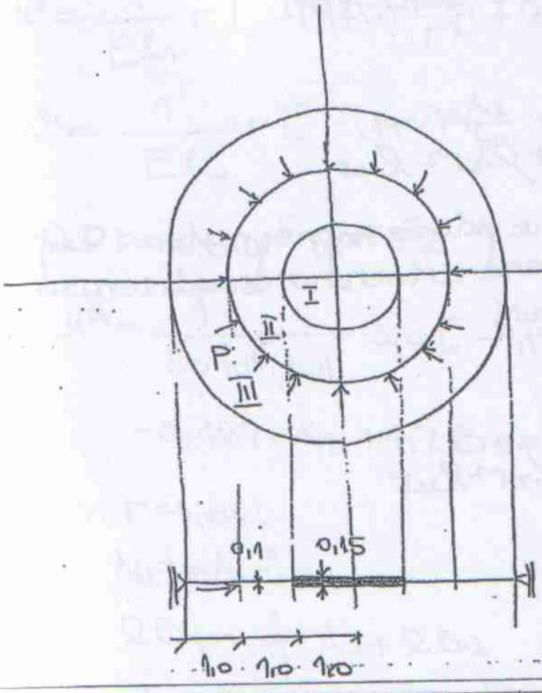
$p_1 \neq 0$

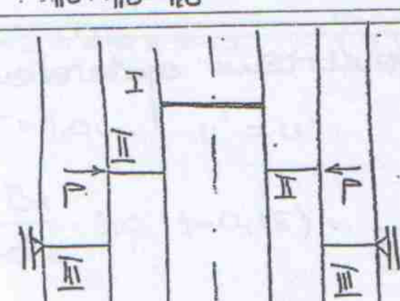


$$N_{\varphi,max} = \frac{b^2 + a^2}{b^2 - a^2} p_1$$

9

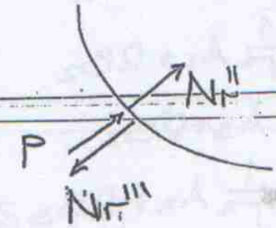
2. Za ploču opterećenu u svojoj ravni sračunati i nacrtati dijagrame presečnih sila.





Granični uslovi:

$r = 3{,}0\,m \; \{ \; u_r^{III} = 0 \quad (1)$



Prelazni uslovi:

$$r = 1{,}0\,m \; \begin{cases} N_r^{I} = N_r^{II} & (2) \\ u_r^{I} = u_r^{II} & (3) \end{cases}$$

$$r = 2{,}0\,m \; \begin{cases} N_r^{II} = N_r^{III} - p & (4) \\ u_r^{II} = u_r^{III} & (5) \end{cases}$$

$$F = \cancel{D}^{\to 0} + A\ln r + Br^2 + Cr^2 \ln r$$

D – ne utiče na naprezanje, pa se usvaja da je $D=0$

$$F_{I} = \cancel{D_1}^{\to 0} + \cancel{A_1}^{\to 0}\ln r + B_1 r^2 + \cancel{C_1}^{\to 0} r^2 \ln r$$

$A_1 = C_1 = 0$ (jer za $r=0$, $\ln r \to \infty$)

$$F_{I} = B_1 r^2$$

$$F_{II} = \cancel{D_2}^{\to 0} + A_2 \ln r + B_2 r^2 + \cancel{C_2}^{\to 0} r^2 \ln r$$

$C_2 = 0$ – važi za kružne pločice napregnute u svojoj ravni i opterećene rotaciono simetričnim optere… po svojoj konturi

$$F_{II} = A_2 \ln r + B_2 r^2$$

$$F_{III} = \cancel{D_3}^{\to 0} + A_3 \ln r + B_3 r^2 + \cancel{C_3}^{\to 0} r^2 \ln r$$

$$F_{III} = A_3 \ln r + B_3 r^2$$

$$\left.\begin{aligned} N_r &= \frac{1}{r}\frac{dF}{dr} \\ N_\varphi &= \frac{d^2F}{dr^2} \\ N_{r\varphi} &= 0 \end{aligned}\right\} \text{rotaciono simetrično opterećenje}$$

$$N_r' = \frac{1}{r} 2B_1 r = 2B_1$$

$$N_\varphi' = 2B_1$$

$$N_r'' = \frac{1}{r^2}A_2 + 2B_2$$

$$N_\varphi'' = -\frac{1}{r^2}A_2 + 2B_2$$

$$N_r''' = \frac{1}{r^2}A_3 + 2B_3$$

$$N_\varphi''' = -\frac{1}{r^2}A_3 + 2B_3$$

$$\varepsilon_\varphi = \frac{u}{r} \Rightarrow u = r \cdot \varepsilon_\varphi$$

$$\varepsilon_\varphi = \frac{1}{Eh}(N_\varphi - \nu N_r)$$

$$u' = \frac{1}{Eh_1} \cdot r\,(2B_1 - \nu\, 2B_1) = \frac{2B_1}{Eh} \cdot r\,(1-\nu)$$

$$u'' = \frac{1}{Eh_2} \cdot r\left(-\frac{A_2}{r^2} + 2B_2 - \nu\,\frac{A_2}{r^2} - \nu\, 2B_2\right)$$

$$u'' = \frac{1}{Eh_2}\, r\left[-(1+\nu)\frac{A_2}{r^2} + 2B_2(1-\nu)\right]$$

$$u''' = \frac{1}{Eh_3}\, r\left[-(1+\nu)\cdot\frac{A_3}{r^2} + 2B_3(1-\nu)\right]$$

(1): $r = 3{,}0\,m \quad u_r''' = 0$

$$u''' = \frac{1}{30\cdot 10^6 \cdot 0{,}1} \cdot 3{,}0\left[-1{,}15 \cdot \frac{A_3}{9} + 1{,}7\, B_3\right] = 0$$

$$-0{,}12\dot{7} \cdot A_3 + 1{,}7\, B_3 = 0 \quad (1)$$

(2): $r = 1{,}0\,m$

$$N_r' = N_r''$$

$$2B_1 = \frac{1}{1} A_2 + 2B_2$$

$$-2B_1 + A_2 + 2B_2 = 0 \quad (2)$$

(3): $r = 1{,}0\,m \quad u' = u''$

$$\frac{2B_1}{E \cdot 0{,}15} \cdot 1{,}0\,(1 - 0{,}15) = \frac{1}{E \cdot 0{,}10} \cdot 1{,}0\left[-1{,}15 \cdot A_2 + 1{,}7 B_2\right]$$

$$11{,}3\dot{3} \cdot B_1 + 11{,}5 \cdot A_2 - 17 B_2 = 0 \quad (3)$$

(4): $r = 2{,}0\,m$

$$N_r'' = N_r''' - p$$

$$\frac{1}{4} A_2 + 2B_2 = \frac{1}{4} A_3 + 2B_3 - p$$

$$\frac{1}{4} A_2 + 2B_2 - \frac{1}{4} A_3 - 2B_3 = -50 \; (4)$$

(5): $r = 2{,}0$

$$u'' = u'''$$

$$-1{,}15\, \frac{A_2}{4} + 1{,}7 B_2 = -1{,}15\, \frac{A_3}{4} + 1{,}7\, B_3$$

$$-0{,}2875\, A_2 + 1{,}7 B_2 + 0{,}2875 \cdot A_3 - 1{,}7 B_3 = 0 \quad (5)$$

$B_1 = -9{,}5409$

$A_2 = -2{,}7033$

$B_2 = -8{,}1893$

$A_3 = 82{,}2967$

$B_3 = 6{,}1857$

$N_r' = -19{,}0819$

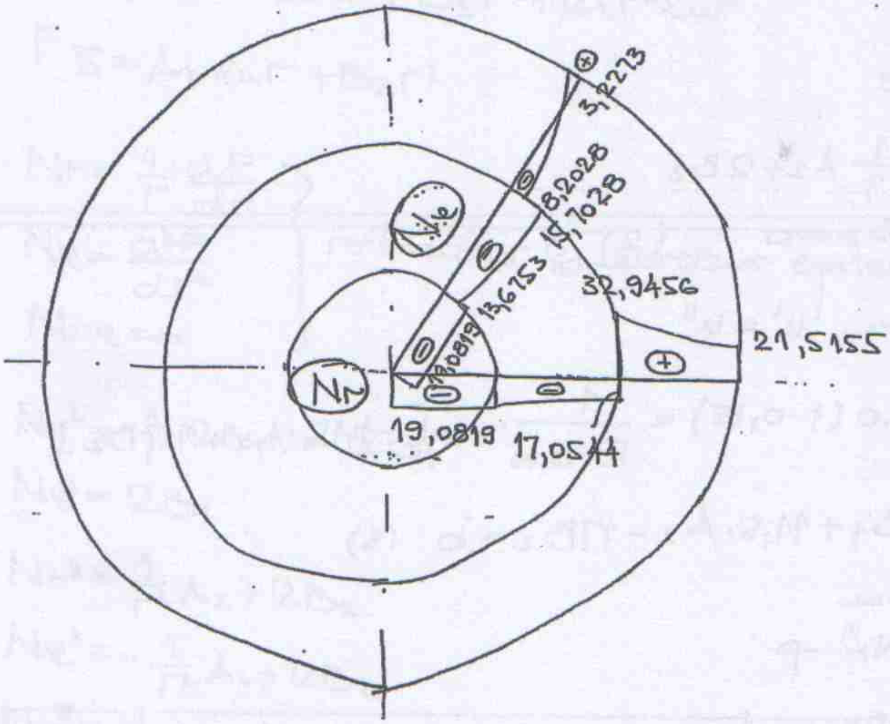
$N_\varphi' = -19{,}0819$

$N_r'' = -\frac{1}{r^2}\, 2{,}7033 - 16{,}3786$

$N_\varphi'' = \frac{1}{r^2} \cdot 2{,}7033 - 16{,}3786$

$N_r''' = \frac{1}{r^2}\, 82{,}2967 + 12{,}3714$

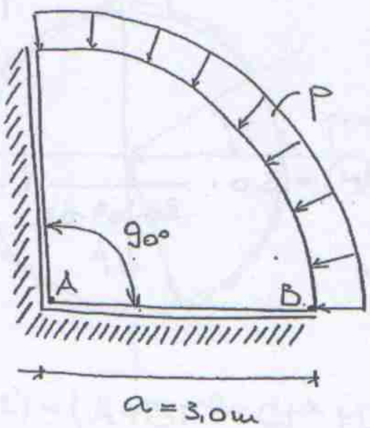
$N_\varphi''' = -\frac{1}{r^2}\, 82{,}2967 + 12{,}3714$



(10)

07.04.2003.

2. Za ploču koja je opterećena u svojoj ravni odrediti izraze za presečne sile, kao i pomeranja u označenim tačkama. Uslovi oslanjanja duž kontura $\varphi=0$ i $\varphi=\pi/2$ sprečavaju pomeranja u pravcu upravnom na konturu.

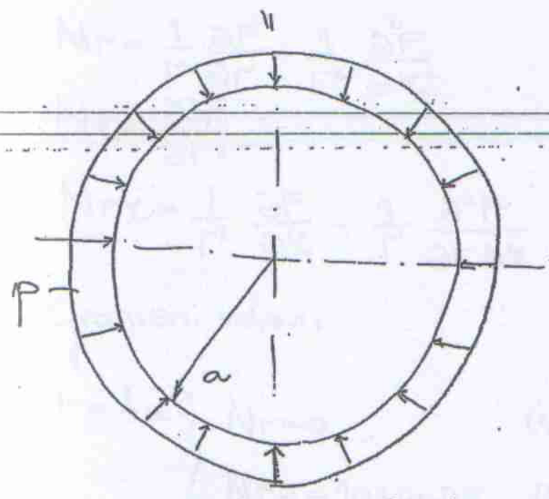


$E = 30\,GPa$

$\nu = 0{,}15$

$h = 0{,}1m$

$p = 20\,KN/m$



D nema uticaja na naprezanje, pa se usvaja $D=0$

$$F = \cancel{D} + \overset{0}{A}\ln r + Br^2 + \overset{0}{C}r^2\ln r$$

~~A=C=0, jer bi za slučaj r=0 ln r→∞~~

$$\left.\begin{aligned} N_r &= \frac{1}{r}\frac{dF}{dr} \\ N_\varphi &= \frac{d^2F}{dr^2} \\ N_{r\varphi} &= 0 \end{aligned}\right\} \text{Rotaciono simetrično opterećenje}$$

Granični uslovi:

$$r = 3{,}0m \left\{ N_r = -p \right.$$

$N_r = \frac{1}{r} \cdot 2Br = 2B$

$2B = -p \Rightarrow B = -10$

$N_r = \frac{1}{r}(-10 \cdot 2r) = -20$

$N_\varphi = -20$

$N_{r\varphi} = 0$

Sprečeno je pomeranje tačke A u obe ortogonalna pravca, tj. tačka A je nepokretna tačka.

$u_A = 0$

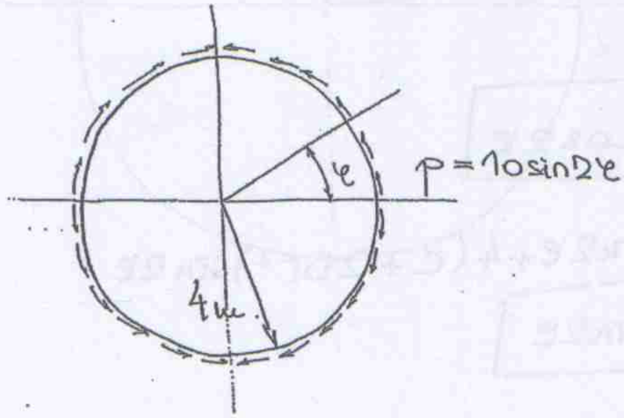
$u_B = r \cdot \varepsilon_\varphi = r_B \cdot \frac{1}{Eh}(N_\varphi - \nu N_r) = 3{,}0 \cdot \frac{1}{30 \cdot 10^6 \cdot 0{,}1}(-20 + 0{,}15 \cdot 20$

$u_B = -1{,}7 \cdot 10^{-5}\ m = -0{,}017\ mm$

11

29.03.2004.

2. Za kružnu ploču poluprečnika a koja je opterećena tangencijalnim opterećenjem, sračunati i nacrtati dijagrame sila u presecima za $\varphi=0$ i $\varphi=\pi/2$.



$$F(r,\varphi)=(A+Br^{-2}+Cr^2+Dr^4)\cos2\varphi$$

$$N_r=\frac{1}{r}\frac{\partial F}{\partial r}+\frac{1}{r^2}\frac{\partial^2 F}{\partial \varphi^2}$$

$$N_\varphi=\frac{\partial^2 F}{\partial r^2}$$

$$N_{r\varphi}=\frac{1}{r^2}\frac{\partial F}{\partial \varphi}-\frac{1}{r}\frac{\partial^2 F}{\partial r\partial\varphi}$$

Granični uslovi:

$$r=4m\begin{cases} N_r=0 & (1)\\ N_{r\varphi}=10\sin2\varphi & (2)\end{cases}$$

$$F=\sum Y_n\cdot\cos2\varphi$$

$$F=(\cancel{A}^{0}r^{-n+2}+\cancel{B}^{0}r^{-n}+Cr^{n}+Dr^{n+2})\cos2\varphi$$

$n=2$

$A=B=0$, jer za $r=0$ $F\to\infty$

$$F=(C\cdot r^2+D\cdot r^4)\cos2\varphi$$

$$\frac{\partial F}{\partial r}=2(C\cdot r+2D\cdot r^3)\cos2\varphi$$

$$\frac{\partial^2 F}{\partial \varphi^2}=-4(C\cdot r^2+D\cdot r^4)\cos2\varphi$$

$$\frac{\partial^2 F}{\partial r^2} = 2(C + 6 \cdot D \cdot r^2)\cos 2\varphi$$

$$\frac{\partial F}{\partial \varphi} = -2(Cr^2 + Dr^4)\sin 2\varphi$$

$$\frac{\partial^2 F}{\partial r \partial \varphi} = -4(Cr + 2Dr^3)\sin 2\varphi$$

$$N_r = 2(C + 2Dr^2)\cos 2\varphi + 4(C + Dr^2)\cos 2\varphi$$

$$\boxed{N_r = -2C \cdot \cos 2\varphi}$$

$$\boxed{N_\varphi = 2(C + 6Dr^2)\cos 2\varphi}$$

$$N_{r\varphi} = -2(C + Dr^2)\sin 2\varphi + 4(C + 2Dr^2)\sin 2\varphi$$

$$\boxed{N_{r\varphi} = (2C + 6Dr^2)\sin 2\varphi}$$

(1): $r = 4{,}0\,m$

$N_r = 0$

$-2C \cdot \cos 2\varphi = 0 \Rightarrow \boxed{C = 0}$

(2): $r = 4{,}0\,m$

$N_{r\varphi} = 10 \sin 2\varphi$

$(\cancel{2C} + 6D \cdot 16) \cdot \sin 2\varphi = 10 \sin 2\varphi$

$96 D = 10$

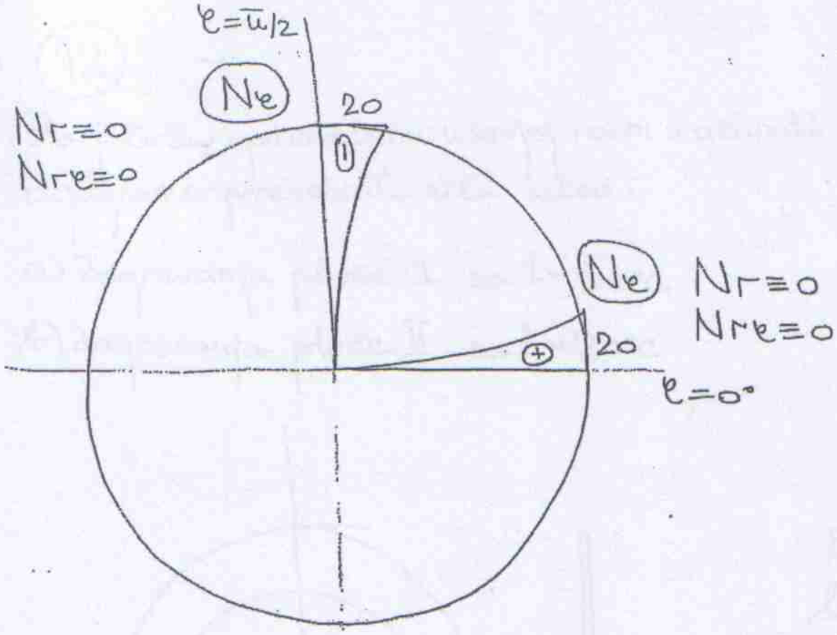
$D = 0{,}1041\dot{6}$

$F = 0{,}1041\dot{6} \cdot r^4 \cdot \cos 2\varphi$

$N_r = 0$

$N_\varphi = 1{,}25 \cdot r^2 \cdot \cos 2\varphi$

$N_{r\varphi} = 0{,}625 \cdot r^2 \cdot \sin 2\varphi$

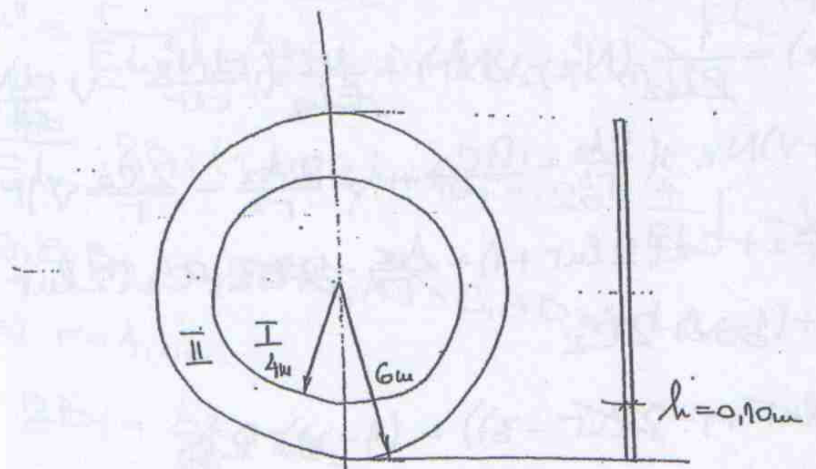

φ=π/2
Nφ
20
Nr≡0
Nrφ≡0
Nφ
Nr≡0
Nrφ≡0
20
φ=0°

(12)

2. Za ploču opterećenu u svojoj ravni sračunati i nacrtati dijagrame presečnih sila usled:

a) Zagrevanja ploče I za $t = 20°C$

b) Zagrevanja ploče II za $t = 20°C$



$E = 30\,GPa$

$h_1 = h_2 = 0{,}10\,m$

$\nu = 0{,}15$

$\alpha_t = 10^{-5}\ 1/°C$

a) $\cancel{D_1} \to 0$ – nema uticaja na naprezanje, pa se usvaja $D_1 = 0$

$$F_I = \cancel{D_1} + \cancel{A_1} \ln r + B_1 r^2 + \cancel{C_1} r^2 \ln r$$

$A_1 = C_1 = 0$ (jer za $r = 0$ $\ln r \to \infty$)

$$F_I = B_1 r^2$$

$$F_{II} = \cancel{D_2} + A_2 \ln r + B_2 r^2 + C_2 r^2 \ln r$$

$\cancel{C_2} \to 0$ – važi za kružne ploče napregnute u svojoj ravni i opterećene rotaciono simetričnim opterećenjem po svojoj konturi

$$N_r = \frac{1}{r}\frac{dF}{dr}$$

$$N_\varphi = \frac{d^2F}{dr^2}$$

$$N_{\varphi r} = 0$$

$$\varepsilon_\varphi' = \frac{1}{E h_1}\left(N_\varphi' - \nu N_r'\right) + \alpha_t \cdot t$$

$$\varepsilon_\varphi'' = \frac{1}{E h_1}\left(N_\varphi'' - \nu N_r''\right)$$

$$\varepsilon_r = \frac{du}{dr}$$

$$\varepsilon_\varphi = \frac{u}{r} \Rightarrow u = r \cdot \varepsilon_\varphi \Rightarrow \frac{du}{dr} = \varepsilon_\varphi + r \cdot \frac{d\varepsilon_\varphi}{dr}$$

$$N_r' = \frac{1}{r} \cdot 2B_1 r = 2B_1$$

$$N_\varphi' = 2B_1$$

$$N_r'' = \frac{1}{r}\left(\frac{A_2}{r} + 2B_2 r + C_2(2r\ln r + r)\right) = \frac{A_2}{r^2} + 2B_2 + C_2(2\ln r$$

$$N_\varphi'' = -\frac{A_2}{r^2} + 2B_2 + C_2(2\ln r + 3)$$

Dokaz da je $C_2 = 0$:

$$\varepsilon_r = \varepsilon_\varphi + r\frac{d\varepsilon_\varphi}{dr}$$

$$\cancel{\frac{1}{Eh_2}}(N_r'' - \nu N_\varphi'') = \cancel{\frac{1}{Eh_2}}(N_\varphi'' - \nu N_r'') + \cancel{\frac{1}{Eh_2}}\left(\frac{dN_\varphi''}{dr} - \nu\frac{dN_r''}{d\varphi}\right)$$

$$(1+\nu)N_r = (1+\nu)N_\varphi + \left(\frac{2A_2}{r^3} + \frac{2C_2}{r} + \nu\frac{2A_2}{r^3} - \frac{2C_2}{r}\nu\right)r$$

$$(1+\nu)\left(\cancel{\frac{A_2}{r^2}} + \cancel{2B_2} + C_2(2\ln r + 1) + \cancel{\frac{A_2}{r^2}} - \cancel{2B_2} - C_2(2\ln r + 3)\right)$$

$$= (1+\nu)\cancel{\frac{2A_2}{r^2}} + (1-\nu)2C_2$$

$$(1+\nu)(C_2(\cancel{2\ln r} + 1 - \cancel{2\ln r} - 3)) = (1-\nu)2C_2$$

$$-2C_2(1+\nu+1-\nu) = 0 \Rightarrow \boxed{C_2 = 0}$$

$$N_r' = 2B_1$$

$$N_\varphi' = 2B_1$$

$$N_r'' = \frac{A_2}{r^2} + 2B_2$$

$$N_\varphi'' = -\frac{A_2}{r^2} + 2B_2$$

Granični uslovi:

$$r = 6{,}0\,m \left\{ N_r'' = 0 \quad (1) \right.$$

Prelazni uslovi:

$$r = 4{,}0\,m \left\{ \begin{array}{ll} u_r' = u_r'' & (2) \\ N_r' = N_r'' & (3) \end{array} \right.$$

(1): $\frac{A_2}{36} + 2B_2 = 0 \qquad (1)$

...măunati i nacrtati

(2) $r = 4{,}0m$

$$u' = r \cdot \varepsilon_{\varphi}' = r \left[\frac{1}{Eh_1} (N_{\varphi}' - \nu N_r') + \alpha_t \cdot t \right]$$

$$u' = \frac{r}{Eh_1} 2B_1(1-\nu) + \alpha_t \cdot t$$

$$u'' = r \cdot \varepsilon_{\varphi}'' = \frac{r}{Eh_2} (N_{\varphi}'' - \nu N_r'') = \frac{r}{Eh_2} \left(-\frac{A_2}{r^2} + 2B_2 - \nu \frac{A_2}{r^2} - \nu 2B_2\right)$$

$$u'' = \frac{r}{Eh_2} \left[-\frac{A_2}{r^2}(1+\nu) + 2B_2(1-\nu) \right]$$

$$\frac{4{,}0}{Eh} \cdot 2B_1(1-0{,}15) + 10^{-5} \cdot 20 = \frac{4}{Eh} \left[-\frac{1{,}15}{16} A_2 + 1{,}7 B_2 \right]$$

$$6{,}8 \cdot B_1 + 0{,}2875 \cdot A_2 - 6{,}8 B_2 = -600 \qquad (2)$$

(3) $r = 4{,}0m$

$$2B_1 = \frac{A_2}{16} + 2B_2$$

$$2B_1 - \frac{A_2}{16} - 2B_2 = 0 \qquad (3)$$

$B_1 = -35{,}8425$

$A_2 = -1200$

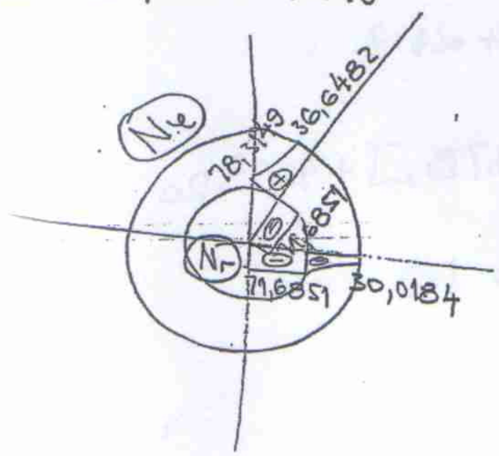
$B_2 = 1{,}6575$

$N_r' = -71{,}6851$

$N_{\varphi}' = -71{,}6851$

$N_r'' = -\frac{1200}{r^2} + 3{,}3149$

$N_{\varphi}'' = +\frac{1200}{r^2} + 3{,}3149$



b) $F_I = \cancel{D_1}^{\to 0} + \cancel{A_1}^{\to 0} \ln r + B_1 r^2 + \cancel{C_1}^{\to 0} r^2 \ln r$

$F_I = B_1 r^2$

$F_{II} = \cancel{D_2}^{\to 0} + A_2 \ln r + B_2 r^2 + \cancel{C_2}^{\to 0} r^2 \ln r$

$\varepsilon_r'' = \frac{du}{dr}$

$u = r \cdot \varepsilon_\varphi'' + \alpha_t \cdot t \Rightarrow \frac{du}{dr} = \varepsilon_\varphi'' + r \frac{d\varepsilon_\varphi''}{dr} \Rightarrow C_2 = 0$

$F_{II} = A_2 \ln r + B_2 r^2$

$N_r' = 2B_1$

$N_\varphi' = 2B_1$

$N_r'' = \frac{A_2}{r^2} + 2B_2$

$N_\varphi'' = -\frac{A_2}{r^2} + 2B_2$

Granični uslovi:

$r = 6{,}0m \; \{ N_r'' = 0 \quad (1)$

Prelazni uslovi:

$r = 4{,}0m \; \begin{cases} u' = u'' & (2) \\ N_r' = N_r'' & (3) \end{cases}$

(1):

$\frac{A_2}{36} + B_2 = 0 \quad (1)$

(2):

$u' = \frac{r}{Eh_1} \cdot 2B_1 (1-\nu)$

$u'' = \frac{r}{Eh_2} \left[-\frac{A_2}{r^2}(1+\nu) + 2B_2(1-\nu) \right] + \alpha t \cdot t$

$\frac{4{,}0}{Eh} \cdot 2B_1 (1-0{,}15) = \frac{4}{Eh} \left[-\frac{1{,}15}{16} A_2 + 1{,}7 B_2 \right] + 10^{-5} \cdot 20$

$6{,}8 B_1 + 0{,}2875 A_2 - 6{,}8 B_2 = 600 \quad (2)$

(3):

$2B_1 - \frac{A_2}{16} - 2B_2 = 0 \quad (3)$

$B_1 = 35,8425$

$A_2 = 1200$

$B_2 = -1,6575$

$N_r' = 71,6851$

$N_\varphi' = 71,6851$

$N_r'' = \frac{1200}{r^2} - 3,3149$

$N_\varphi'' = -\frac{1200}{r^2} - 3,3149$