NAVIER

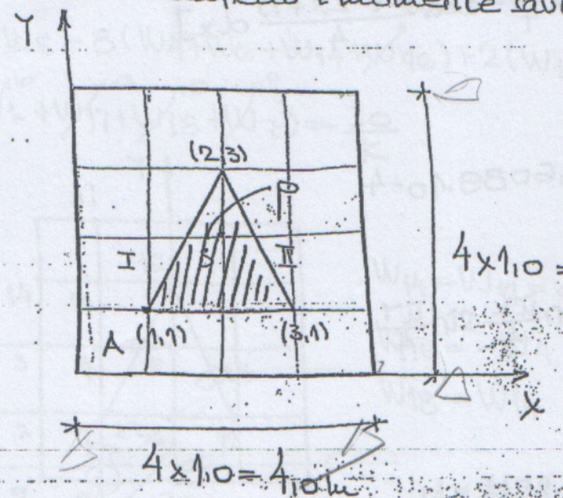
DIFERENCNI

12

18.03.2000.

1. a) Rešiti problem savijanja koristeći prvi član reda usvojenog rešenja.

b) Koristeći diferencni postupak odrediti ugibe u označenim tačkama, kao i momente savijanja u sredini ploče.



$E = 30\,GPa$

$h = 15\,cm$

$\nu = 0{,}20$

$p = 30\,kN/m^2$

$$K = \frac{E\cdot h^3}{12(1-\nu^2)} = 8789{,}0625$$

I: $x=1 \quad Y=1 \Rightarrow 1 = a+b$

$x=2 \quad Y=3 \Rightarrow 3 = 2a+b \Rightarrow 3 = 2a+1-a \Rightarrow a=2$

$b=-1$

$Y = 2x-1$

II: $x=2 \quad Y=3 \Rightarrow 3 = 2a+b$

$x=3 \quad Y=1 \Rightarrow 1 = 3a+b \Rightarrow 1 = 3a+3-2a \Rightarrow a=-2$

$b=7$

$Y = -2x+7$

$$W(x,Y) = A_{11}\cdot \sin\frac{\pi x}{4}\sin\frac{\pi Y}{4}$$

$$A_{11}=\frac{Z_{11}}{K\cdot\pi^4\cdot\left(\frac{1}{a^2}+\frac{1}{a^2}\right)^2}$$

$$Z_{11}=\frac{4P}{16}\left[\int_1^2 \sin\frac{\pi x}{4}dx\int_1^{2x-1}\sin\frac{\pi y}{4}dy+\int_2^3\sin\frac{\pi x}{4}dx\int_1^{-2x+7}\sin\frac{\pi y}{4}dy\right]$$

$$Z_{11}=\frac{4}{16}P\cdot\frac{4}{\pi}\left[\int_1^2\sin\frac{\pi x}{4}\left(\cos\frac{\pi}{4}-\cos\frac{\pi(2x-1)}{4}\right)dx+\right.$$
$$\left.+\int_2^3\sin\frac{\pi x}{4}\left(\cos\frac{\pi}{4}-\cos\frac{\pi(-2x+7)}{4}\right)dx\right]$$

$$Z_{11}=12{,}8539$$

$$A_{11}=\frac{12{,}8539}{8789{,}0625\cdot\pi^4\left(\frac{2}{4^2}\right)^2}=9{,}6089\cdot10^{-4}$$

$$W(x,y)=9{,}6089\cdot10^{-4}\cdot\sin\frac{\pi x}{4}\sin\frac{\pi y}{4}$$

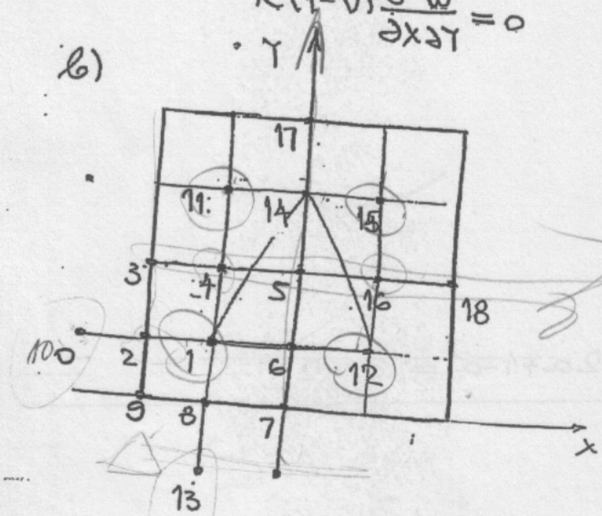
$$W_A=4{,}8044\cdot10^{-4}$$

$$M_{x,s}=-K\left(\frac{\partial^2 w}{\partial x^2}+\nu\frac{\partial^2 w}{\partial y^2}\right)=6{,}2514\,KNm$$

$$M_{y,s}=-K\left(\frac{\partial^2 w}{\partial y^2}+\nu\frac{\partial^2 w}{\partial x^2}\right)=6{,}2514\,KNm$$

$$M_{xy,s}=-K(1-\nu)\frac{\partial^2 w}{\partial x\partial y}=0$$

b)



$$W_2=W_3=W_7=W_8=W_9=0$$

$$W_{10}=-W_1 \qquad W_{12}=W_1$$

$$W_{13}=-W_1$$

НАПОН

$$\frac{F_{10}-F_1}{2S}=0 \qquad \boxed{F_{10}=-F_1}$$

$$\frac{W_{10}-2W_2+W_1}{S^2}=0 \qquad \boxed{W_{10}=-W_1}$$

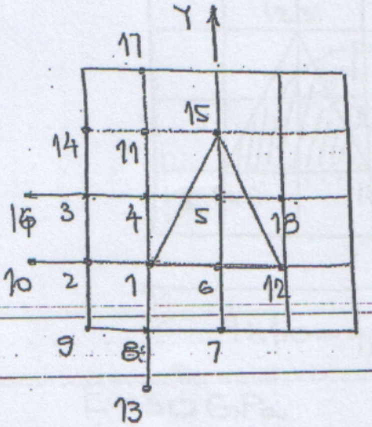
$k=1$

$$20\cdot w_1-8(\cancelto{0}{w_2}+w_4+w_6+\cancelto{0}{w_8})+2(\cancelto{0}{w_3}+w_5+\cancelto{0}{w_7}+\cancelto{0}{w_9})+\cancelto{-w_1}{w_{10}}+w_{11}+$$
$$+\cancelto{w_1}{w_{12}}+\cancelto{-w_1}{w_{13}}=\frac{2k\cdot s^4}{k}=\frac{30\cdot 1^4}{k}$$

$$19w_1-8(w_4+w_6)+2w_5+w_{11}=\frac{30}{k}$$

$k=5$

$$20w_5-8(w_4+w_6+w_{14}+\cancelto{w_4}{w_{16}})+2(w_1+w_{11}+\cancelto{w_1}{w_{12}}+\cancelto{w_{11}}{w_{15}})+$$
$$+(\cancelto{0}{w_3}+\cancelto{0}{w_{17}}+\cancelto{0}{w_{18}}+\cancelto{0}{w_7})=\frac{30}{k}$$



$w_{14}=w_{17}=0$

$w_{16}=-w_4$

$w_{18}=w_{14}$

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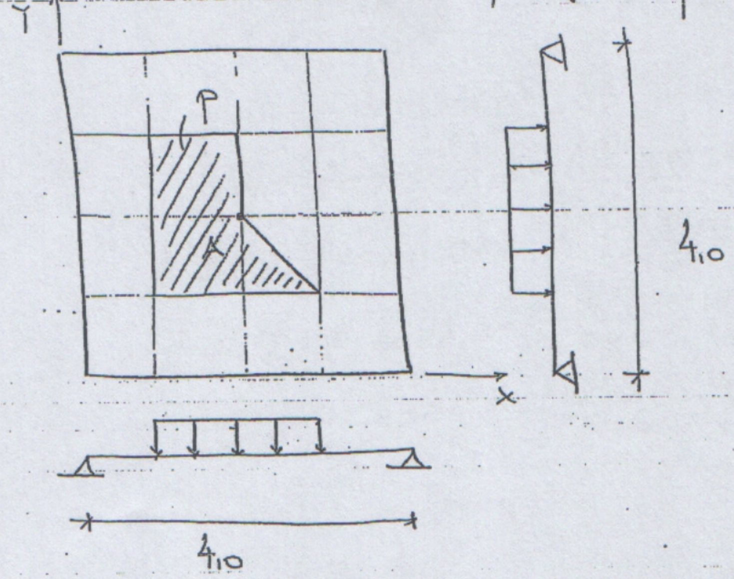
(11)

1. Napisati izraz za ugib i izračunati momente u sredini ploče koristeći prvi član reda usvojenog rešenja.



$p = 20\,kN/m$

$\nu = 0{,}15$

$E = 30\,GPa$

$h = 0{,}15\,m$

$$K = \frac{E \cdot h^3}{12(1-\nu^2)} = 8631{,}7136$$

$$W(x,y) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} A_{mn} \cdot \sin\frac{m\pi x}{a}\,\sin\frac{n\pi y}{b}$$

$m=1,\ n=1$

$$A_{11} = \frac{Z_{11}}{K\pi^2\left(\frac{1}{a^2}+\frac{1}{b^2}\right)^2}$$

$$Z_{11} = \frac{4}{ab}\int_0^a\int_0^b Z(x,y)\sin\frac{\pi x}{a}\sin\frac{\pi y}{b}\,dx\,dy$$

$$Z_{11} = \frac{4p}{16}\left[\int_1^2 \sin\frac{\pi x}{4}dx\int_1^3 \sin\frac{\pi y}{4}dy + \int_2^3 \sin\frac{\pi x}{4}dx\int_1^{-x+4}\sin\frac{\pi y}{4}dy\right]$$

$$Z_{11} = \frac{4p}{16}\cdot\frac{4}{\pi}\left[\int_1^2 \sin\frac{\pi x}{4}\left(\cos\frac{\pi}{4}-\cos\frac{3\pi}{4}\right)dx + \int_2^3 \sin\frac{\pi x}{4}\left(\cos\frac{\pi}{4}-\cos\frac{\pi(-x+4)}{4}\right)d\right.$$

$Z_{11} = 10,1321$

$$A_{11} = \frac{Z_{11}}{K \cdot \pi^4 \cdot \left(\frac{1}{a^2} + \frac{1}{b^2}\right)^2} = 7,7123 \cdot 10^{-4}$$

$$W(x,y) = 7,7123 \cdot 10^{-4} \cdot \sin\frac{\pi x}{4} \sin\frac{\pi y}{4}$$

$$M_x = -K\left(\frac{\partial^2 W}{\partial x^2} + \nu \frac{\partial^2 W}{\partial y^2}\right) = 4,7223 \cdot \sin\frac{\pi x}{4} \sin\frac{\pi y}{4}$$

$$M_y = -K\left(\frac{\partial^2 W}{\partial y^2} + \nu \frac{\partial^2 W}{\partial x^2}\right) = 4,7223 \cdot \sin\frac{\pi x}{4} \sin\frac{\pi y}{4}$$

$$M_{xy} = -K(1-\nu)\frac{\partial^2 W}{\partial x \partial y} = -3,4904 \cdot \cos\frac{\pi x}{4} \cos\frac{\pi y}{4}$$

$$T_x = -K\left(\frac{\partial^3 W}{\partial x^3} + \frac{\partial^3 W}{\partial x \partial y^2}\right) = 6,4503 \cdot \cos\frac{\pi x}{4} \sin\frac{\pi y}{4}$$

$$T_y = -K\left(\frac{\partial^3 W}{\partial y^3} + \frac{\partial^3 W}{\partial x^2 \partial y}\right) = 6,4503 \cdot \sin\frac{\pi x}{4} \cos\frac{\pi y}{4}$$

$A(2,0;2,0)$

$M_{x,A} = 4,7223\ KNm$

$M_{y,A} = 4,7223\ KNm$

$M_{xy,A} = 0$

$T_{x,A} = 0$

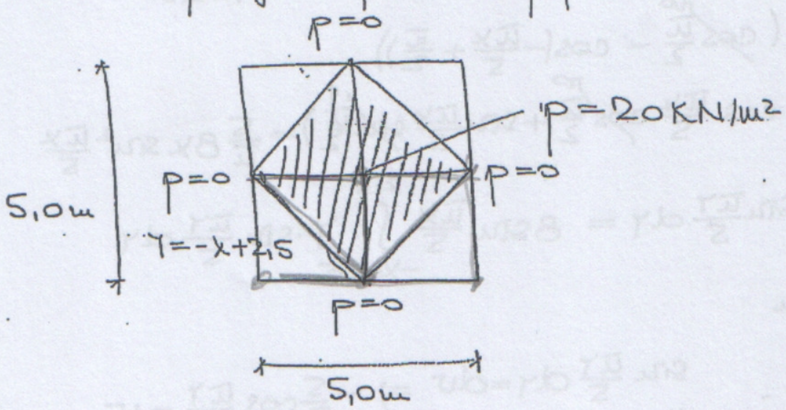
$T_{y,A} = 0$

NAVIER

(20)

31.05.2008.

1. Za kvadratnu ploču opterećenu opterećenjem prema slici odrediti momente savijanja i ugib u sredini ploče. Koristiti prvi član reda usvojenog rešenja. Ploča je po konturi slobodno oslonjena.



$E = 30\,GPa$

$\nu = 0{,}25$

$h = 25\,cm$

$$W(x,y) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} A_{mn} \sin\frac{m\pi x}{a} \sin\frac{n\pi y}{b}$$

$$A_{mn} = \frac{Z_{mn}}{K\pi^4\left(\frac{m^2}{a^2}+\frac{n^2}{b^2}\right)^2}$$

$$Z_{mn} = \frac{4}{ab}\int_0^a\int_0^b Z(x,y) \sin\frac{m\pi x}{a} \sin\frac{n\pi y}{b}\,dx\,dy$$

$m=1,\ n=1$

$$Z_{11} = \frac{4}{25}\int_0^a\int_0^b Z(x,y) \sin\frac{\pi x}{5} \sin\frac{\pi y}{5}\,dx\,dy$$

$$Z(x,y) = Ax + By + C$$

$$\left.\begin{array}{l} x=2{,}5 \\ y=0 \\ z=0 \end{array}\right\} \Rightarrow 0 = 2{,}5A + C \Rightarrow C = -2{,}5A$$

$x=0 \quad y=2{,}5 \quad z=0 \Rightarrow 0 = 2{,}5B + C \Rightarrow B = A$

$x=2{,}5 \quad y=2{,}5 \quad z=20 \Rightarrow 20 = 2{,}5A + 2{,}5B + C$

$$20 = 2{,}5A + 2{,}5A - 2{,}5A$$

$A = 8 \quad B = 8 \quad C = -20$

$$Z(x,y) = 8x + 8y - 20$$

$$z_{11}=\frac{4}{25}\cdot 4\int_{0}^{2,5}\int_{-x+2,5}^{2,5}(8x+8y-20)\sin\frac{\pi x}{5}\sin\frac{\pi y}{5}\,dx\,dy$$

$$z_{11}=\frac{16}{25}\int_{0}^{2,5}\int_{-x+2,5}^{2,5}\left[8x\sin\frac{\pi x}{5}\sin\frac{\pi y}{5}+8y\sin\frac{\pi x}{5}\sin\frac{\pi y}{5}-20\sin\frac{\pi x}{5}\sin\frac{\pi y}{5}\right]dx\,dy$$

$$J_1=\int_{-x+2,5}^{2,5}8x\sin\frac{\pi x}{5}\cdot\sin\frac{\pi y}{5}\,dy=-\frac{5}{\pi}8x\sin\frac{\pi x}{5}\cos\frac{\pi y}{5}\Big|_{-x+2,5}^{2,5}$$

$$J_1=-\frac{5}{\pi}8x\sin\frac{\pi x}{5}\left(\cancel{\cos\frac{\pi}{2}}^{\,0}-\cos\left(-\frac{\pi x}{5}+\frac{\pi}{2}\right)\right)$$

$$J_1=+\frac{5}{\pi}8x\sin\frac{\pi x}{5}\left(\cos\frac{\pi x}{5}\cancel{\cos\frac{\pi}{2}}^{\,0}+\sin\frac{\pi x}{5}\cancel{\sin\frac{\pi}{2}}^{\,1}\right)=\frac{5}{\pi}8x\sin^2\frac{\pi x}{5}$$

$$J_2=\int_{-x+2,5}^{2,5}8y\sin\frac{\pi x}{5}\sin\frac{\pi y}{5}\,dy=8\sin\frac{\pi x}{5}\int_{-x+2,5}^{2,5}y\cdot\sin\frac{\pi y}{5}\,dy$$

$$\int u\cdot dv=u\cdot v-\int v\,du$$

$$y=u\Rightarrow dy=du\qquad\qquad \sin\frac{\pi y}{5}dy=dv\Rightarrow-\frac{5}{\pi}\cos\frac{\pi y}{5}=v$$

$$J_2=8\cdot\sin\frac{\pi x}{5}\left[-y\cdot\frac{5}{\pi}\cos\frac{\pi y}{5}\Big|_{-x+2,5}^{2,5}+\int_{-x+2,5}^{2,5}\frac{5}{\pi}\cos\frac{\pi y}{5}\,dy\right]$$

$$J_2=8\cdot\sin\frac{\pi x}{5}\Big[+(-x+2,5)\cdot\frac{5}{\pi}\cos\frac{\pi(-x+2,5)}{5}-2,5\cdot\frac{5}{\pi}\cancel{\cos\frac{\pi}{2}}^{\,0}+$$
$$+\frac{25}{\pi^2}\left(\sin\frac{\pi}{2}-\sin\frac{\pi(2,5-x)}{5}\right)\Big]$$

$$\cos\frac{\pi(2,5-x)}{5}=\cancel{\cos\frac{\pi}{2}}^{\,0}\cos\frac{\pi x}{5}+\cancel{\sin\frac{\pi}{2}}^{\,1}\sin\frac{\pi x}{5}=\sin\frac{\pi x}{5}$$

$$\sin\frac{\pi(2,5-x)}{5}=\cancel{\sin\frac{\pi}{2}}^{\,1}\cos\frac{\pi x}{5}-\cancel{\cos\frac{\pi}{2}}^{\,0}\sin\frac{\pi x}{5}=\cos\frac{\pi x}{5}$$

$$J_2=8\cdot\sin\frac{\pi x}{5}\left[(2,5-x)\cdot\frac{5}{\pi}\cdot\sin\frac{\pi x}{5}+\frac{25}{\pi^2}\left(1-\cos\frac{\pi x}{5}\right)\right]$$

$$J_3=-20\sin\frac{\pi x}{5}\int_{2,5-x}^{2,5}\sin\frac{\pi y}{5}\,dy=+\frac{100}{\pi}\sin\frac{\pi x}{5}\left(\cancel{\cos\frac{\pi}{2}}^{\,0}-\sin\frac{\pi x}{5}\right)=20\sin^2\frac{\pi x}{5}$$

$$z_{11}=\frac{16}{25}\int_{0}^{2,5}\Big[\cancel{\frac{40}{\pi}\cdot x\sin^2\frac{\pi x}{5}}+\frac{100}{\pi}\sin^2\frac{\pi x}{5}-\cancel{\frac{40}{\pi}\cdot x\sin^2\frac{\pi x}{5}}+\frac{200}{\pi^2}\sin\frac{\pi x}{5}-$$
$$-\frac{200}{\pi^2}\sin\frac{\pi x}{5}\cos\frac{\pi x}{5}+\frac{100}{\pi}\sin^2\frac{\pi x}{5}\Big]dx$$

$$z_{11}=61,2501$$

$$A_{11}=\frac{61,2501}{k\cdot\pi^4\left(\frac{1}{a^2}+\frac{1}{b^2}\right)^2}=2,3580\cdot10^{-3}$$

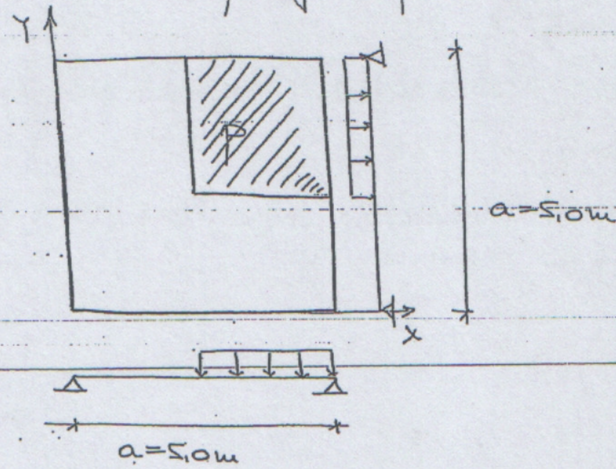
$$W(x,y) = 2{,}3580 \cdot 10^{-3} \cdot \sin\frac{\pi x}{5}\,\sin\frac{\pi y}{5}$$

$$W(2{,}5\,;\,2{,}5) = 2{,}3580 \cdot 10^{-3}\,\mathrm{m}$$

NAVIER

(3)
05.01.2005.

1. Zadata je slobodno oslonjena kvadratna ploča opterećena prema skici. Odrediti ugib i presečne sile u sredini ploče koristeći prvi član reda usvojenog rešenja.



$E = 3{,}15 \cdot 10^7\ KN/m^2$ $h = 0{,}12m$

$\nu = 0{,}20$ $p = 20KN/m^2$

$$K = \frac{E \cdot h^3}{12(1-\nu^2)} = 4725$$

$$W(x,y) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} A_{mn} \sin\frac{m\pi x}{a} \sin\frac{n\pi y}{b}$$

$$A_{mn} = \frac{Z_{mn}}{K\left(\frac{m^2\pi^2}{a^2} + \frac{n^2\pi^2}{b^2}\right)^2}$$

$$Z_{mn} = \frac{4}{ab}\int_0^a\int_0^b Z(x,y) \sin\frac{m\pi x}{a} \sin\frac{n\pi y}{b}\,dx\,dy$$

$$Z_{11} = \frac{4}{25}\int_{2,5}^{5}\int_{2,5}^{5} 20 \cdot \sin\frac{\pi x}{a} \sin\frac{\pi y}{b}\,dx\,dy = 8{,}1057$$

$$A_{11} = \frac{8{,}1057}{4725\left(\frac{\pi^2}{5^2} + \frac{\pi^2}{5^2}\right)^2} = 2{,}75175 \cdot 10^{-3}$$

$$W(x,y) = 2{,}75175 \cdot 10^{-3} \cdot \sin\frac{\pi x}{5} \sin\frac{\pi y}{5}$$

$$W(2{,}5;2{,}5) = 2{,}75175 \cdot 10^{-3}$$

$$M_x = -K\left(\frac{\partial^2 W}{\partial x^2} + \nu \frac{\partial^2 W}{\partial y^2}\right) = -4725\left[-1{,}0863 \cdot \sin\frac{\pi x}{5} \sin\frac{\pi y}{5} - 0{,}2\, 1{,}0863 \cdot \sin\frac{\pi x}{5} \sin\frac{\pi y}{5}\right]$$

$$M_x = 6{,}1596 \sin\frac{\pi x}{5} \sin\frac{\pi y}{5}$$

$$M_y = -K\left(\frac{\partial^2 W}{\partial y^2} + \nu \frac{\partial^2 W}{\partial y^2}\right) = 6{,}1596 \cdot \sin\frac{\pi x}{5} \sin\frac{\pi y}{5}$$

$$M_{xy} = -K(1-\nu)\frac{\partial^2 W}{\partial x \partial y} = -4725(1-0{,}2) \cdot 1{,}0863 \cdot 10^{-3} \cos\frac{\pi x}{5} \cos\frac{\pi y}{5} =$$

$$= -4{,}1064 \cdot \cos\frac{\pi x}{5} \cos\frac{\pi y}{5}$$

$$T_x = -K\left(\frac{\partial^3 W}{\partial x^3} + \frac{\partial^3 W}{\partial x \partial y^2}\right) = -4725 \cdot \left(-6{,}8257 \cdot 10^{-4} \cos\frac{\pi x}{5} \sin\frac{\pi y}{5} - 6{,}8257 \cdot 10^{-4} \cdot \cos\frac{\pi x}{5} \sin\frac{\pi y}{5}\right.$$

$$= 6{,}4503 \cdot \cos\frac{\pi x}{5} \sin\frac{\pi y}{5}$$

$$T_y = -K\left(\frac{\partial^3 W}{\partial y^3} + \frac{\partial^3 W}{\partial x^2 \partial y}\right) = -4725\left(-6{,}8257 \cdot 10^{-4} \sin\frac{\pi x}{5} \cos\frac{\pi y}{5} - 6{,}8257 \cdot 10^{-4} \cdot \sin\frac{\pi x}{5} \cos\frac{\pi y}{5}\right) =$$

$$= 6{,}4503 \cdot \sin\frac{\pi x}{5} \cos\frac{\pi y}{5}$$

$x = 2{,}5 \quad y = 2{,}5$

$M_x = 6{,}1596$ KNm

$M_y = 6{,}1596$ KNm

$M_{xy} = 0$

$T_x = 0$

$T_y = 0$

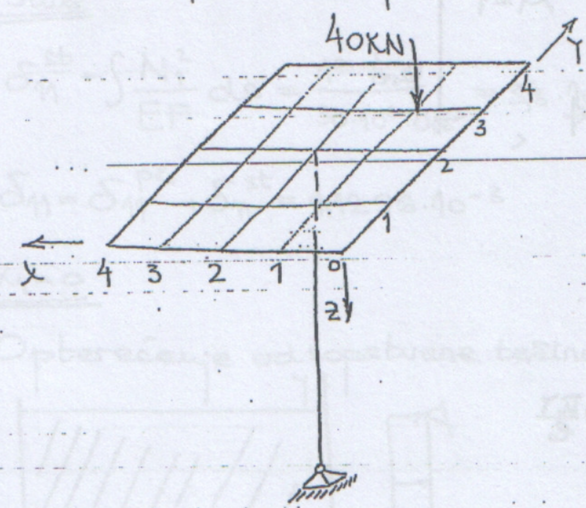
sin ax

NAVIER

9

08.04.1996.

1. Odrediti normalnu silu u stubu i f-nu ugiba ploče prikazane na slici. Ploča je po konturi slobodno oslonjena. Pored koncentrisanog opterećenja na ploču deluje i sopstvena težina ploče. Pri proračunu koristiti prvi član Furijeovog reda



Ploča:

$\gamma = 70\,KN/m^3$, $h = 0{,}02\,m$

$E_p = 210\,GN/m^2$, $\nu = 0{,}20$

Stub:

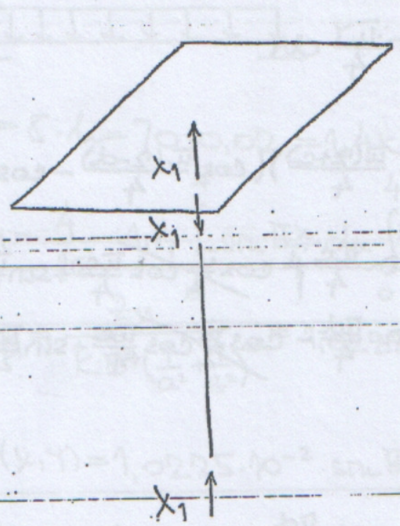
$b/d = 20/20\,cm$

$l = 4{,}0\,m$

$E_s = 30\,GN/m^2$

$$K = \frac{E \cdot h^3}{12(1-\nu^2)} = 145{,}8\dot{3}$$

* Dekompozicija:



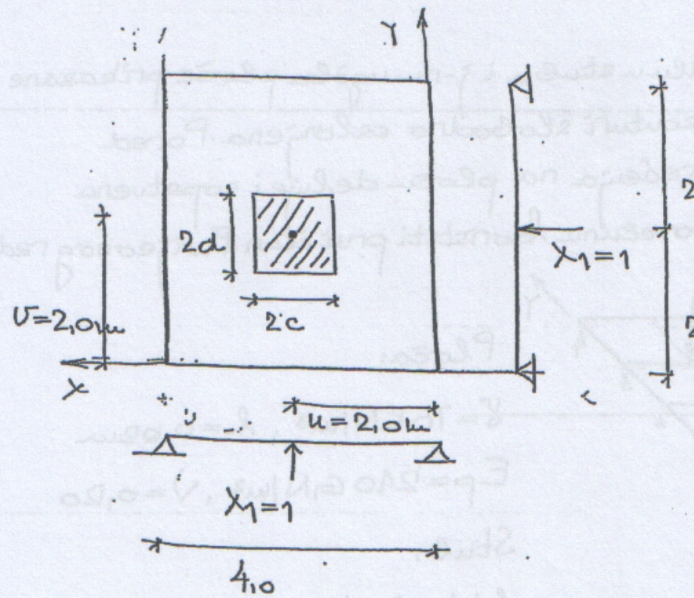
$$\delta_{11} \cdot X_1 + \delta_{10} = 0$$

$$\delta_{11} = \delta_{11}^{pl} + \delta_{11}^{st}$$

$$\delta_{10} = \delta_{10}^{pl}$$

$X_1 = 1$

Ploča



$$W(x,y) = A_{11} \cdot \sin\frac{\pi x}{a} \cdot \sin\frac{\pi y}{b}$$

$$A_{11} = \frac{Z_{11}}{K \cdot \pi^4 \left(\frac{1}{a^2}+\frac{1}{b^2}\right)^2}$$

$$Z_{11} = \frac{4}{ab}\int_0^a\int_0^b Z(x,y)\sin\frac{\pi x}{a}\sin\frac{\pi y}{b}\,dx\,dy$$

$$2d \cdot 2d \cdot Z_0 = 1 = X_1$$

$$Z_{11} = \frac{4}{16} \cdot \int_{2-c}^{2+c} Z_0 \cdot \sin\frac{\pi x}{4}\,dx \cdot \int_{2-d}^{2+d} \sin\frac{\pi y}{4}\,dy$$

$$Z_{11} = \frac{4Z_0}{16} \cdot \frac{16}{\pi^2} \cdot \left(\cos\frac{\pi(2-c)}{4} - \cos\frac{\pi(2+c)}{4}\right)\left(\cos\frac{\pi(2-d)}{4} - \cos\frac{\pi(2+d)}{4}\right)$$

$$Z_{11} = \frac{4Z_0}{\pi^2}\left(\cancel{\cos\frac{\pi}{2}\cos\frac{\pi c}{4}} + \sin\frac{\pi}{2}\sin\frac{\pi c}{4} - \cancel{\cos\frac{\pi}{2}\cos\frac{\pi c}{4}} + \sin\frac{\pi}{2}\sin\frac{\pi c}{4}\right) \cdot \left(\cancel{\cos\frac{\pi}{2}\cos\frac{\pi d}{4}} + \sin\frac{\pi}{2}\sin\frac{\pi d}{4} - \cancel{\cos\frac{\pi}{2}\cos\frac{\pi c}{4}} + \sin\frac{\pi}{2}\sin\frac{\pi d}{4}\right)$$

$$Z_{11} = \frac{4Z_0}{\pi^2}\, 2\sin\frac{\pi c}{4} \cdot 2\sin\frac{\pi d}{4}$$

$$\lim_{\substack{c\to 0\\ d\to 0}} Z_{11} = \lim_{\substack{c\to 0\\ d\to 0}} \frac{4Z_0}{\pi^2} \cdot 4\, \frac{\sin\frac{\pi c}{4}}{\frac{\pi c}{4}} \cdot \frac{\pi c}{4} \cdot \frac{\sin\frac{\pi d}{4}}{\frac{\pi d}{4}} \cdot \frac{\pi d}{4} = \frac{4Z_0 cd}{4} = \frac{1}{4}$$

$$A_{11}=\frac{0{,}25}{145{,}833\cdot\pi^4\left(\frac{1}{4^2}\cdot 2\right)^2}=1{,}1263\cdot 10^{-3}$$

$$W(x,y)=1{,}1263\cdot 10^{-3}\sin\frac{\pi x}{4}\sin\frac{\pi y}{4}$$

$$\delta_{11}^{pl}=W(2,2)=1{,}1263\cdot 10^{-3}$$

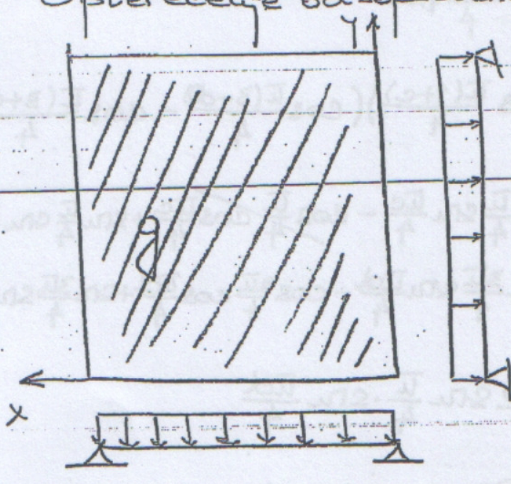
Stub

$$\delta_{11}^{st}=\int\frac{N_1^2}{EF}ds=\frac{1^2\cdot 4{,}0}{30\cdot 10^6\cdot 0{,}2^2}=3{,}\dot{3}\cdot 10^{-6}$$

$$\delta_{11}=\delta_{11}^{pl}+\delta_{11}^{st}=1{,}1296\cdot 10^{-3}$$

$X_1=0$

Opterećenje od sopstvene težine:



$$g=\gamma\cdot h=70\cdot 0{,}02=1{,}4\,KN/m^2$$

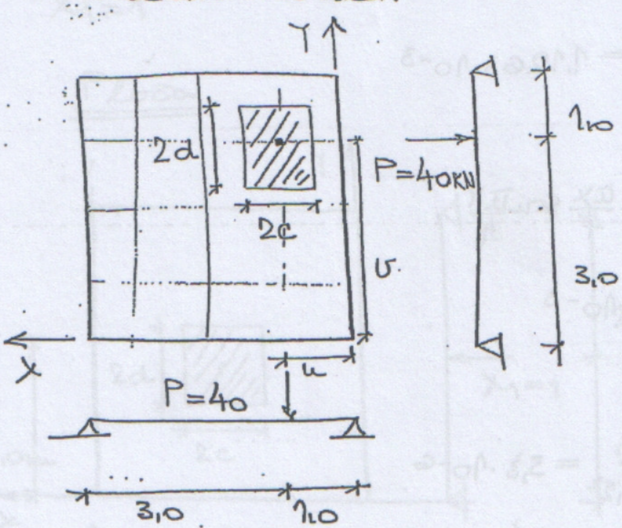
$$Z_{11}=\frac{4}{16}\cdot 1{,}4\int_0^4\sin\frac{\pi x}{4}dx\int_0^4\sin\frac{\pi y}{4}dy=2{,}2696$$

$$A_{11}=\frac{Z_{11}}{K\pi^4\left(\frac{1}{a^2}+\frac{1}{b^2}\right)^2}=1{,}0225\cdot 10^{-2}$$

$$W(x,y)=1{,}0225\cdot 10^{-2}\cdot\sin\frac{\pi x}{4}\sin\frac{\pi y}{4}$$

$$\delta_{10}^{g}=W(2,2)=1{,}0225\cdot 10^{-2}$$

Koncentrisana sila:



$$2c\cdot 2d\cdot p_0 = 40 = P$$

$$Z_{11} = \frac{4}{16}\int_{u-c}^{u+c} p_0\cdot \sin\frac{\pi x}{4}\,dx \int_{v-d}^{v+d} \sin\frac{\pi y}{4}\,dx$$

$$Z_{11} = \frac{4p_0}{16}\cdot\frac{16}{\pi^2}\left(\cos\frac{\pi(1-c)}{4} - \cos\frac{\pi(1+c)}{4}\right)\left(\cos\frac{\pi(3-d)}{4} - \cos\frac{\pi(3+d)}{4}\right)$$

$$Z_{11} = \frac{4p_0}{\pi^2}\left(\cancel{\cos\frac{\pi}{4}\cos\frac{\pi c}{4}} + \sin\frac{\pi}{4}\sin\frac{\pi c}{4} - \cancel{\cos\frac{\pi}{4}\cos\frac{\pi c}{4}} + \sin\frac{\pi}{4}\sin\frac{\pi c}{4}\right)\cdot$$
$$\cdot\left(\cos\frac{3\pi}{4}\cos\frac{\pi d}{4} + \sin\frac{3\pi}{4}\sin\frac{\pi d}{4} - \cos\frac{3\pi}{4}\cos\frac{\pi d}{4} + \sin\frac{3\pi}{4}\sin\frac{\pi d}{4}\right)$$

$$Z_{11} = \frac{4\cdot 1\cdot p_0}{\pi^2}\cdot 2\sin\frac{\pi}{4}\sin\frac{\pi c}{4}\cdot 2\sin\frac{\pi}{4}\cdot\sin\frac{\pi d}{4}$$

$$Z_{11} = \frac{16\cdot 1\cdot p_0}{\pi^2}\cdot\frac{\sqrt{2}}{2}\cdot\frac{\sqrt{2}}{2}\cdot\sin\frac{\pi c}{4}\sin\frac{\pi d}{4} = \frac{8p_0}{\pi^2}\cdot\sin\frac{\pi c}{4}\sin\frac{\pi d}{4}$$

$$\lim_{\substack{c\to 0\\ d\to 0}} Z_{11} = \lim_{\substack{c\to 0\\ d\to 0}} \frac{8p_0}{\pi^2}\cdot\frac{\sin\frac{\pi c}{4}}{\frac{\pi c}{4}}\cdot\frac{\pi c}{4}\cdot\frac{\sin\frac{\pi d}{4}}{\frac{\pi d}{4}}\cdot\frac{\pi d}{4} = \frac{1}{2}p_0\cdot c\cdot d$$

$$Z_{11} = \frac{1}{8}\cdot 40 = 5{,}0$$

$$A_{11} = \frac{5{,}0}{145{,}83\cdot\pi^4\left(\frac{1}{4^2}+\frac{1}{4^2}\right)^2} = 2{,}2527\cdot 10^{-2}$$

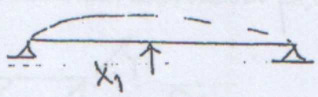
$$W = 2{,}2527\cdot 10^{-2}\cdot\sin\frac{\pi x}{4}\sin\frac{\pi y}{4}$$

$$\delta_{10}^{P} = -W(2,2) = -2{,}2527 \cdot 10^{-2}$$

$$\delta_{10}^{pl} = \delta_{10}^{g} + \delta_{10}^{P} = -3{,}2752 \cdot 10^{-2}$$

$$\delta_{11} \cdot X_1 + \delta_{10} = 0 \Rightarrow X_1 = +28{,}9926\,\text{KN} \rightarrow \text{Sila u stubu}$$

$$Z = Z_0 + Z_1 \cdot X_1$$



$$W = W^{g} + W^{P} - W_1 \cdot X_1 = \left(1{,}0225 \cdot 10^{-2} + 2{,}2527 \cdot 10^{-2} - 1{,}1263 \cdot 10^{-3} \cdot X_1\right) \sin\frac{\pi x}{4} \sin\frac{\pi y}{4}$$

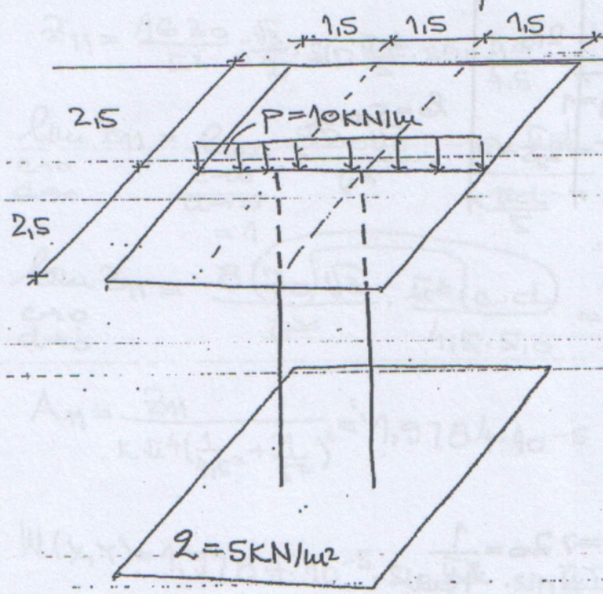
$$W = 9{,}6642 \cdot 10^{-5} \cdot \sin\frac{\pi x}{4} \cdot \sin\frac{\pi y}{4}$$

NAVIER

8

23.01.2006.

1. Za konstrukciju prikazanu na slici odrediti sile u stubovima i ugibe u sredini ploča. Koristiti prvi član reda usvojenog rešenja. Ploče su po konturi slobodno oslonjene.



$E = 30\,GPa$

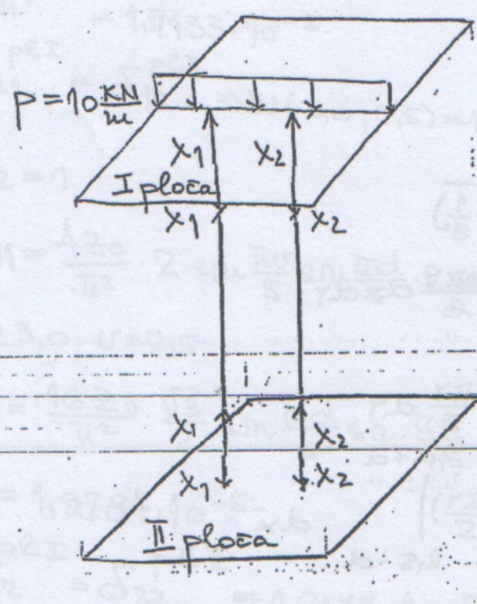
$\nu = 0$

$h = 0{,}20\,m$

Stubovi: $b/d = 0{,}5/0{,}5$

Visina stubova: $l = 3{,}0\,m$

*Dekompozicija

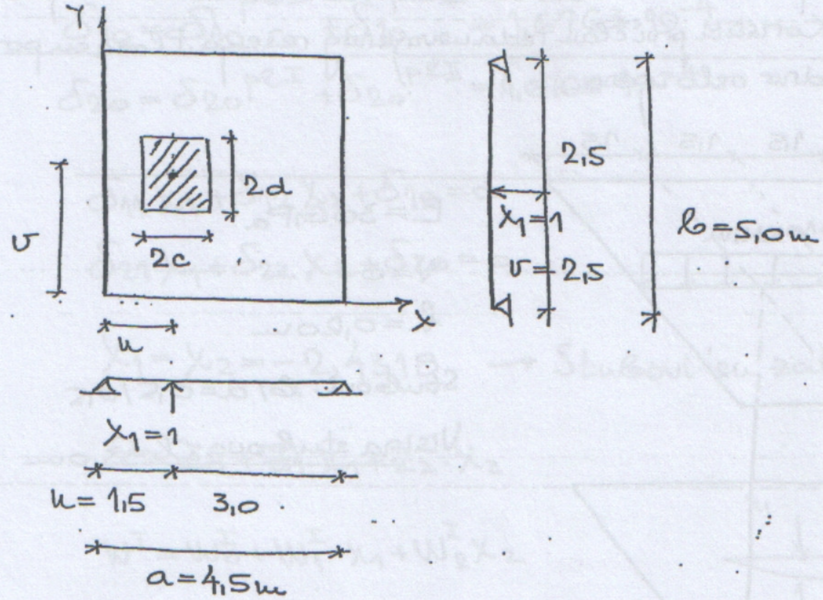


$$\delta_{11}\cdot X_1 + \delta_{12}\cdot X_2 + \delta_{10} = 0$$

$$\delta_{21}\cdot X_1 + \delta_{22}\cdot X_2 + \delta_{20} = 0$$

$$K=\frac{E\cdot h^3}{12(1-\nu^2)}=20000$$

I ploča



$$1=2c\cdot 2d\cdot Z_0=4cd\cdot Z_0 \Rightarrow Z_0=\frac{1}{4cd}$$

$$W(x,y)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}A_{mn}\cdot\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b}$$

$m=1 \quad n=1$

$$W(x,y)=A_{11}\cdot\sin\frac{\pi x}{a}\sin\frac{\pi y}{b}$$

$$A_{mn}=\frac{Z_{mn}}{K\cdot\pi^4\left(\frac{m^2}{a^2}+\frac{n^2}{b^2}\right)} \Rightarrow A_{11}=\frac{Z_{11}}{K\cdot\pi^4\left(\frac{1}{a^2}+\frac{1}{b^2}\right)}$$

$$Z_{mn}=\frac{4}{ab}\int_0^a\int_0^b Z(x,y)\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b}\,dx\,dy$$

$$Z_{11}=\frac{4}{4{,}5\cdot 5}\cdot Z_0\int_{u-c}^{u+c}\sin\frac{\pi x}{4{,}5}\,dx\int_{v-d}^{v+d}\sin\frac{\pi y}{5}\,dy$$

$$Z_{11}=\frac{4Z_0}{22{,}5}\int_{u-c}^{u+c}\sin\frac{\pi x}{4{,}5}\cdot\frac{5}{\pi}\cdot\left(-\cos\frac{\pi y}{5}\right)\Big|_{2,5-d}^{2,5+d}\,dx$$

$$Z_{11}=\frac{4Z_0}{22{,}5}\cdot\frac{4{,}5\cdot 5}{\pi^2}\left[\left(\cos\frac{\pi(2{,}5-d)}{5}-\cos\frac{\pi(2{,}5+d)}{5}\right)\cdot\left(\cos\frac{\pi(1{,}5-c)}{4{,}5}-\cos\frac{\pi(1{,}5+c)}{4{,}5}\right)\right]$$

$$Z_{11} = \frac{4z_0}{\pi^2}\left[\left(\cancel{\cos\frac{\pi}{2}\cos\frac{\pi d}{5}} + \sin\frac{\pi}{2}\sin\frac{\pi d}{5} - \cancel{\cos\frac{\pi}{2}\cos\frac{\pi d}{5}} + \sin\frac{\pi}{2}\sin\frac{\pi d}{5}\right)\cdot\left(\cancel{\cos\frac{\pi}{3}\cos\frac{\pi c}{4{,}5}} + \sin\frac{\pi}{3}\sin\frac{\pi c}{4{,}5} - \cancel{\cos\frac{\pi}{3}\cos\frac{\pi c}{4{,}5}} + \sin\frac{\pi}{3}\sin\frac{\pi c}{4{,}5}\right)\right]$$

$$Z_{11} = \frac{4z_0}{\pi^2}\left[2\cdot\sin\frac{\pi}{2}\sin\frac{\pi d}{5}\cdot 2\cdot\sin\frac{\pi}{3}\cdot\sin\frac{\pi\cdot c}{4{,}5}\right]$$

$$Z_{11} = \frac{16z_0}{\pi^2}\cdot\frac{\sqrt{3}}{2}\cdot\sin\frac{\pi d}{5}\cdot\sin\frac{\pi c}{4{,}5}$$

$$\lim_{\substack{c\to 0\\ d\to 0}} Z_{11} = \lim_{\substack{c\to 0\\ d\to 0}} \frac{8z_0\sqrt{3}}{\pi^2}\cdot\underbrace{\frac{\sin\frac{\pi d}{5}}{\frac{\pi d}{5}}\cdot\frac{\pi d}{5}\cdot\frac{\sin\frac{\pi c}{4{,}5}}{\frac{\pi c}{4{,}5}}}_{=1}\cdot\frac{\pi c}{4{,}5}$$

$$\lim_{\substack{c\to 0\\ d\to 0}} Z_{11} = \frac{8\,z_0\sqrt{3}}{\cancel{\pi^2}}\cdot\frac{\cancel{\pi^2}\,c\cdot d}{4{,}5\cdot 5{,}0} = \frac{8\sqrt{3}}{4{,}5\cdot 5{,}0} = 0{,}3079$$

$$A_{11} = \frac{Z_{11}}{K\cdot\pi^4\left(\frac{1}{4{,}5^2}+\frac{1}{5^2}\right)^2} = 1{,}9784\cdot 10^{-5}$$

$$W(x,y) = 1{,}9784\cdot 10^{-5}\cdot\sin\frac{\pi x}{4{,}5}\cdot\sin\frac{\pi y}{5{,}0}$$

$$\delta_{11}^{pl\,I} = W(1{,}5;2{,}5) = 1{,}7133\cdot 10^{-5}$$

$$\delta_{11}^{pl\,II} = 1{,}7133\cdot 10^{-5}$$

$$\delta_{21}^{pl\,I} = \delta_{12}^{pl\,I} = W(3{,}0;2{,}5) = 1{,}7133\cdot 10^{-5} = \delta_{21}^{pl\,II} = \delta_{12}^{pl\,II}$$

$X_2 = 1$

$$Z_{11} = \frac{4z_0}{\pi^2}\cdot 2\sin\frac{\pi v}{5}\sin\frac{\pi d}{5}\cdot 2\sin\frac{\pi u}{4{,}5}\sin\frac{\pi c}{4{,}5}$$

$u = 3{,}0 \quad v = 2{,}5$

$$Z_{11} = \frac{16z_0}{\pi^2}\cdot\frac{\sqrt{3}}{2}\sin\frac{\pi d}{5}\sin\frac{\pi c}{4{,}5}$$

$$A_{11} = 1{,}9784\cdot 10^{-5}$$

$$\delta_{22}^{pl\,I} = \delta_{22}^{pl\,II} = 1{,}7133\cdot 10^{-5}$$

* Stub

$$\delta_{11}^{st} = \int \frac{N_1^2}{EF} ds = \frac{1^2 \cdot 3{,}0}{30 \cdot 10^6 \cdot 0{,}5^2} = 0{,}4 \cdot 10^{-6} = \delta_{22}^{st}$$

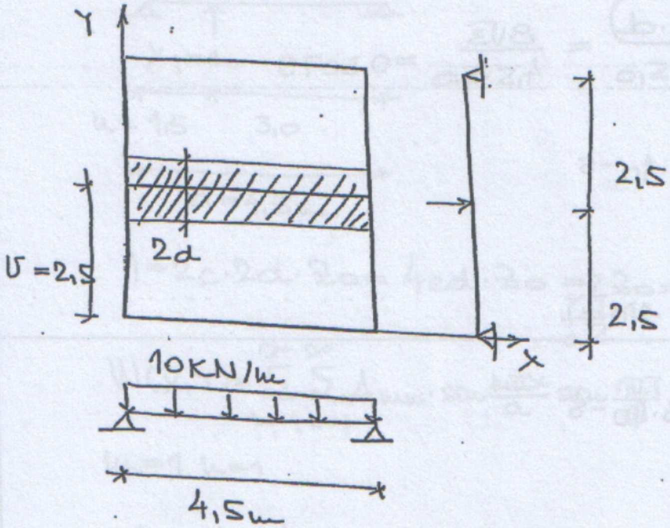
$$\delta_{11} = \delta_{11}^{pl\,I} + \delta_{11}^{st} + \delta_{11}^{pl\,II} = 3{,}4665 \cdot 10^{-5}$$

$$\delta_{12} = \delta_{21} = \delta_{12}^{pl\,I} + \delta_{12}^{pl\,II} = 3{,}42653 \cdot 10^{-5}$$

$$\delta_{22} = \delta_{22}^{pl\,I} + \delta_{22}^{st} + \delta_{22}^{pl\,II} = 3{,}4665 \cdot 10^{-5}$$

* Spoljašnje opterećenje

I ploča



$$W(x,y) = A_{11} \sin\frac{\pi x}{a} \sin\frac{\pi y}{b}$$

$$A_{11} = \frac{Z_{11}}{K \cdot \pi^4 \left(\frac{1}{a^2} + \frac{1}{b^2}\right)^2}$$

$$Z_{11} = \frac{4}{ab} \int_0^a \int_0^b Z(x,y) \sin\frac{\pi x}{a} \sin\frac{\pi y}{b} dx dy$$

$$Z_{11} = \frac{4}{4{,}5 \cdot 5} \int_0^{4{,}5} p_0 \sin\frac{\pi x}{a} dx \int_{v-d}^{v+d} \sin\frac{\pi y}{b} dy$$

$$2d \cdot a \cdot p_0 = P$$

$$Z_{11} = \frac{4}{22{,}5} \int_0^{4{,}5} p_0 \cdot \sin\frac{\pi x}{4{,}5} \cdot \left(\cos\frac{\pi(2{,}5-d)}{5} - \cos\frac{\pi(2{,}5+d)}{5}\right) dx \cdot \frac{a \cdot b}{\pi^2}$$

$$Z_{11}=\frac{4}{22{,}5}\cdot p_0\cdot(\cos 0-\cos\pi)\left(\cancel{\cos\frac{\pi}{2}\cos\frac{\pi d}{5}}+\sin\frac{\pi}{2}\sin\frac{\pi d}{5}-\right.$$

$$\left.-\cancel{\cos\frac{\pi}{2}\cos\frac{\pi d}{5}}+\sin\frac{\pi}{2}\sin\frac{\pi d}{5}\right)\cdot\frac{22{,}5}{\pi^2}$$

$$Z_{11}=\frac{4p_0}{22{,}5}\cdot 2\cdot 2\sin\frac{\pi d}{5}=\frac{16p_0}{\pi^2}\sin\frac{\pi d}{5}$$

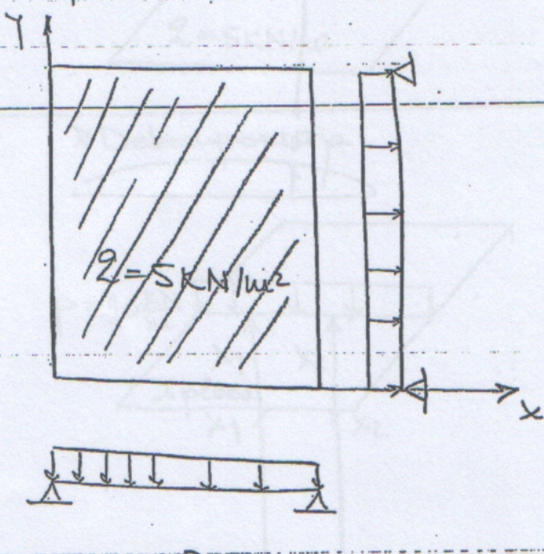
$$\lim_{d\to 0} Z_{11}=\lim_{d\to 0}\frac{16p_0}{\pi^2}\cdot\frac{\sin\frac{\pi d}{5}}{\frac{\pi d}{5}}\cdot\frac{\pi d}{5}=\frac{8\cdot 10}{5\cdot\pi}=5{,}0930$$

$$A_{11}=\frac{5{,}0930}{20000\cdot\pi^4\left(\frac{1}{4{,}5^2}+\frac{1}{5^2}\right)^2}=3{,}2722\cdot 10^{-4}$$

$$W(x,y)=3{,}2722\cdot 10^{-4}\cdot\sin\frac{\pi x}{4{,}5}\sin\frac{\pi y}{5}$$

$$\delta_{10}^{pl\,I}=-W(1{,}5;2{,}5)=-2{,}8338\cdot 10^{-4}=\delta_{20}^{pl\,I}$$

$\underline{II}$ ploča



$$A_{11}=\frac{Z_{11}}{K\pi^4\left(\frac{1}{a^2}+\frac{1}{b^2}\right)^2}$$

$$Z_{11}=\frac{4}{4{,}5\cdot 5}\int_0^{4{,}5}5\cdot\sin\frac{\pi x}{4{,}5}\,dx\int_0^{5}\sin\frac{\pi y}{5}\,dy=8{,}1057$$

$$A_{11}=5{,}2078\cdot 10^{-4}$$

$$W(x,y)=5{,}2078\cdot 10^{-4}\cdot\sin\frac{\pi x}{4{,}5}\cdot\sin\frac{\pi y}{5}$$

$$\delta_{10}^{pl\,II} = +W(1{,}5;2{,}5) = 4{,}5101\cdot 10^{-4}$$

$$\delta_{20}^{pl\,II} = +W(3{,}0;2{,}5) = 4{,}5101\cdot 10^{-4}$$

$$\delta_{10} = \delta_{10}^{pl\,I} + \delta_{10}^{pl\,II} = 1{,}6763\cdot 10^{-4}$$

$$\delta_{20} = \delta_{20}^{pl\,I} + \delta_{20}^{pl\,II} = 1{,}6763\cdot 10^{-4}$$

$$\delta_{11}X_1 + \delta_{12}X_2 + \delta_{10} = 0$$

$$\delta_{21}X_1 + \delta_{22}X_2 + \delta_{20} = 0$$

$X_1 = X_2 = -2{,}4318 \rightarrow$ Stubovi su zategnuti

$$Z = Z_0 + Z_1\cdot X_1 + Z_2\cdot X_2$$

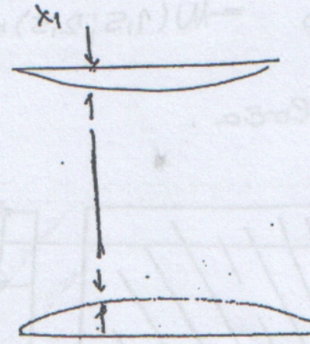
$$W^{I} = W_0^{I} + W_1^{I}\cdot X_1 + W_2^{I} X_2$$

$x = 2{,}25 \quad y = 2{,}5$

$$W^{I} = 4{,}2344\cdot 10^{-4}\ m$$

$$W^{II} = W_0^{II} + W_1^{II}\cdot X_1 + W_2^{II}\cdot X_2$$

$$W^{II} = 4{,}2456\cdot 10^{-4}\ m$$

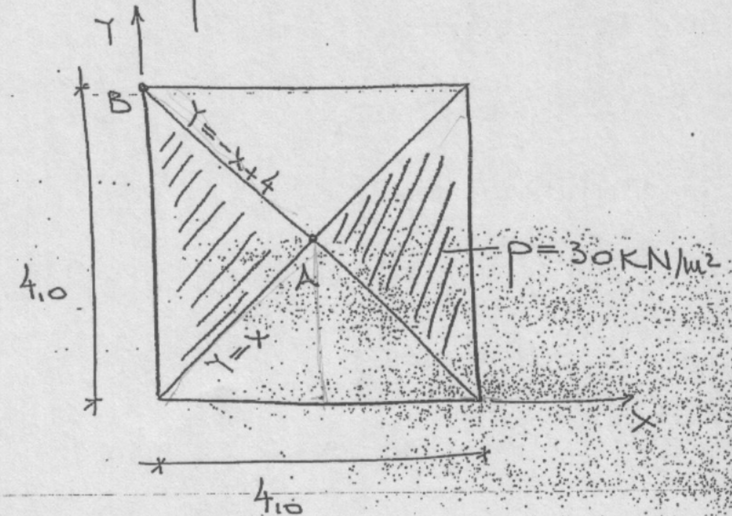


NAVIER

13

24.06.2002.

1. Za pravougaonu ploču opterećenu prema slici sračunati presečne sile u označenim tačkama. Koristiti prvi član reda usvojenog rešenja. Ploča je po konturi slobodno oslonjena.



E=30GPa

$\nu = 0,20$

$h = 0,12m$

$$K = \frac{E \cdot h^3}{12(1-\nu^2)} = 4500$$

I: $\left.\begin{matrix} x=0 & Y=0 \\ x=4 & Y=4 \end{matrix}\right\} \Rightarrow Y = x$

II $\left.\begin{matrix} x=0 & Y=4 \\ x=4 & Y=0 \end{matrix}\right\} \Rightarrow Y = -X+4$

$$W(x,Y) = A_{11} \sin\frac{\pi x}{a} \sin\frac{\pi Y}{b}$$

$$A_{11} = \frac{Z_{11}}{K \cdot \pi^4 \left(\frac{1}{a^2}+\frac{1}{b^2}\right)^2}$$

$$Z_{11} = \frac{4}{a \cdot b} \int_0^a \int_0^b Z(x,Y) \sin\frac{\pi x}{a} \sin\frac{\pi Y}{b} dx$$

$$Z_{11} = \frac{4}{16} \cdot p \left[\int_0^2 \sin\frac{\pi x}{4} dx \int_x^{-x+4} \sin\frac{\pi Y}{4} dY + \int_2^4 \sin\frac{\pi x}{4} dx \int_{-x+4}^{x} \sin\frac{\pi Y}{4} dY \right]$$

$$Z_{11}=\frac{4\cdot 30}{16}\frac{4}{\pi}\left[\int_0^2 \sin\frac{\pi x}{4}\left(\cos\frac{\pi x}{4}-\cos\frac{\pi(-x+4)}{4}\right)dx+\right.$$

$$\left.+\int_2^4 \sin\frac{\pi x}{4}\cdot\left(\cos\frac{\pi(-x+4)}{4}-\cos\frac{\pi x}{4}\right)dx\right]$$

$Z_{11}=24{,}3171$

$A_{11}\doteq 3{,}5504\cdot 10^{-3}$

$W(x,y)=3{,}5504\cdot 10^{-3}\sin\frac{\pi x}{4}\sin\frac{\pi y}{4}$

$M_x=-K\left(\frac{\partial^2 W}{\partial x^2}+\nu\frac{\partial^2 W}{\partial y^2}\right)=11{,}8264\cdot\sin\frac{\pi x}{4}\sin\frac{\pi y}{4}$

$M_y=-K\left(\frac{\partial^2 W}{\partial y^2}+\nu\frac{\partial^2 W}{\partial x^2}\right)=11{,}8264\cdot\sin\frac{\pi x}{4}\sin\frac{\pi y}{4}$

$M_{xy}=-K(1-\nu)\frac{\partial^2 W}{\partial x\partial y}=-7{,}8843\cdot\cos\frac{\pi x}{4}\cos\frac{\pi y}{4}$

$T_x=-K\left(\frac{\partial^3 W}{\partial x^3}+\frac{\partial^3 W}{\partial x\partial y^2}\right)=15{,}4807\cdot\cos\frac{\pi x}{4}\cdot\sin\frac{\pi y}{4}$

$T_y=-K\left(\frac{\partial^3 W}{\partial y^3}+\frac{\partial^3 W}{\partial x^2\partial y}\right)=15{,}4807\cdot\sin\frac{\pi x}{4}\cos\frac{\pi y}{4}$

$A(2{,}0;\ 2{,}0)$

$B(0;\ 4{,}0)$

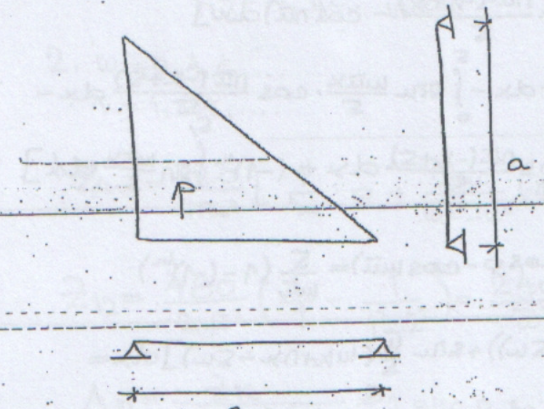
$M_{x,A}=11{,}8264$	$M_{x,B}=0$
$M_{y,A}=11{,}8264$	$M_{y,B}=0$
$M_{xy,A}=0$	$M_{xy,B}=7{,}8843$
$T_{x,A}=0$	$T_{x,B}=0$
$T_{y,A}=0$	$T_{y,B}=0$

NAVIER

(14)

28.02.2004.

1.2 Za trougaonu ploču opterećenju prema slici odrediti rešenje za ugib u proizvoljnoj tački ploče. Koristeći prvi član koji je različit od nule, sračunati max ugib ploče. Ploča je po konturi slobodno oslonjena.



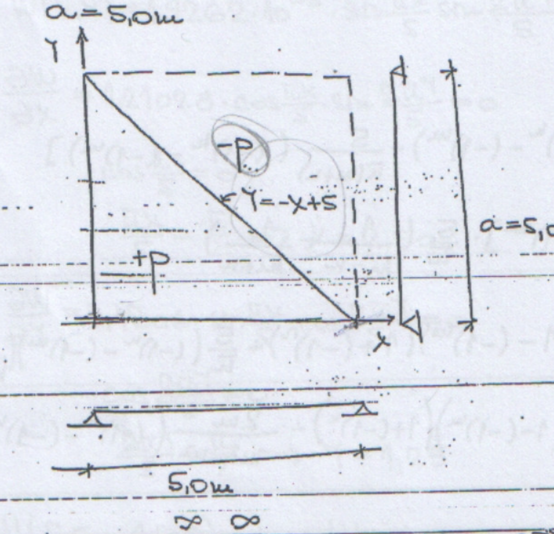
$E = 30 GPa$

$\nu = 0$

$h = 0{,}12 m$

$p = 30 kN/m^2$

$a = 5{,}0 m$



$$\int_0^5\int_0^{y=-x+5} - \int_0^5\int_{x+1}$$

$$W(x,y)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty} A_{mn}\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b}$$

$$A_{mn}=\frac{Z_{mn}}{K\cdot\pi^4\left(\frac{m^2}{a^2}+\frac{n^2}{b^2}\right)^2}$$

$$Z_{mn}=\frac{4}{ab}\int_0^a\int_0^b z(x,y)\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b}\,dx\,dy$$

$$Z_{mn}=\frac{4}{25}\cdot 30\left[\int_0^5\sin\frac{m\pi x}{a}dx\int_0^{-x+5}\sin\frac{n\pi y}{b}dy-\int_0^5\sin\frac{m\pi x}{a}dx\int_{-x+5}^5\sin\frac{n\pi y}{b}dy\right]$$

$$Z_{mn}=\frac{120}{25}\cdot\frac{5}{n\pi}\left[\int_0^5\sin\frac{m\pi x}{5}\left(\cos 0-\cos\frac{n\pi(-x+5)}{5}\right)dx-\right.$$

$$\left.-\int_0^5\sin\frac{m\pi x}{5}\left(\cos\frac{n\pi(-x+5)}{5}-\cos n\pi\right)dx\right]$$

$$Z_{mn}=\frac{24}{n\pi}\left[\int_0^5\sin\frac{m\pi x}{5}dx-\int_0^5\sin\frac{m\pi x}{5}\cdot\cos\frac{n\pi(-x+5)}{5}dx-\right.$$

$$\left.-\int_0^5\sin\frac{m\pi x}{5}\cdot\cos\frac{n\pi(-x+5)}{5}dx+(-1)^n\int_0^5\sin\frac{m\pi x}{5}dx\right]$$

$$J_1=\int_0^5\sin\frac{m\pi x}{5}dx=\frac{5}{m\pi}(\cos 0-\cos m\pi)=\frac{5}{m\pi}(1-(-1)^m)$$

$$J_2=\int_0^5\frac{1}{2}\left[\sin\left(\frac{\pi}{5}(mx-nx+5n)\right)+\sin\frac{\pi}{5}(mx+nx-5n)\right]dx=$$

$$=\frac{1}{2}\left[\int_0^5\sin\left(\frac{\pi x(m-n)}{5}+\pi n\right)dx+\int_0^5\sin\left(\frac{\pi x(m+n)}{5}-n\pi\right)dx\right]=$$

$$=\frac{1}{2}\left[\frac{5}{\pi(m-n)}(\cos\pi n-\cos\pi(m-n+n))+\frac{5}{\pi(m+n)}(\cos(-n\pi)-\cos m\pi)\right]$$

$$\frac{\pi x(m-n)}{5}+\pi n=t$$

$$\frac{\pi(m-n)}{5}dx=dt$$

$$J_2=\frac{1}{2}\left[\frac{5}{\pi(m-n)}((-1)^n-(-1)^m)+\frac{5}{\pi(m+n)}((-1)^n-(-1)^m)\right]$$

$$J_2=\frac{1}{2}\left[(-1)^n-(-1)^m\right]\cdot\frac{5}{\pi}\left(\frac{1}{m-n}+\frac{1}{m+n}\right)$$

$$Z_{mn}=\frac{24}{n\pi}\left[\frac{5}{m\pi}(1-(-1)^m)(1+(-1)^n)+\frac{5}{\pi}((-1)^n-(-1)^m)\left(\frac{1}{m-n}+\frac{1}{m+n}\right)\right]$$

$$Z_{mn}=\frac{120}{n\pi^2}\left[\frac{1}{m}\left(1-(-1)^m\right)\left(1+(-1)^n\right)+\frac{2m}{m^2-n^2}\left((-1)^n-(-1)^m\right)\right]$$

Ako su m i n oba parna ili oba neparna $\Rightarrow Z_{mn}=0$

1. $m = 2k+1 \qquad , k = 0,1,\dots \infty$
   $n = 2k$

$$Z_{mn} = \frac{120}{n\pi^2}\left[\frac{1}{m}(1+1)(1+1) + \frac{2m}{m^2-n^2}(1+1)\right]$$

$$Z_{mn} = \frac{120}{n\pi^2}\left[\frac{4}{m} - \frac{4m}{m^2-n^2}\right] = \frac{480}{n\pi^2}\left(\frac{1}{m} - \frac{m}{m^2-n^2}\right)$$

2. $m = 2,4,6,\dots$
   $n = 1,3,5,\dots$

$$Z_{mn} = \frac{120}{n\pi^2}\left[\frac{1}{m}\cdot 0 + \frac{2m}{m^2-n^2}(-1-1)\right] = \frac{480\cdot m}{n\pi^2(m^2-n^2)}$$

$$Z_{12} = \frac{480}{2\pi^2}\left(\frac{1}{1} - \frac{1}{1^2-2^2}\right) = \frac{240}{\pi^2}\left(1+\frac{1}{3}\right) = 32{,}4228$$

$$A_{12} = \frac{Z_{12}}{K\cdot\pi^4\left(\frac{1^2}{5^2}+\frac{2^2}{5^2}\right)^2} = 1{,}9262\cdot 10^{-3}$$

$$K = \frac{30\cdot 10^6\cdot 0{,}12^3}{12(1-0)} = 4320$$

$$W(x,y) = 1{,}9262\cdot 10^{-3}\cdot \sin\frac{\pi x}{5}\sin\frac{2\pi y}{5}$$

$$\frac{\partial W}{\partial x} = 1{,}21028\cdot\cos\frac{\pi x}{5}\cdot\sin\frac{2\pi y}{5} = 0$$

$$\cos\frac{\pi x}{5} = 0$$

$$\frac{\pi x}{5} = \frac{\pi}{2} \Rightarrow x = 2{,}50$$

$$\frac{\partial W}{\partial y} = 2{,}4206\cdot\sin\frac{\pi x}{5}\cdot\cos\frac{2\pi y}{5} = 0$$

$$\cos\frac{2\pi y}{5} = 0$$

$$\frac{2\pi y}{5} = \frac{\pi}{2} \Rightarrow y = 1{,}25$$

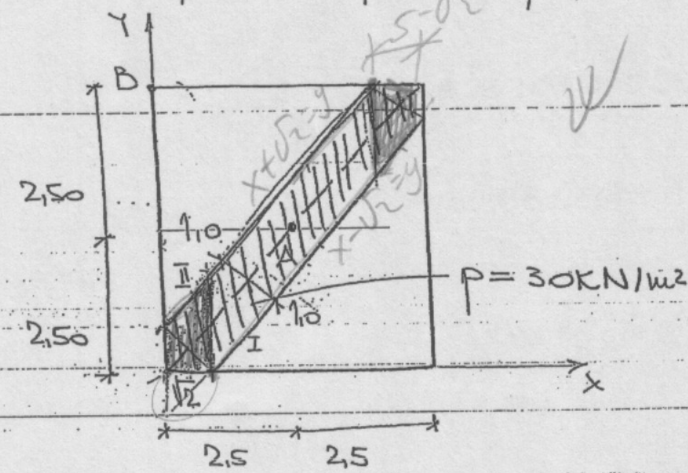
$$W(2{,}50;1{,}25) = \max W = 1{,}9262\cdot 10^{-3}\,m$$

NAVIER X2

(10)

02.09.2002.

1. Za pravougaonu ploču opterećenu prema slici sračunati presečne sile u označenim tačkama. Koristiti prvi član reda usvojenog rešenja. Ploča je po konturi slobodno oslonjena.



$E = 30\ GPa$

$\nu = 0{,}2$

$h = 0{,}20m$

$$K = \frac{30 \cdot 10^6 \cdot 0{,}2^3}{12(1-0{,}2^2)} = 20833{,}\dot{3}$$

$Y = kx + n$

$k = 1$

$I: Y = x - \sqrt{2}$

$II: Y = x + \sqrt{2}$

$$W(x,Y) = A_{11} \cdot \sin\frac{\pi x}{5} \sin\frac{\pi Y}{5}$$

$$A_{11} = \frac{Z_{11}}{K\pi^4\left(\frac{1}{a^2}+\frac{1}{b^2}\right)^2}$$

$$Z_{11} = \frac{4}{ab}\int_0^a\int_0^b Z(x,Y)\sin\frac{\pi x}{5}\sin\frac{\pi Y}{5}\,dx\,dY$$

$$Z_{11} = \frac{4}{25}\cdot 30\left[\int_0^{\sqrt{2}}\sin\frac{\pi x}{5}dx\int_0^{x+\sqrt{2}}\sin\frac{\pi Y}{5}dY + \int_{\sqrt{2}}^{5-\sqrt{2}}\sin\frac{\pi x}{5}dx\int_{x-\sqrt{2}}^{x+\sqrt{2}}\sin\frac{\pi Y}{5}dx + \right.$$

$$\left. + \int^{5}\sin\frac{\pi x}{5}dx\int\sin\frac{\pi Y}{5}dY\right]$$

$$Z_{11}=\frac{120}{25}\cdot\frac{5}{\pi}\Big[\int_0^{\sqrt{2}}\sin\frac{\pi x}{5}\cdot\Big(\cos 0-\cos\frac{\pi(x+\sqrt{2})}{5}\Big)dx+$$

$$+\int_{\sqrt{2}}^{5-\sqrt{2}}\sin\frac{\pi x}{5}\Big(\cos\frac{\pi(x-\sqrt{2})}{5}-\cos\frac{\pi(x+\sqrt{2})}{5}\Big)dx+$$

$$+\int_{5-\sqrt{2}}^{5}\sin\frac{\pi x}{5}\Big(\cos\frac{\pi(x-\sqrt{2})}{5}-\cos\pi\Big)dx$$

$$Z_{11}=\frac{120}{5\pi}[0{,}5323+2{,}8946+0{,}5323]=30{,}2468$$

$$A_{11}=\frac{30{,}2468}{20833{,}3\cdot\pi^4\left(\frac{2}{5^2}\right)^2}=2{,}3288\cdot 10^{-3}$$

$$W(x,y)=2{,}3288\cdot 10^{-3}\cdot\sin\frac{\pi x}{5}\sin\frac{\pi y}{5}$$

Tačka A:

$$W(2{,}5;2{,}5)=2{,}3288\cdot 10^{-3}$$

Tačka B:

$$W(0;5{,}0)=0$$

Presečne sile:

$$M_x=-K\left(\frac{\partial^2 W}{\partial x^2}+\nu\frac{\partial^2 W}{\partial y^2}\right)=22{,}9848\cdot\sin\frac{\pi x}{5}\sin\frac{\pi y}{5}$$

$$M_y=-K\left(\frac{\partial^2 W}{\partial y^2}+\nu\frac{\partial^2 W}{\partial x^2}\right)=22{,}9848\cdot\sin\frac{\pi x}{5}\sin\frac{\pi y}{5}$$

$$M_{xy}=-K(1-\nu)\frac{\partial^2 W}{\partial x\partial y}=-15{,}3232\cdot\cos\frac{\pi x}{5}\cos\frac{\pi y}{5}$$

$$T_x=-K\left(\frac{\partial^3 W}{\partial x^3}+\frac{\partial^3 W}{\partial x\partial y^2}\right)=24{,}0696\cdot\cos\frac{\pi x}{5}\sin\frac{\pi y}{5}$$

$$T_y=-K\left(\frac{\partial^3 W}{\partial y^3}+\frac{\partial^3 W}{\partial x^2\partial y}\right)=24{,}0696\cdot\sin\frac{\pi x}{5}\cos\frac{\pi y}{5}$$

$M_{x,A}=22{,}9848$ KNm $M_{x,B}=0$

$M_{y,A}=22{,}9848$ KNm $M_{y,B}=0$

$M_{xy,A}=0$ $M_{xy,B}=15{,}3232$ KNm

$T_{x,A}=0$ $T_{x,B}=0$

$T_{y,A}=0$ $T_{y,B}=0$

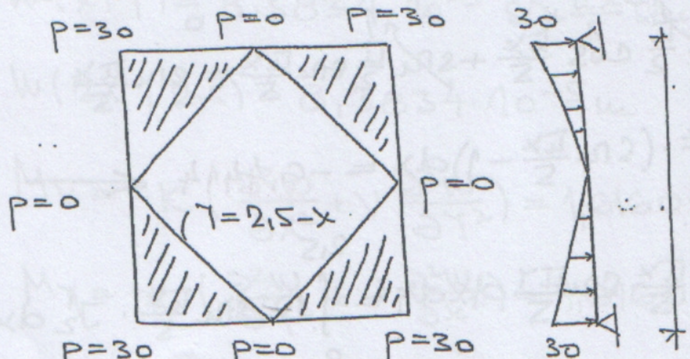
NT

NAVIER

(21) GREŠKA JER MORA $4 \cdot Z_{mn}$ jer se 4 puta integrali ista figura

20.04.2006.

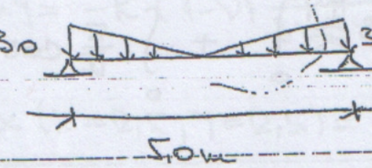
1. Za kvadratnu ploču opterećenu opterećenjem na slici odrediti momente savijanja i ugibu sredini ploče. Koristiti prvi član reda usvojenog rešenja. Ploča je po konturi slobodno oslonjena.



$E = 30\,GPa$

$\nu = 0{,}2$

$h = 0{,}25\,m$



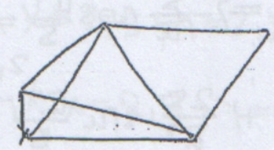
$Y = aX + b$

$X=0 \quad Y=2{,}5 \Rightarrow 2{,}5 = b$

$X=2{,}5 \quad Y=0 \Rightarrow 0 = 2{,}5a + b$

$a = -1$

$Y = -X + 2{,}5$



$Z(X,Y) = AX + BY + C$

$X=0 \quad Y=0 \quad Z=30 \Rightarrow 30 = C$

$X=2{,}5 \quad Y=0 \quad Z=0 \Rightarrow 0 = 2{,}5A + C \Rightarrow A = -12$

$X=0 \quad Y=2{,}5 \quad Z=0 \Rightarrow 0 = 2{,}5B + C \Rightarrow B = -12$

$Z(X,Y) = -12X - 12Y + 30$

$$W(X,Y) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} A_{mn} \sin\frac{m\pi X}{a} \sin\frac{n\pi Y}{b}$$

$$A_{mn} = \frac{Z_{mn}}{K \cdot \pi^4 \left(\frac{m^2}{a^2} + \frac{n^2}{b^2}\right)^2}$$

$$Z_{mn} = \frac{4}{a \cdot b} \int_0^a \int_0^b Z(X,Y) \sin\frac{m\pi X}{a} \sin\frac{n\pi Y}{b}\, dX\, dY$$

$m=1 \quad n=1$

$$Z_{11}=\frac{4\cdot 4}{25}\int_0^{2,5}\int_0^{2,5-x}(-12x-12y+30)\sin\frac{\pi x}{5}\sin\frac{\pi y}{5}\,dy\,dy$$

$$Z_{11}=\frac{4}{25}\cdot(J_1+J_2+J_3)$$

$$J_1=\int_0^{2,5}\int_0^{2,5-x}-12x\sin\frac{\pi x}{5}\sin\frac{\pi y}{5}\,dx\,dy=+12\,\frac{5}{\pi}\int_0^{2,5}x\cdot\sin\frac{\pi x}{5}\cos\frac{\pi y}{5}\Big|_0^{2,5-x}dx$$

$$\cos\frac{\pi(2,5-x)}{5}=\cancel{\cos\frac{\pi}{2}}^{\,0}\cos\frac{\pi x}{5}+\cancel{\sin\frac{\pi}{2}}^{\,1}\sin\frac{\pi x}{5}=\sin\frac{\pi x}{5}$$

$$J_1=\frac{60}{\pi}\int_0^{2,5}x\cdot\sin\frac{\pi x}{5}\cdot\left(\sin\frac{\pi x}{5}-1\right)dx=-6,4414 \longleftarrow$$

$$J_2=\int_0^{2,5}\int_0^{2,5-x}-12y\sin\frac{\pi x}{5}\sin\frac{\pi y}{5}\,dx\,dy=-\int_0^{2,5}12\cdot\sin\frac{\pi x}{5}\cdot J_2'\,dx$$

$$J_2'=\int_0^{2,5-x}y\cdot\sin\frac{\pi y}{5}\,dy=-\frac{5}{\pi}y\cdot\cos\frac{\pi y}{5}\Big|_0^{2,5-x}+\int_0^{2,5-x}\frac{5}{\pi}\cos\frac{\pi y}{5}\,dy$$

$$\int u\,dv=u\cdot v-\int v\,du$$

$$u=y \Rightarrow du=dy \qquad \sin\frac{\pi y}{5}dy=dv \Rightarrow -\frac{5}{\pi}\cos\frac{\pi y}{5}=v$$

$$J_2'=-\frac{5}{\pi}(2,5-x)\cdot\sin\frac{\pi x}{5}+\cancel{\frac{5}{\pi}0\cdot\cos 0}+\frac{25}{\pi^2}\sin\frac{\pi y}{5}\Big|_0^{2,5-x}$$

$$\sin\frac{\pi(2,5-x)}{5}=\cancel{\sin\frac{\pi}{2}}^{\,1}\cos\frac{\pi x}{5}+\cancel{\cos\frac{\pi}{2}}^{\,0}\sin\frac{\pi x}{5}=\cos\frac{\pi x}{5}$$

$$J_2'=-\frac{5}{\pi}(2,5-x)\sin\frac{\pi x}{5}+\frac{25}{\pi^2}\cdot\cos\frac{\pi x}{5}$$

$$J_2=\int_0^{2,5}\left(\frac{60}{\pi}((2,5-x)\cdot\sin\frac{\pi x}{5}-\frac{300}{\pi^2}\sin\frac{\pi x}{5}\cos\frac{\pi x}{5}\right)dx$$

$$J_2=3,4249$$

$$J_3=\int_0^{2,5}\int_0^{2,5-x}30\sin\frac{\pi x}{5}\sin\frac{\pi y}{5}\,dx\,dy=\int_0^{2,5}-\frac{150}{\pi}\sin\frac{\pi x}{5}\cos\frac{\pi y}{5}\Big|_0^{2,5-x}dx$$

$$J_3=-\frac{150}{\pi}\int_0^{2,5}\sin\frac{\pi x}{5}\left(\cos\frac{\pi(2,5-x)}{5}-\cancel{\cos 0}^{\,1}\right)dx=-\frac{150}{\pi}\int_0^{2,5}\sin\frac{\pi x}{5}\left(\sin\frac{\pi x}{5}-1\right)dx$$

$$J_3=16,3078$$

$$z_{11} = 2{,}1266$$

$$A_{11} = \frac{2{,}1266}{K \cdot \pi^4 \left(\frac{1}{25} + \frac{1}{25} \right)^2} = 8{,}3834 \cdot 10^{-5}$$

$$K = 40690{,}1042$$

$$W(x,y) = 8{,}3834 \cdot 10^{-5} \sin\frac{\pi x}{5} \sin\frac{\pi y}{5}$$

$$W(2{,}5;\, 2{,}5) = 8{,}3834 \cdot 10^{-5}\, m$$

$$M_x = -K\left(\frac{\partial^2 W}{\partial x^2} + \nu \frac{\partial^2 W}{\partial y^2} \right) = 1{,}6160 \cdot \sin\frac{\pi x}{5} \sin\frac{\pi y}{5}$$

$$M_y = -K\left(\frac{\partial^2 W}{\partial y^2} + \nu \frac{\partial^2 W}{\partial x^2} \right) = 1{,}6160 \cdot \sin\frac{\pi x}{5} \sin\frac{\pi y}{5}$$

$$M_{xy} = -K(1-\nu) \frac{\partial^2 W}{\partial x \partial y} = -1{,}0774 \cdot \cos\frac{\pi x}{5} \cos\frac{\pi y}{5}$$

$$M_x(x=2{,}5;\, y=2{,}5) = 1{,}6160$$

$$M_y(x=2{,}5;\, y=2{,}5) = 1{,}6160$$

$$M_{xy}(x=2{,}5;\, y=2{,}5) = 0$$