(5)

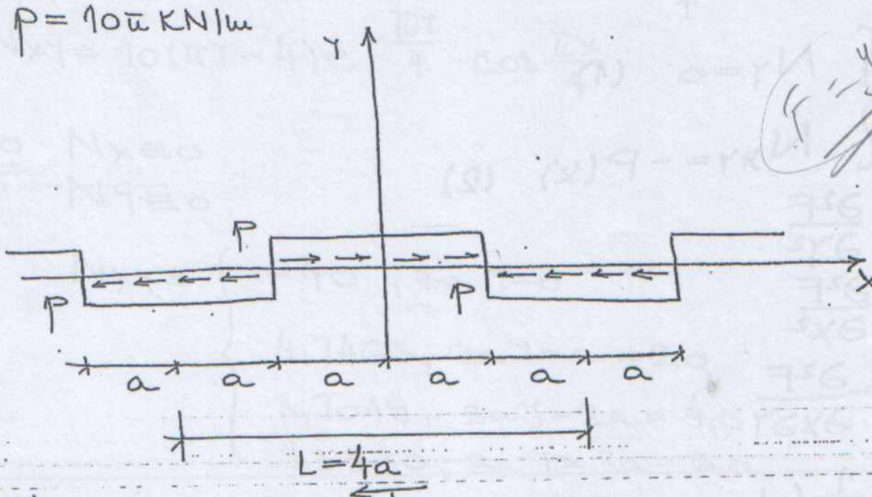
26.11.2005.

2. Za poluravan koja je po konturi opterećena linijskim opterećenjem p u pravcu X ose, sračunati i nacrtati dijagrame presečnih sila u presecima $X=0$ i $X=2a$.

Koristiti prvi član reda usvojenog rešenja.

$a = 2{,}0\,m$

$p = 10\pi\ KN/m$



$L = 4a$

$p(-x) = p(x) \rightarrow$ Opterećenje je parna f-ja

$$P(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\frac{2n\pi x}{L}$$

$n = 1$

$$P(x) = \frac{a_0}{2} + a_1 \cos\frac{2\pi x}{L}$$

$a_0 = 0$, jer je opterećenje na konturi ravnotežno

$$a_1 = \frac{4}{L}\int_0^{L/2} p(x)\cos\frac{2\pi x}{L}\,dx$$

$$p(x) = \begin{cases} 10\pi, & 0 \le x \le a \\ -10\pi, & a \le x \le 2a \end{cases}$$

$$a_1 = \frac{4}{8}\left[\int_0^2 10\pi\cos\frac{2\pi x}{8}\,dx - \int_2^4 10\pi\cos\frac{2\pi x}{8}\,dx\right] = 40$$

$$P(x) = 40\cdot\cos\frac{\pi x}{4}$$

$$Y=(A_1+\tfrac{\pi}{4}YB_1)e^{-\frac{\pi}{4}Y}+\underbrace{(C_1+\tfrac{\pi}{4}YD_1)e^{\frac{\pi}{4}Y}}$$

$\rightarrow\infty$ za $Y\rightarrow\infty \Rightarrow C_1=D_1=0$

$$\alpha_1=\frac{\pi}{4}$$

$$F=\frac{1}{\alpha_1^2}(A_1+\alpha_1 YB_1)e^{-\alpha_1 Y}\cdot\underline{\sin\alpha_1 x}$$

zato što je $p(x)=40\cdot\cos\frac{\pi x}{4}$, a za $Y=0$ $N_{xy}=-p(x)$, pa da bi u izrazu za N_{xy} bio cos

Granični uslovi:

$$Y=0\begin{cases}N_y=0 & (1)\\ N_{xy}=-P(x) & (2)\end{cases}$$

$$N_x=\frac{\partial^2F}{\partial y^2}$$

$$N_y=\frac{\partial^2F}{\partial x^2}$$

$$N_{xy}=-\frac{\partial^2F}{\partial x\partial y}$$

$$\frac{\partial F}{\partial x}=\frac{1}{\alpha_1}(A_1+\alpha_1YB_1)e^{-\alpha_1Y}\cos\alpha_1x$$

$$\frac{\partial^2F}{\partial x^2}=-(A_1+\alpha_1YB_1)e^{-\alpha_1Y}\cdot\sin\alpha_1x$$

$$\frac{\partial^2F}{\partial x\partial y}=-(A_1+\alpha_1YB_1-B_1)e^{-\alpha_1Y}\cdot\cos\alpha_1x$$

$$\frac{\partial F}{\partial y}=-\frac{1}{\alpha_1}(A_1+\alpha_1YB_1-B_1)e^{-\alpha_1Y}\cdot\sin\alpha_1x$$

$$\frac{\partial^2F}{\partial y^2}=(A_1+\alpha_1YB_1-2B_1)e^{-\alpha_1Y}\cdot\sin\alpha_1x$$

(1): $Y=0$

$$N_y=-(A_1+0)e^0\cdot\sin\alpha_1x=0\Rightarrow A_1=0$$

(2) $Y=0$

$$N_{xy}=+(\cancel{A_1}^{0}+0-B_1)e^0\cdot\cos\alpha_1x=+40\cdot\cos\alpha_1x$$

$$-B=+40\Rightarrow B=-40 \qquad \boxed{B=-40}$$

$$F=\frac{4}{\pi}\cdot40\cdot Y\cdot e^{-\frac{\pi}{4}Y}\cdot\sin\frac{\pi x}{4}$$

$$F=\frac{160}{\pi}\cdot y\cdot e^{-\frac{\pi y}{4}}\cdot\sin\frac{\pi x}{4}$$

$$N_x=\left(0+\frac{\pi}{4}\cdot 40\cdot y-80\right)e^{-\frac{\pi y}{4}}\cdot\sin\frac{\pi x}{4}$$

$$\longrightarrow N_x=10(\pi y-8)e^{-\frac{\pi y}{4}}\sin\frac{\pi x}{4}$$

$$\longrightarrow N_y=-10\pi y\, e^{-\frac{\pi y}{4}}\cdot\sin\frac{\pi x}{4}$$

$$N_{xy}=+(0+10\pi y-40)e^{-\frac{\pi y}{4}}\cdot\cos\frac{\pi x}{4}$$

$$\longrightarrow N_{xy}=10(\pi y-4)e^{-\frac{\pi y}{4}}\cdot\cos\frac{\pi x}{4}$$

$\underline{x=0}$ $\quad N_x\equiv 0$

$N_y\equiv 0$

$$N_{xy}=\begin{cases}-40, & \text{za } y=0\\ 4{,}7463, & \text{za } y=a=2{,}0\\ 3{,}7019, & \text{za } y=2a=4{,}0\\ 1{,}3340, & \text{za } y=3a=6{,}0\end{cases}$$

$\underline{x=2a}=4{,}0\,\text{m}$

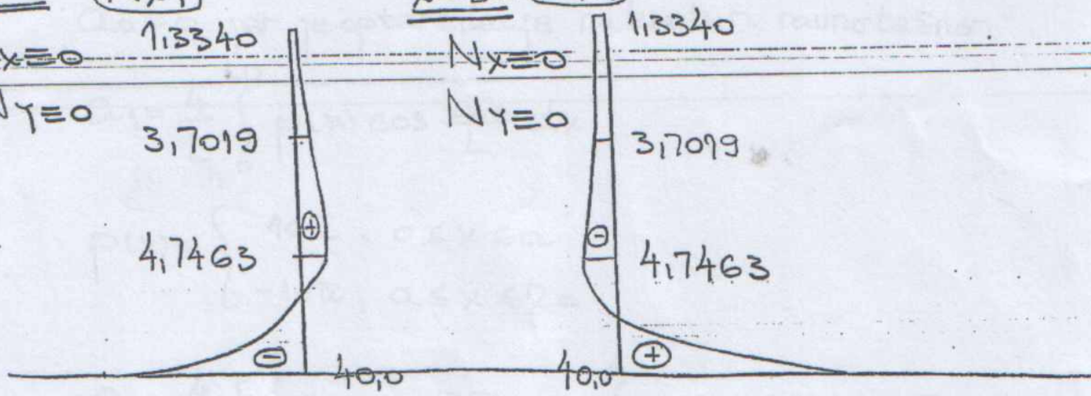
$N_x=N_y\equiv 0$

$$N_{xy}=\begin{cases}40, & \text{za } y=0\\ -4{,}7463, & \text{za } y=a\\ -3{,}7019, & \text{za } y=2a\\ -1{,}3340, & \text{za } y=3a\end{cases}$$

$\underline{x=0}$ $\quad N_{xy}$ $\qquad\qquad$ $\underline{x=2a}$ $\quad N_{xy}$

$N_x\equiv 0$ $\qquad\qquad$ $N_x\equiv 0$

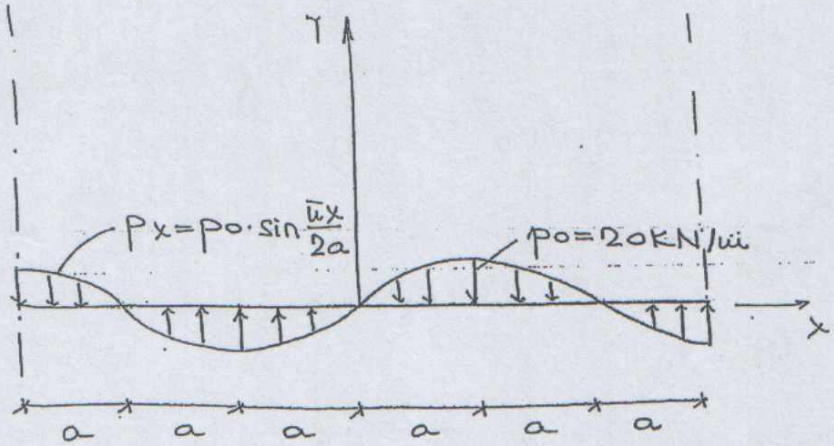
$N_y\equiv 0$ $\qquad\qquad$ $N_y\equiv 0$



(19)

25.08.2008.

2. Za poluravan opterećenu prema slici odrediti izraze i nacrtati dijagrame za sile u preseku $x=0$ i $x=a$. Za crtanje koristiti vrednosti $y=0, a, 2a, 3a$.



$L = 4a$

симусна ф-ја

$p(x) = p_0 \cdot \sin\frac{\pi x}{2a}$

$F = Y_1 \cdot \sin\frac{\pi x}{2a}$

$Y_1 = \left(A_1 + \frac{\pi y}{2a} \cdot B_1\right) e^{-\frac{\pi y}{2a}} + \underbrace{\left(C_1 + \frac{\pi y}{2a} D_1\right) e^{\frac{\pi y}{2a}}}$

$\rightarrow \infty$ kada $y \rightarrow \infty \Rightarrow C_1 = D_1 = 0$

$Y_1 = \left(A_1 + \frac{\pi y}{2a} B_1\right) e^{-\frac{\pi y}{2a}}$

$F = \frac{1}{\alpha_1^2}\left(A_1 + \alpha_1 \cdot y B_1\right) e^{-\alpha_1 y} \cdot \sin\alpha_1 x$

Granični uslovi:

$$y=0 \begin{cases} N_y = p(x) & (1) \\ N_{yx} = 0 & (2) \end{cases}$$

$N_x = \frac{\partial^2 F}{\partial y^2}$

$N_y = \frac{\partial^2 F}{\partial x^2}$

$N_{xy} = -\frac{\partial^2 F}{\partial x \partial y}$

$$\frac{\partial F}{\partial x} = \frac{1}{\alpha_1}(A_1+\alpha_1 \cdot y B_1)e^{-\alpha_1 y}\cos\alpha_1 x$$

$$\frac{\partial^2 F}{\partial x^2} = -(A_1+\alpha_1 y B_1)e^{-\alpha_1 y}\cdot\sin\alpha_1 x$$

$$\frac{\partial^2 F}{\partial x \partial y} = -(A_1+\alpha_1 y B_1 - B_1)e^{-\alpha_1 y}\cos\alpha_1 x$$

(1): $y=0$

$$N_y = -(A_1+\cancel{\alpha_1} 0 B_1)e^{-0}\sin\alpha_1 x = p_0 \sin\alpha_1 x$$

$$A_1 = -p_0 = -20$$

(2): $y=0$

$$N_{xy} = +(A_1 - B_1)e^{0}\cos\alpha_1 x = 0$$

$$B_1 = A_1 = -20$$

$$F = \frac{4a^2}{\pi^2}\left(-20-20\frac{\pi}{2a}y\right)e^{-\frac{\pi}{2a}y}\cdot\sin\frac{\pi x}{2a}$$

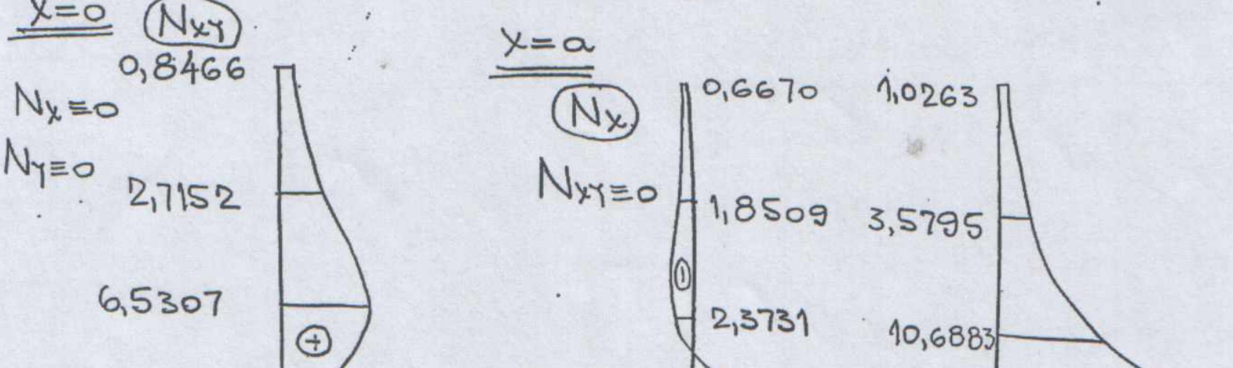
$$\frac{\partial F}{\partial y} = -\frac{1}{\alpha_1}(A_1+\alpha_1 y B_1 - B_1)e^{-\alpha_1 y}\sin\alpha_1 x$$

$$\frac{\partial^2 F}{\partial y^2} = (A_1+\alpha_1 y B_1 - 2B_1)e^{-\alpha_1 y}\sin\alpha_1 x$$

$$N_x = \left(-20-20\frac{\pi}{2a}y+40\right)e^{-\alpha_1 y}\sin\alpha_1 x = 20\left(1-\frac{\pi}{2a}y\right)e^{-\frac{\pi y}{2a}}\cdot\sin\frac{\pi x}{2a}$$

$$N_y = 20\left(1+\frac{\pi}{2a}y\right)e^{-\frac{\pi y}{2a}}\cdot\sin\frac{\pi x}{2a}$$

$$N_{xy} = -20\cdot\frac{\pi}{2a}y\,e^{-\frac{\pi y}{2a}}\cdot\cos\frac{\pi x}{2a}$$



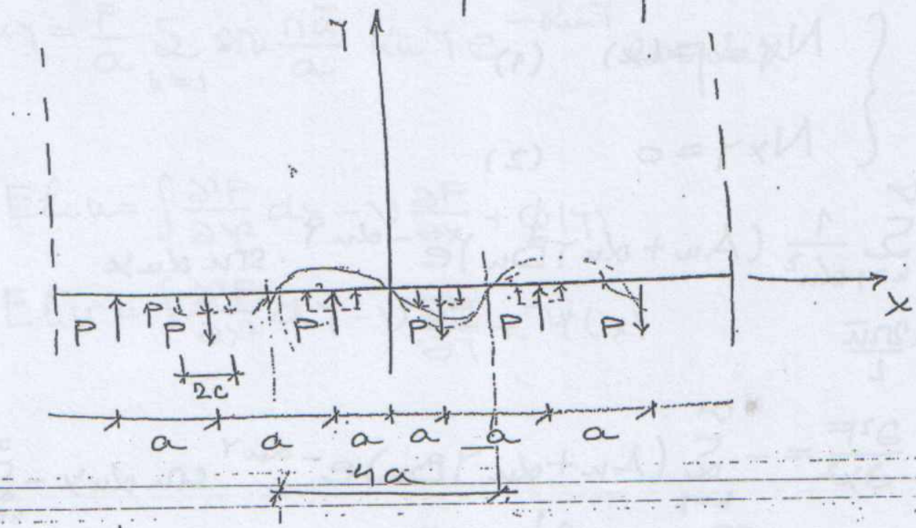
(6)

25.05.2002.

2. Za poluravan opterećenu periodičnu antimetričnu opterećenjem:

a) Odrediti izraze za sile u preseku

b) Odrediti izraze za pomeranja



$$L = 4a$$

$$p = \frac{P}{2c}$$

$$p(x) = \begin{cases} 0, \text{ za } 0 \le x \le a-c \wedge a+c \le x \le 2a \\ p \ldots \end{cases}$$

$$p(x) = \sum_{n=1}^{\infty} b_n \cdot \sin\frac{2n\pi x}{L}$$

$$b_n = \frac{4}{L}\int_0^{L/2} p(x)\cdot \sin\frac{2n\pi x}{L}\,dx = \frac{4}{4a}\int_{a-c}^{a+c} p\cdot \sin\frac{2n\pi x}{{}_2 4a}\,dx$$

$$b_n = \frac{1}{a}\cdot p\left(\cos\frac{n\pi x}{2a}(a+c) - \cos\frac{n\pi x}{2a}(a+c)\right)\cdot\frac{2a}{n\pi}$$

$$b_n = \frac{2}{n\pi}p\left(\cancel{\cos\frac{n\pi}{2}\cos\frac{n\pi c}{2a}} + \sin\frac{n\pi}{2}\sin\frac{n\pi c}{2a} - \cancel{\cos\frac{n\pi}{2}\cos\frac{n\pi c}{2a}} + \sin\frac{n\pi}{2}\sin\frac{n\pi c}{2a}\right)$$

$$b_n = \frac{4p}{n\pi}\sin\frac{n\pi}{2}\sin\frac{n\pi c}{2}$$

$$\lim_{c\to 0} b_n = \lim_{c\to 0} \frac{4p}{n\pi} \cdot \frac{\cancel{\sin\frac{n\pi c}{2a}}^{\,1}}{\cancel{\frac{n\pi c}{2a}}} \cdot \frac{n\pi c}{2a} \cdot \sin\frac{n\pi}{2}$$

$$b_n = \frac{^2 4pc}{2a} \cdot \sin\frac{n\pi}{2} = \frac{P}{a} \cdot \sin\frac{n\pi}{2}$$

$$p(x) = \sum_{n=1}^{\infty} \frac{P}{a} \sin\frac{n\pi}{2} \cdot \sin\alpha_n x$$

granični uslovi:

$$Y=0 \begin{cases} N_Y = p(x) & (1) \\ N_{XY} = 0 & (2) \end{cases}$$

$$F = \sum_{n=1}^{\infty} \frac{1}{\alpha_n^2} (A_n + \alpha_n Y B_n) e^{-\alpha_n Y} \cdot \sin\alpha_n x$$

$$\alpha_n = \frac{2n\pi}{L}$$

(1):

$$N_Y = \frac{\partial^2 F}{\partial x^2} = -\sum_{n=1}^{\infty} (A_n + \alpha_n Y B_n) e^{-\alpha_n Y} \sin\alpha_n x = \sum_{n=1}^{\infty} \frac{P}{a} \sin\frac{n\pi}{2} \sin\alpha_n x$$

$$\sum_{n=1}^{\infty} -(A_n + \alpha_n \cancel{Y} B_n) \cancel{e^{-\alpha_n Y}}^{\,1} = \sum_{n=1}^{\infty} \frac{P}{a} \sin\frac{n\pi}{a}$$

$$A_n = -\frac{P}{a} \sin\frac{n\pi}{a}$$

(2):

$$N_{XY} = -\frac{\partial^2 F}{\partial x \partial Y} = -\sum_{n=1}^{\infty} \left[(A_n + \alpha_n Y B_n) \cdot e^{-\alpha_n Y} \cdot (-\alpha_n) + \alpha_n B_n e^{-\alpha_n Y} \right] \cdot \cos\alpha_n x \cdot \frac{1}{\alpha_n}$$

$$N_{XY} = +\sum_{n=1}^{\infty} (A_n - B_n + \alpha_n Y B_n) e^{-\alpha_n Y} \cos\alpha_n x = 0$$

$$-\frac{P}{a} \sin\frac{n\pi}{a} + B_n (\cancel{\alpha_n Y}^{\,0} - 1) = 0$$

$$B_n = -\frac{P}{a} \sin\frac{n\pi}{a}$$

$$F = -\frac{P}{a} \sum_{n=1}^{\infty} \frac{1}{\alpha_n^2} \sin\frac{n\pi}{a} (1 + \alpha_n Y) e^{-\alpha_n Y} \sin\alpha_n x$$

$$N_y = \frac{\partial^2 F}{\partial x^2} = +\frac{p}{a}\sum_{n=1}^{\infty} \sin\frac{n\pi}{a}(1+\alpha_n y)e^{-\alpha_n y} \sin\alpha_n x$$

$$N_{xy} = -\frac{p}{a}\sum_{n=1}^{\infty}\frac{1}{\alpha_n^2}\sin\frac{n\pi}{a}\left[\alpha_n e^{-\alpha_n y} - \alpha_n(1+\alpha_n y)e^{-\alpha_n y}\right] \cdot \alpha_n \cos\alpha_n x$$

$$N_{xy} = -\frac{p}{a}\sum_{n=1}^{\infty}\sin\frac{n\pi}{a}(\cancel{1}-\cancel{1}-\alpha_n y)e^{-\alpha_n y} \cdot \cos\alpha_n x$$

$$N_{xy} = \frac{p}{a}\sum_{n=1}^{\infty}\sin\frac{n\pi}{a}\alpha_n y e^{-\alpha_n y}\cos\alpha_n x$$

б)

$$Ehu = \int\frac{\partial^2 F}{\partial y^2}dx - \nu\frac{\partial F}{\partial x} + \phi(y)$$

$$Ehv = \int\frac{\partial^2 F}{\partial x^2}dy - \nu\frac{\partial F}{\partial y} + \psi(x)$$

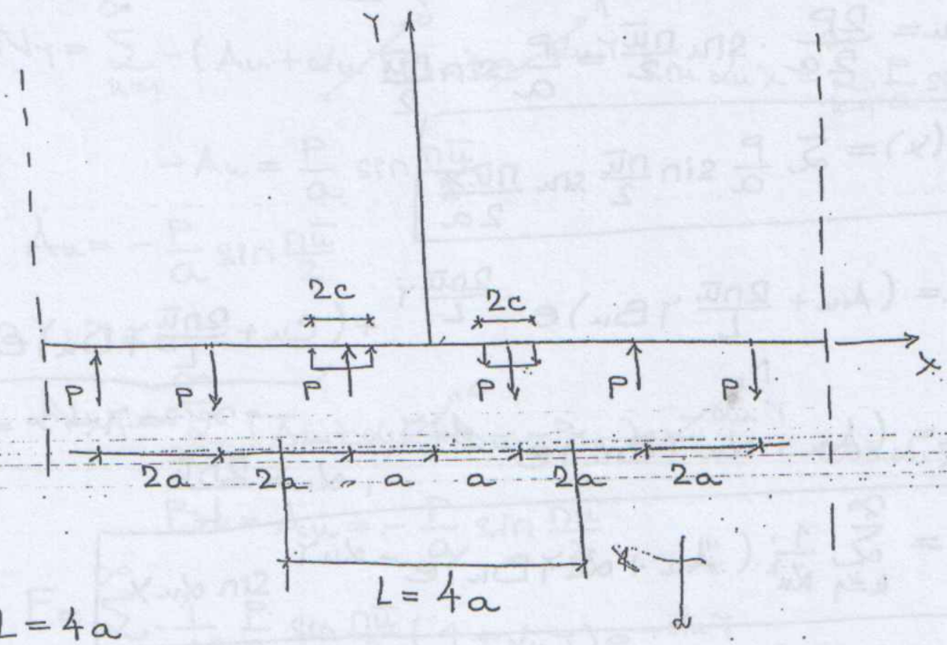
6 2

25.05.2002.

2. Za poluravan opterećenu periodičnim antimetričnim opterećenjem:

a) Odrediti izraze za sile u preseku

b) Odrediti izraze za pomeranja.



$L = 4a$

$p = \frac{P}{2c}$

Antimetrično opterećenje $\rightarrow$ razvija se u sinusni red

$$P(x) = \sum_{n=1}^{\infty} b_n \cdot \sin\frac{2n\pi x}{L}$$

$$b_n = \frac{4}{L}\int_0^{L/2} p(x)\cdot \sin\frac{2n\pi x}{L}$$

$$p(x) = \begin{cases} 0 & , 0 \le x \le a-c \wedge a+c \le x \le 2a \\ p = \frac{P}{2c} & , a-c \le x \le a+c \end{cases}$$

$$b_n = \frac{1}{a}\int_{a-c}^{a+c} p\cdot \sin\frac{2n\pi x}{L}dx = \frac{1}{a}\int_{a-c}^{a+c} p\cdot \sin\frac{n\pi x}{2a}dx$$

$$b_n = -\frac{1}{a}\cdot\frac{2a}{n\pi}p\cdot\cos\frac{n\pi x}{2a}\Big|_{a-c}^{a+c}$$

$n = 1$

$$b_n = \frac{2p}{n\pi} \cdot \left(\cos\frac{n\pi}{2a}(a-c) - \cos\frac{n\pi}{2a}(a+c) \right)$$

$$b_n = \frac{2p}{n\pi} \left(\cancel{\cos\frac{n\pi}{2}\cos\frac{n\pi c}{2a}} + \sin\frac{n\pi}{2}\sin\frac{n\pi c}{2a} - \cancel{\cos\frac{n\pi}{2}\cos\frac{n\pi c}{2a}} + \sin\frac{n\pi}{2}\sin\frac{n\pi c}{2a} \right)$$

$$b_n = \frac{4p}{n\pi} \cdot \sin\frac{n\pi}{2}\sin\frac{n\pi c}{2a}$$

$$\lim_{c\to 0} b_n = \lim_{c\to 0} \frac{4p}{n\pi} \cdot \sin\frac{n\pi}{2} \frac{\sin\frac{n\pi c}{2a}}{\cancel{\frac{n\pi c}{2a}}} \cdot \frac{n\pi c}{2a} \quad 2P \quad 1$$

$$b_n = \frac{2P}{2a} \cdot \sin\frac{n\pi}{2} = \frac{P}{a}\sin\frac{n\pi}{2}$$

$$\boxed{P(x) = \sum \frac{P}{a}\sin\frac{n\pi}{2}\sin\frac{n\pi x}{2a}}$$

$$Y_n = \left(A_n + \frac{2n\pi}{L} Y B_n\right)e^{-\frac{2n\pi}{L}Y} + \underbrace{\left(C_n + \frac{2n\pi}{L} Y D_n\right)e^{\frac{2n\pi}{L}Y}}_{\to\infty \;\; 3aY\to\infty \Rightarrow C_n = D_n = 1}$$

$$Y_n = (A_n + \alpha_n Y B_n)e^{-\alpha_n Y}, \quad \alpha_n = \frac{2n\pi}{L}$$

$$\boxed{F = \sum_{n=1}^{\infty} \frac{1}{\alpha_n^2}(A_n + \alpha_n Y B_n)e^{-\alpha_n Y} \cdot \sin\alpha_n x}$$

granični uslovi:

$$Y=0 \begin{cases} N_Y = P(x) & (1) \\ N_{XY} = 0 & (2) \end{cases}$$

$$N_X = \frac{\partial^2 F}{\partial Y^2}$$

$$N_Y = \frac{\partial^2 F}{\partial X^2}$$

$$N_{XY} = -\frac{\partial^2 F}{\partial X \partial Y}$$

$$\frac{\partial F}{\partial x} = \sum_{n=1}^{\infty} \frac{1}{\alpha_n}(A_n + \alpha_n Y B_n)e^{-\alpha_n Y} \cdot \cos\alpha_n x$$

$$\frac{\partial^2 F}{\partial x^2} = \sum_{n=1}^{\infty} -(A_n + \alpha_n Y B_n)e^{-\alpha_n Y} \cdot \sin\alpha_n x$$

$$\frac{\partial^2 F}{\partial x \partial y} = \sum_{n=1}^{\infty} -(A_n + \alpha_n y B_n - B)e^{-\alpha_n y} \cdot \cos\alpha_n x$$

$$\frac{\partial F}{\partial y} = \sum_{n=1}^{\infty} -\frac{1}{\alpha_n}(A_n + \alpha_n y B_n - B)e^{-\alpha_n y} \cdot \sin\alpha_n x$$

2,0933

$$\frac{\partial^2 F}{\partial y^2} = \sum_{n=1}^{\infty} (A_n + \alpha_n y B_n - 2B_n)e^{-\alpha_n y} \cdot \sin\alpha_n x$$

$\frac{7 \cdot 3,14 \cdot 3}{9}$

(1): $y = 0$

$$N_y = \sum_{n=1}^{\infty} -(A_n + \cancel{\alpha_n y}^{0} B_n)\cancel{e^{-\alpha_n y}}^{1} \cdot \sin\alpha_n x = \sum_{n=1}^{\infty} \frac{P}{a}\sin\frac{n\pi}{2}\sin\alpha_n x$$

$$-A_n = \frac{P}{a}\sin\frac{n\pi}{2}$$

$$A_n = -\frac{P}{a}\sin\frac{n\pi}{2}$$

(2): $y = 0$

$$N_{xy} = \sum_{n=1}^{\infty} (A_n + \cancel{\alpha_n y}^{0} B_n - B_n)\cancel{e^{-\alpha_n y}}^{1} \cdot \cos\alpha_n x = 0$$

$$B_n = A_n = -\frac{P}{a}\sin\frac{n\pi}{2}$$

$$F = \sum_{n=1}^{\infty} -\frac{1}{\alpha_n^2}\frac{P}{a}\sin\frac{n\pi}{2}(1 + \alpha_n y)e^{-\alpha_n y} \cdot \sin\alpha_n x$$

$$N_x = \sum_{n=1}^{\infty} +\frac{P}{a}\sin\frac{n\pi}{2}(1 - \alpha_n y)e^{-\alpha_n y} \cdot \sin\alpha_n x$$

$$N_y = \sum_{n=1}^{\infty} \frac{P}{a}\sin\frac{n\pi}{2}(1 + \alpha_n y)e^{-\alpha_n y} \cdot \sin\alpha_n x$$

$$N_{xy} = \sum_{n=1}^{\infty} \frac{P}{a}\sin\frac{n\pi}{2}\alpha_n y e^{-\alpha_n y} \cdot \cos\alpha_n x$$

b) $u = \int_0^x \frac{\partial u}{\partial x} dx + \int_0^y \frac{\partial u}{\partial y} dy$

$v = \int_0^x \frac{\partial v}{\partial x} dx + \int_0^y \frac{\partial v}{\partial y} dy$

$\frac{\partial u}{\partial x} = \varepsilon_x \quad \frac{\partial v}{\partial y} = \varepsilon_y$

$\varepsilon_x = \frac{1}{Eh}(N_x - \nu N_y)$

$\varepsilon_y = \frac{1}{Eh}(N_y - \nu N_x)$

$Ehu = \int (N_x - \nu N_y) dx + \phi(y)$

$Ehv = \int (N_y - \nu N_x) dy + \psi(x)$

$Ehu = \int \frac{\partial^2 F}{\partial y^2} dx - \nu \frac{\partial F}{\partial x} + \phi(y) \qquad (1)$

$Ehv = \int \frac{\partial^2 F}{\partial x^2} dy - \nu \frac{\partial F}{\partial y} + \psi(x) \qquad (2)$

Uslov koji povezuje f-je $\phi(y)$ i $\psi(x)$:

$\gamma_{xy} = \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)$

$Eh\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) = Eh\gamma_{xy} = 2(1+\nu)N_{xy} = -2(1+\nu)\frac{\partial^2 F}{\partial x \partial y}$

$Eh\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) = \int \frac{\partial^3 F}{\partial y^3} dx - \nu \frac{\partial^2 F}{\partial x \partial y} + \frac{d\phi(y)}{dy} + \int \frac{\partial^3 F}{\partial x^3} dy - \nu \frac{\partial^2 F}{\partial x \partial y} + \frac{d\psi(x)}{dx}$

$Eh\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) = \int \frac{\partial^3 F}{\partial y^3} dx + \int \frac{\partial^3 F}{\partial x^3} dy + \frac{d\phi(y)}{dy} + \frac{d\psi(x)}{dx} = (-2-2\nu+2\nu)\frac{\partial^2 F}{\partial x \partial y}$

$\int \frac{\partial^3 F}{\partial y^3} dx + \int \frac{\partial^3 F}{\partial x^3} dy + \frac{d\phi(y)}{dy} + \frac{d\psi(x)}{dx} = -2\frac{\partial^2 F}{\partial x \partial y} \qquad (3)$

$Ehu = \int \sum \frac{P}{a} \sin\frac{n\pi}{2}(1-\alpha_n y)e^{-\alpha_n y} \cdot \sin \alpha_n x \, dx +$

$+ \nu \sum \frac{1}{\alpha_n} \frac{P}{a} \sin\frac{n\pi}{2}(1+\alpha_n y)e^{-\alpha_n y} \cdot \cos \alpha_n x + \phi(y)$

$Ehu = \sum_{n=1}^{\infty} \frac{1}{\alpha_n} \frac{P}{a} \sin\frac{n\pi}{2}[\nu(1+\alpha_n y) - (1-\alpha_n)]e^{-\alpha_n y} \cos \alpha_n x + \phi(y)$

$$EhV = \int \sum \frac{p}{a} \sin\frac{n\pi}{2}(1+\alpha_n y)e^{-\alpha_n y} \sin\alpha_n x \, dy -$$

$$-\nu \sum \frac{1}{\alpha_n} \frac{p}{a} \sin\frac{n\pi}{2} e^{-\alpha_n y} \cdot \sin\alpha_n x + \Psi(x)$$

$$J = \int (1+\alpha_n y)e^{-\alpha_n y} dy = \int e^{-\alpha_n y} dy + \alpha_n \int y e^{-\alpha_n y} dy$$

$$J_1 = \int e^{-\alpha_n y} dy = -\frac{1}{\alpha_n} e^{-\alpha_n y}$$

$$-\alpha_n y = t \Rightarrow -\alpha_n dy = dt$$

$$J_2 = \alpha_n \int y e^{-\alpha_n y} dy$$

$$\int u \, dv = u \cdot v - \int v \, du$$

$$u = y \Rightarrow du = dy$$

$$e^{-\alpha_n y} dy = dv \Rightarrow -\frac{1}{\alpha_n} e^{-\alpha_n y} = v$$

$$J_2 = \alpha_n \left[-\frac{1}{\alpha_n} y \cdot e^{-\alpha_n y} + \int \frac{1}{\alpha_n} e^{-\alpha_n y} dy \right]$$

$$J_2 = \alpha_n \left[-\frac{1}{\alpha_n} y e^{-\alpha_n y} - \frac{1}{\alpha_n^2} e^{-\alpha_n y} \right]$$

$$J_2 = -e^{-\alpha_n y}\left(1 + \frac{1}{\alpha_n}\right)$$

$$J = -e^{-\alpha_n y}\left(\frac{1}{\alpha_n} + 1 + \frac{1}{\alpha_n}\right) = -e^{-\alpha_n y}\left(1 + \frac{2}{\alpha_n}\right)$$

$$EhV = \sum_{n=1}^{\infty} -\frac{p}{a} \sin\frac{n\pi}{2} e^{-\alpha_n y}\left(1 + \frac{2}{\alpha_n} + \frac{\nu}{\alpha_n}\right) \sin\alpha_n x + \Psi(x)$$

F-je $\Phi(y)$ i $\Psi(x)$ određujemo iz j-ne (3) uz zadovoljen graničnih uslova

NP

③ 2

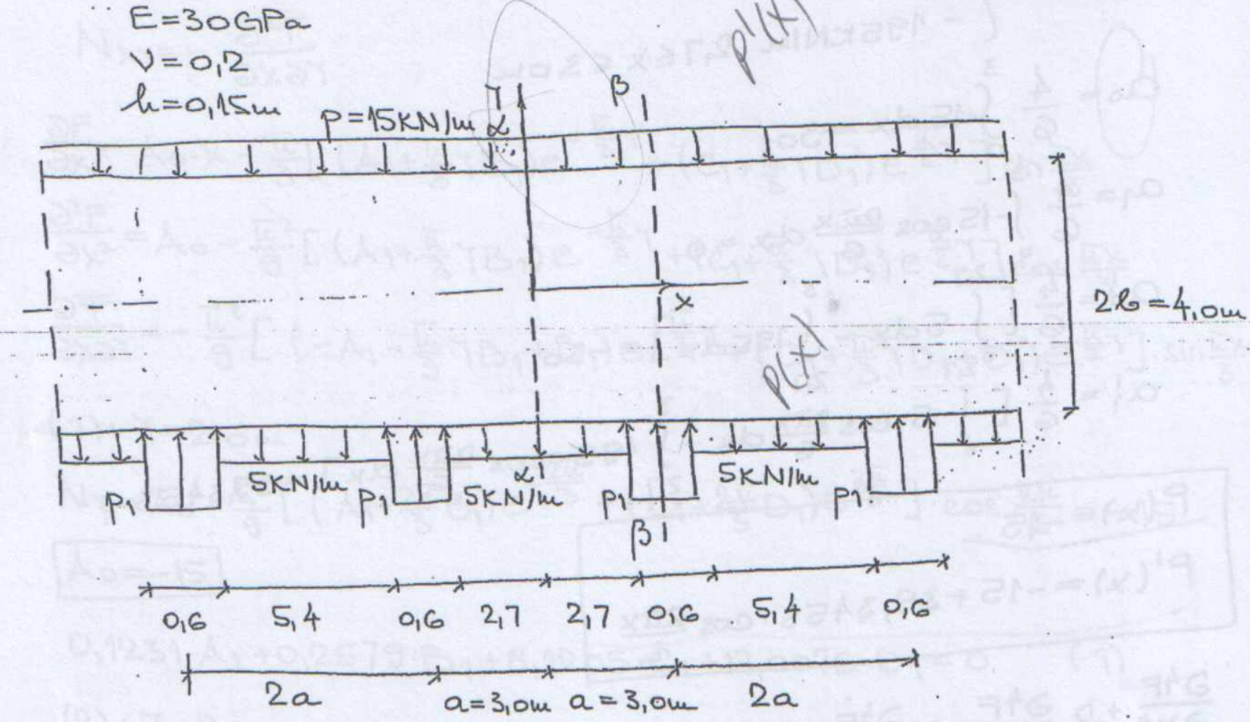
29.11.2003.

2. Za kontinualni zidni nosač opterećen periodičnim linijskim opterećenjem prema slici:

a) Sračunati i nacrtati dijagrame normalnih sila N_x u presecima $\alpha-\alpha$ i $\beta-\beta$. Odrediti ukupnu zatežuću silu u tim presecima. Koristiti prvi član reda.

b) Pretpostavljajući linearnu raspodelu normalnih napona po visini zida, odrediti ukupne zatežuće sile u presecima $\alpha-\alpha$ i $\beta-\beta$ i uporediti ih sa rešenjem dobijenim u tački a.

$E = 30\,GPa$
$\nu = 0{,}2$
$h = 0{,}15\,m$



$L = 2a = 6{,}0\,m$
$B = 4{,}0\,m$
$0{,}45 < \frac{B}{L} = 0{,}\dot{6} < 1{,}0$

$0{,}6 \cdot p_1 = 5 \cdot 5{,}4 + 6 \cdot 15 \Rightarrow p_1 = 195\,KN/m$

* Napomena: Opterećenje na gornjoj konturi nije u ravnoteži samo sa sobom pa se javlja i član $\frac{a_0}{2}$, što važi i za donju konturu!

$$\left.\begin{aligned} p(x) &= \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\frac{2n\pi x}{L} \\ p'(x) &= \frac{a_0'}{2} + \sum_{n=1}^{\infty} a_n' \cos\frac{2n\pi x}{L} \end{aligned}\right\}$$ Simetrično opterećenje $\Rightarrow$ razvijamo kao parnu f-ju.

$n=1$

$$P(x)=\frac{a_0}{2}+a_1\cos\frac{2\pi x}{L}$$

$$P'(x)=\frac{a_0'}{2}+a_1'\cos\frac{2\pi x}{L}$$

$$a_0=\frac{4}{L}\int_0^{L/2}p(x)\,dx$$

$$a_1=\frac{4}{L}\int_0^{L/2}p(x)\cos\frac{2\pi x}{L}\,dx$$

$$p(x)=-15\,\text{KN/m}\qquad 0\le x\le 3{,}0\text{m}$$

$$p'(x)=\begin{cases}5\,\text{KN/m} & 0\le x\le 2{,}7\text{m}\\ -195\,\text{KN/m} & 2{,}7\le x\le 3{,}0\text{m}\end{cases}$$

$$a_0=\frac{4}{6}\int_0^3 -15\,dx=-30$$

$$a_1=\frac{4}{6}\int_0^3 -15\cos\frac{2\pi x}{6}\,dx=0$$

$$a_0'=\frac{4}{6}\left[\int_0^{2,7}5\,dx-\int_{2,7}^{3}195\,dx\right]=-30$$

$$a_1'=\frac{4}{6}\left[\int_0^{2,7}5\cdot\cos\frac{2\pi x}{6}\,dx-\int_{2,7}^{3}195\cdot\cos\frac{2\pi x}{6}\,dx\right]=39{,}3453$$

$$\boxed{\begin{aligned}&P(x)=-15.\\&P'(x)=-15+39{,}3453\cdot\cos\frac{2\pi x}{L}\end{aligned}}$$

$$\frac{\partial^4F}{\partial x^4}+2\frac{\partial^4F}{\partial x^2\partial y^2}+\frac{\partial^4F}{\partial y^4}=0$$

$$F=\frac{A_0}{2}x^2+\sum_{n=1}^{\infty}Y_n\cdot\cos\frac{2n\pi x}{L}$$

$$Y_n=\left(A_n+\frac{2n\pi}{L}\cdot yB_n\right)e^{-\frac{2n\pi}{L}\cdot y}+\left(C_n+\frac{2n\pi}{L}yD_n\right)e^{\frac{2n\pi}{L}y}$$

$n=1$

$$Y_1=\left(A_1+\frac{\pi}{3}yB_1\right)e^{-\frac{\pi}{3}y}+\left(C_1+\frac{\pi}{3}yD_1\right)e^{\frac{\pi}{3}y}$$

$$F=\frac{A_0'}{2}x^2+\left[\left(A_1+\frac{\pi}{3}yB_1\right)e^{-\frac{\pi}{3}y}+\left(C_1+\frac{\pi}{3}yD_1\right)e^{\frac{\pi}{3}y}\right]\cdot\cos\frac{\pi x}{3}$$

granični uslovi:

$$Y=b=2{,}0 \begin{cases} N_Y = P(x) & (1) \\ N_{XY}=0 & (2) \end{cases}$$

$$Y=-b=-2{,}0 \begin{cases} N_Y = P'(x) & (3) \\ N_{XY}=0 & (4) \end{cases}$$

$$N_x = \frac{\partial^2 F}{\partial Y^2}$$

$$N_Y = \frac{\partial^2 F}{\partial x^2}$$

$$N_{XY} = -\frac{\partial^2 F}{\partial x \partial Y}$$

$$\frac{\partial F}{\partial x} = A_0 \cdot x - \frac{\pi}{3}\left[(A_1+\frac{\pi}{3}YB_1)e^{-\frac{\pi}{3}Y} + (C_1+\frac{\pi}{3}YD_1)e^{\frac{\pi}{3}Y}\right]\sin\frac{\pi x}{3}$$

$$\frac{\partial^2 F}{\partial x^2} = A_0 - \frac{\pi^2}{9}\left[(A_1+\frac{\pi}{3}YB_1)e^{-\frac{\pi}{3}Y} + (C_1+\frac{\pi}{3}YD_1)e^{\frac{\pi}{3}Y}\right]\cos\frac{\pi x}{3}$$

$$\frac{\partial^2 F}{\partial x \partial Y} = -\frac{\pi^2}{9}\left[(-A_1-\frac{\pi}{3}YB_1+B_1)e^{-\frac{\pi}{3}Y} + (C_1+\frac{\pi}{3}YD_1+D_1)e^{\frac{\pi}{3}Y}\right]\cdot\sin\frac{\pi x}{3}$$

(1): $Y=2{,}0\,m$

$$N_Y = A_0 - \frac{\pi^2}{9}\left[(A_1+\frac{2\pi}{3}B_1)e^{-\frac{2\pi}{3}} + (C_1+\frac{2\pi}{3}D_1)e^{\frac{2\pi}{3}}\right]\cdot\cos\frac{\pi x}{3} = -15$$

$\boxed{A_0=-15}$

$$0{,}1231\cdot A_1 + 0{,}2579\cdot B_1 + 8{,}1205\cdot C_1 + 17{,}0076\cdot D_1 = 0 \qquad (1)$$

(2): $Y=2{,}0$

$$N_{XY} = +\frac{\pi^2}{9}\left[(-A_1-\frac{2\pi}{3}B_1+B_1)e^{-\frac{2\pi}{3}} + (C_1+\frac{2\pi}{3}D_1+D_1)e^{\frac{2\pi}{3}}\right]\sin\frac{\pi x}{3}=0$$

$$-0{,}1231\cdot A_1 - 0{,}1348\cdot B_1 + 8{,}1205\cdot C_1 + 25{,}1281\cdot D_1 = 0 \qquad (2)$$

(3): $Y=-2{,}0$

$$N_Y = A_0 - \frac{\pi^2}{9}\left[(A_1-\frac{2\pi}{3}B_1)e^{+\frac{2\pi}{3}} + (C_1-\frac{2\pi}{3}D_1)e^{-\frac{2\pi}{3}}\right]\cos\frac{\pi x}{3} = -15+39{,}3453\cdot\cos\frac{\pi x}{3}$$

$$8{,}1205\cdot A_1 - 17{,}0076\cdot B_1 + 0{,}1231\cdot C_1 - 0{,}2579\cdot D_1 = -35{,}8786 \qquad (3)$$

(4): $Y=-2{,}0$

$$N_{XY} = +\frac{\pi^2}{9}\left[(-A_1+\frac{2\pi}{3}B_1+B_1)e^{\frac{2\pi}{3}} + (C_1-\frac{2\pi}{3}D_1+D_1)e^{-\frac{2\pi}{3}}\right]\cdot\sin\frac{\pi x}{3}=0$$

$$-8{,}1205\cdot A_1 + 25{,}1281\cdot B_1 + 0{,}1231\cdot C_1 - 0{,}1348\cdot D_1 = 0$$

$$A_0 = -15$$

$$A_1 = -13{,}8899 \qquad C_1 = 1{,}6917$$

$$B_1 = -4{,}5005 \qquad D_1 = -0{,}6389$$

$$F = -7{,}5 \cdot x^2 + \left[\left(-13{,}8899 + \frac{\pi}{3} \cdot 4{,}5005 \cdot y\right) e^{-\frac{\pi}{3} y} + \left(1{,}6917 - \frac{\pi}{3} \cdot 0{,}6389 \cdot y\right) e^{\frac{\pi}{3} y} \right] \cdot \cos \frac{\pi x}{3}$$

$$N_x = \frac{\partial^2 F}{\partial y^2}$$

$$N_x = -\left[(5{,}3614 + 5{,}1682 \cdot y) \cdot e^{-\frac{\pi}{3} y} + (-0{,}4588 + 0{,}7337 \cdot y) e^{\frac{\pi}{3} y} \right] \cdot \cos \frac{\pi x}{3}$$

1. Presek $\alpha - \alpha$

$x = 0$

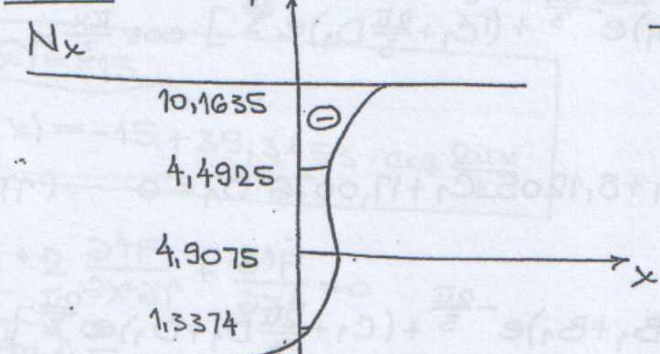
$$N_x = -\left[(5{,}3614 + 5{,}1682 \cdot y) e^{-\frac{\pi}{3} y} + (-0{,}4588 + 0{,}7337 \cdot y) e^{\frac{\pi}{3} y} \right] \cdot \cancel{\cos 0}^{\,1}$$

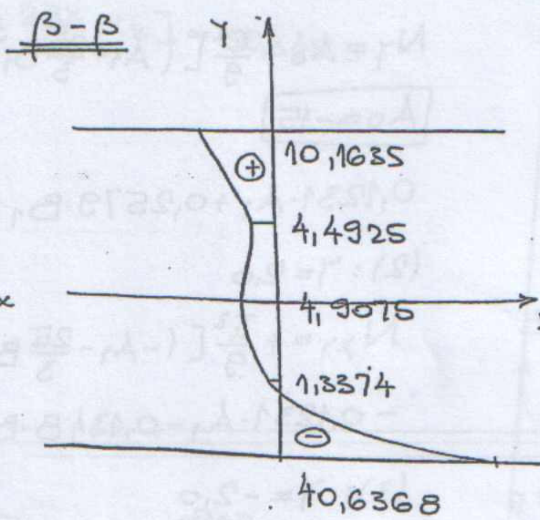
2. Presek $\beta - \beta$

$x = 3{,}0\,m$

$$N_x = (5{,}3614 + 5{,}1682 \cdot y) e^{-\frac{\pi}{3} y} + (-0{,}4588 + 0{,}7337 y) e^{\frac{\pi}{3} y}$$

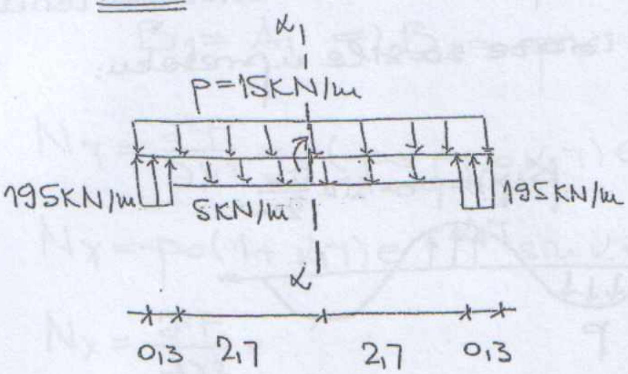
$\alpha - \alpha$





b)

α-α

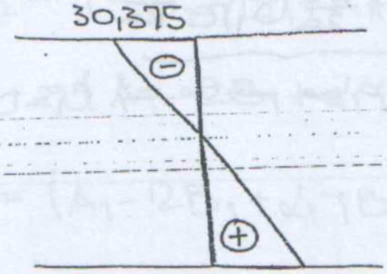


$$M_{\alpha-\alpha} = 195\cdot 0{,}3(2{,}7+0{,}15) - 15\cdot 3\cdot 1{,}5 - 5\cdot 2{,}7\cdot 1{,}35 = 81\,kNm$$

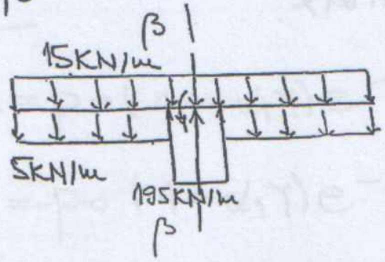
$$\sigma_x = \frac{M_{\alpha-\alpha}}{W} = \frac{81}{\frac{1}{6}\cdot h\cdot 4^2} = 30{,}375\,\frac{1}{h}$$

$$N_x = \sigma_x \cdot h = 30{,}375$$

$N_{x,\alpha-\alpha}$



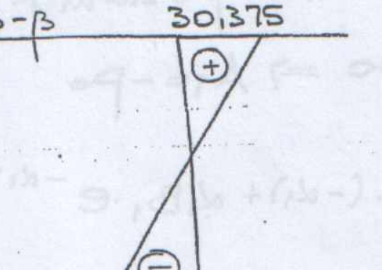
β-β



$$M_{\beta-\beta} = 195\cdot 0{,}3\cdot 0{,}15 - 15\cdot 3\cdot 1{,}5 - 5\cdot 2{,}7\cdot(0{,}3+1{,}35) = -81\,kNm$$

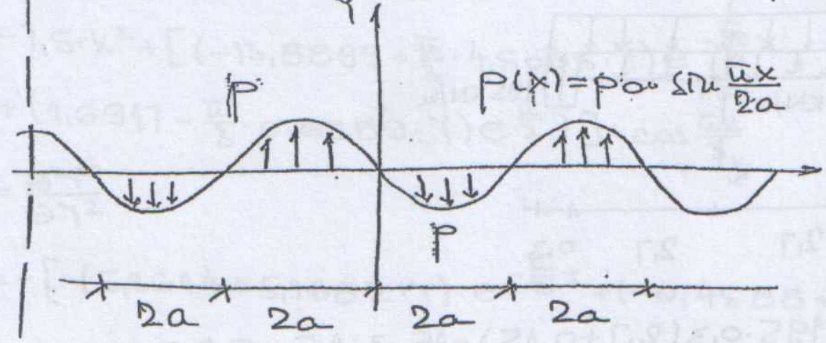
$$N_x = \sigma_x \cdot h = -30{,}375$$

$N_{x,\beta-\beta}$



④

2. Za poluravan opterećenu periodičnom antimetričnom opterećenjem odrediti izraze za sile u presecima:



$L = 4a$

$p(x) = p_0 \cdot \sin \frac{\pi x}{2a}$

$$Y_1 = \left(A_1 + \frac{\pi y}{2a} B_1\right) e^{-\frac{\pi}{2a} y} + \underbrace{\left(C_1 + \frac{\pi y}{2a} D_1\right) e^{\frac{\pi y}{2a}}}$$

$y \to \infty \quad \to \infty \Rightarrow C_1 = D_1 = 0$

$$Y_1 = \left(A_1 + \frac{\pi y}{2a} B_1\right) e^{-\frac{\pi y}{2a}}$$

$\alpha_1 = \frac{\pi}{2a}$

$$F = \frac{1}{\alpha_1^2} (A_1 + \alpha_1 y B_1) e^{-\alpha_1 y} \sin \alpha_1 x$$

granični uslovi:

$$y = 0 \begin{cases} N_y = p(x) \\ N_{xy} = 0 \end{cases}$$

$$\frac{\partial^2 F}{\partial x^2} = -(A_1 + \alpha_1 y B_1) e^{-\alpha_1 y} \sin \alpha_1 x = p_0 \sin \alpha_1 x$$

$$-(A_1 + \cancel{\alpha_1 y}^{\,0} B_1) \cancel{e^{-\alpha_1 y}}^{\,1} = p_0 \Rightarrow A_1 = -p_0$$

$$\frac{\partial^2 F}{\partial x \partial y} = +\frac{1}{\alpha_1} \left[(A_1 + \alpha_1 y B_1) \cdot e^{-\alpha_1 y} \cdot (-\alpha_1) + \alpha_1 B_1 \cdot e^{-\alpha_1 y} \right] \cdot \cos \alpha_1 x = 0$$

$$(A_1+\alpha_1 \overset{0}{\cancel{Y}} B_1)e^{-\alpha_1 Y} - B_1 e^{-\alpha_1 Y} = 0$$

$$B_1 = A_1 \Rightarrow B_1 = -p_0$$

$$N_Y = \frac{\partial^2 F}{\partial X^2} = +(-p_0 - p_0 \alpha_1 Y)e^{-\alpha_1 Y} \sin \alpha_1 X$$

$$N_Y = -p_0(1+\alpha_1 Y)e^{-\alpha_1 Y} \sin \alpha_1 X$$

$$N_X = \frac{\partial^2 F}{\partial Y^2}$$

$$\frac{\partial F}{\partial Y} = \frac{1}{\alpha_1^2}\left[\alpha_1 B_1 \cdot e^{-\alpha_1 Y} - \alpha_1(A_1+\alpha_1 Y B_1)e^{-\alpha_1 Y}\right]\cdot \sin \alpha_1 X =$$

$$= \frac{1}{\alpha_1}\cdot(B_1 - A_1 - \alpha_1 Y B_1)e^{-\alpha_1 Y} \cdot \sin \alpha_1 X$$

$$\frac{\partial^2 F}{\partial Y^2} = (-B_1 - B_1 + A_1 + \alpha_1 Y B_1)e^{-\alpha_1 Y} \sin \alpha_1 X$$

$$= (A_1 - 2B_1 + \alpha_1 Y B_1)e^{-\alpha_1 Y} \sin \alpha_1 X$$

$$N_X = (A_1 - 2B_1 + \alpha_1 Y B_1)e^{-\alpha_1 Y} \sin \alpha_1 X$$

$$N_{XY} = -(A_1 - B_1 + \alpha_1 Y B_1)e^{-\alpha_1 Y} \cos \alpha_1 X$$

$$N_X = p_0(1-\alpha_1 Y)e^{-\alpha_1 Y} \sin \alpha_1 X$$

$$N_Y = -p_0(1+\alpha_1 Y)e^{-\alpha_1 Y} \sin \alpha_1 X$$

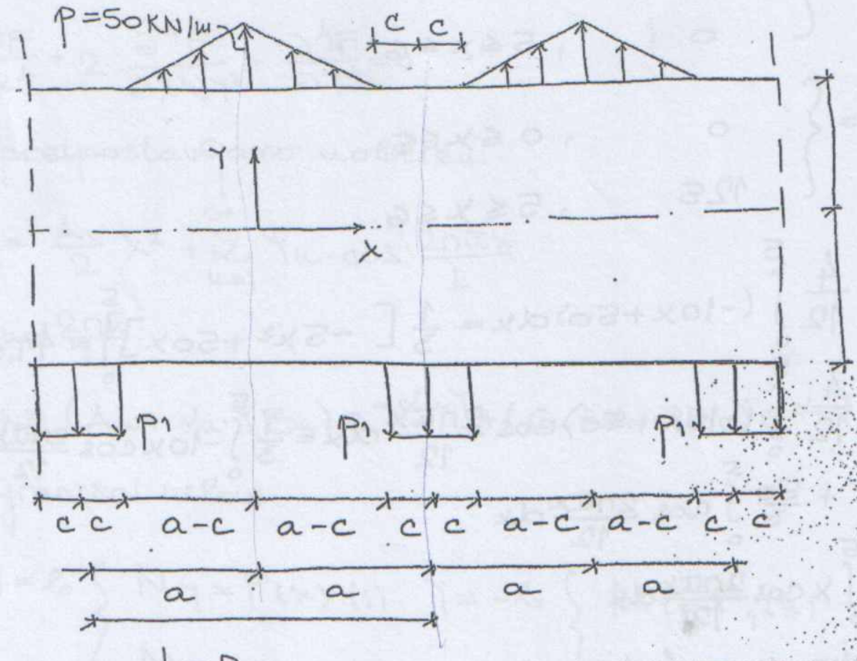
$$N_{XY} = p_0 \cdot \alpha_1 Y B_1 e^{-\alpha_1 Y} \cos \alpha_1 X$$

NP

(2) 2

01.09.1997.

2. Odrediti presečne sile koristeći prvi član.



$a=6{,}0m$

$b=3{,}0m$

$c=1{,}0m$

$L=2a=12{,}0m$

$B=2b=6{,}0m$

$0{,}45<\frac{B}{L}=\frac{6}{12}=0{,}5<1{,}0 \Rightarrow$ Zidni nosač

$2\cdot c\cdot p_1=2\cdot\frac{1}{2}\cdot p(a-c)=50(6-1)=250$

$p_1=125\,KN/m$

* Razvijamo kao parnu f-ju:

$$P(x)=\frac{a_0}{2}+\sum a_n\cdot\cos\frac{2n\pi x}{L}$$

$$P_1(x)=\frac{a_0'}{2}+\sum a_n'\cdot\cos\frac{2n\pi x}{L}$$

$$a_0=\frac{4}{L}\int_0^{L/2}p(x)dx$$

$$a_n=\frac{4}{L}\int_0^{L/2}p(x)\cos\frac{2n\pi x}{L}dx$$

$Y = ax + b$

$x = 0,\ Y = 50 \Rightarrow 50 = a\cdot 0 + b \Rightarrow b = 50$

$x = 5,\ Y = 0 \Rightarrow 0 = a\cdot 5 + 50 \Rightarrow a = -10$

$Y = -10x + 50$

$$p(x) = \begin{cases} -10x + 50 & ,\ 0 \le x \le 5 \\ 0 & ,\ 5 \le x \le 6 \end{cases}$$

$$p'(x) = \begin{cases} 0 & ,\ 0 \le x \le 5 \\ 125 & ,\ 5 \le x \le 6 \end{cases}$$

$$a_0 = \frac{4}{12}\int_0^5 (-10x + 50)\,dx = \frac{1}{3}\left[-5x^2 + 50x\right]\Big|_0^5 = 41,\dot{6}$$

$$a_n = \frac{4}{12}\int_0^5 (-10x+50)\cos\frac{2n\pi x}{12}\,dx = \frac{1}{3}\int_0^5 -10x\cos\frac{2n\pi x}{12}\,dx +$$

$$+ \frac{50}{3}\int_0^5 \cos\frac{2n\pi x}{12}\,dx$$

$$J_1 = \int_0^5 x\cos\frac{2n\pi x}{12}\,dx$$

$$\int u\,dv = u\cdot v - \int v\,du$$

$u = x \Rightarrow du = dx$

$$\cos\frac{2n\pi x}{12}\,dx = dv \Rightarrow \frac{12}{2n\pi}\sin\frac{2n\pi x}{12} = v \Rightarrow \frac{6}{n\pi}\sin\frac{n\pi x}{6} = v$$

$$J_1 = \frac{6}{n\pi}\cdot x\sin\frac{n\pi x}{6}\Big|_0^5 - \int_0^5 \frac{6}{n\pi}\sin\frac{n\pi x}{6}\,dx = \frac{30}{n\pi}\cdot\sin\frac{5n\pi}{6} +$$

$$+ \frac{36}{n^2\pi^2}\cos\frac{n\pi x}{6}\Big|_0^5$$

$$J_1 = \frac{30}{n\pi}\sin\frac{5n\pi}{6} + \frac{36}{n^2\pi^2}\left(\cos\frac{5n\pi}{6} - 1\right)$$

$$a_n = -\frac{10}{3}\left[\cancel{\frac{30}{n\pi}\sin\frac{5n\pi}{6}} + \frac{36}{n^2\pi^2}\left(\cos\frac{5n\pi}{6} - 1\right)\right] + \cancel{\frac{300}{3}}\sin\frac{5n\pi}{6}$$

$$a_n = -\frac{120}{n^2\pi^2}\left(\cos\frac{5n\pi}{6} - 1\right)$$

$$a_0' = \frac{4}{L}\int_0^{L/2} p'(x)\,dx = \frac{4}{12}\int_5^6 125\,dx = 41,\dot{6}$$

$$a_n' = \frac{4}{L}\int_0^{L/2} p'(x)\cos\frac{2n\pi x}{L}\,dx = \frac{500}{\cancel{12}}\cdot\frac{6}{n\pi}\sin\frac{n\pi x}{6}\Big|_5^6$$

$$a_n' = \frac{250}{n\pi}\left(\cancelto{0}{\sin n\pi} - \sin\frac{5n\pi}{6}\right)$$

$$a_n' = -\frac{250}{n\pi}\cdot\sin\frac{5n\pi}{6}$$

$$P(x) = 41{,}\dot{6} - \sum_{n=1}^{\infty} \frac{120}{n^2\pi^2}\left(\cos\frac{5n\pi}{6} - 1\right)\cdot \cos\frac{2n\pi x}{12}$$

$$P_1(x) = 41{,}\dot{6} - \sum_{n=1}^{\infty} \frac{250}{n\pi} \sin\frac{5n\pi}{6}\cdot \cos\frac{2n\pi x}{12}$$

$$\frac{\partial^4 F}{\partial x^4} + 2\,\frac{\partial^4 F}{\partial x^2 \partial y^2} + \frac{\partial^4 F}{\partial y^4} = 0$$

F pretpostavljamo u obliku:

$$F = \frac{A_0}{2}x^2 + \sum_{n=1}^{\infty} Y_n \cdot \cos\frac{2n\pi x}{L}$$

$$\alpha_n = \frac{2n\pi}{L}$$

$$Y_n = (A_n + \alpha_n \cdot y B_n)e^{-\alpha_n y} + (C_n + \alpha_n \cdot y D_n)e^{\alpha_n y}$$

granični uslovi:

$$y = b \begin{cases} N_y = P(x) & (1) \\ N_{yx} = 0 & (2) \end{cases} \qquad y = -b \begin{cases} N_y = P_1(x) & (3) \\ N_{yx} = 0 & (4) \end{cases}$$

$$N_x = \frac{\partial^2 F}{\partial y^2}$$

$$N_y = \frac{\partial^2 F}{\partial x^2}$$

$$N_{xy} = -\frac{\partial^2 F}{\partial x \partial y}$$

$$F = \frac{A_0}{2}x^2 + \sum_{n=1}^{\infty}\left[(A_n + \alpha_n y B_n)e^{-\alpha_n y} + (C_n + \alpha_n y D_n)e^{\alpha_n y}\right]\cos\alpha_n x$$

$$\frac{\partial F}{\partial x} = A_0 x - \sum_{n=1}^{\infty} \alpha_n\left[(A_n + \alpha_n y B_n)e^{-\alpha_n y} + (C_n + \alpha_n y D_n)e^{+\alpha_n y}\right]\sin\alpha_n x$$

$$\frac{\partial^2 F}{\partial x^2} = A_0 - \sum_{n=1}^{\infty} \alpha_n^2\left[(A_n + \alpha_n y B_n)e^{-\alpha_n y} + (C_n + \alpha_n y D_n)e^{+\alpha_n y}\right]\cos\alpha_n x$$

$$\frac{\partial^2 F}{\partial x \partial y} = -\sum_{n=1}^{\infty} \alpha_n^2\left[(-A_n - \alpha_n y B_n + \alpha_n B_n)e^{-\alpha_n y} + (C_n + \alpha_n y D_n + \alpha_n D_n)e^{\alpha_n y}\right]\cdot \sin\alpha_n x$$

(1):
$$A_0 - \sum_{n=1}^{\infty} \alpha_n^2\left[(A_n + \alpha_n y B_n)e^{-\alpha_n y} + (C_n + \alpha_n y D_n)e^{\alpha_n y}\right]\cos\alpha_n x =$$
$$= 41{,}6 - \sum_{n=1}^{\infty} \frac{120}{n^2\pi^2}\left(\cos\frac{5n\pi}{6} - 1\right)\cos\frac{2n\pi x}{12}$$

(2):
(3): $\Rightarrow A_n, B_n, C_n, D_n \Rightarrow N_x, N_y, N_{xy}$
(4):

(17)

15.07.2002.

2. Za zidno platno opterećeno sopstvenom težinom primenom diferencnog postupka odrediti:

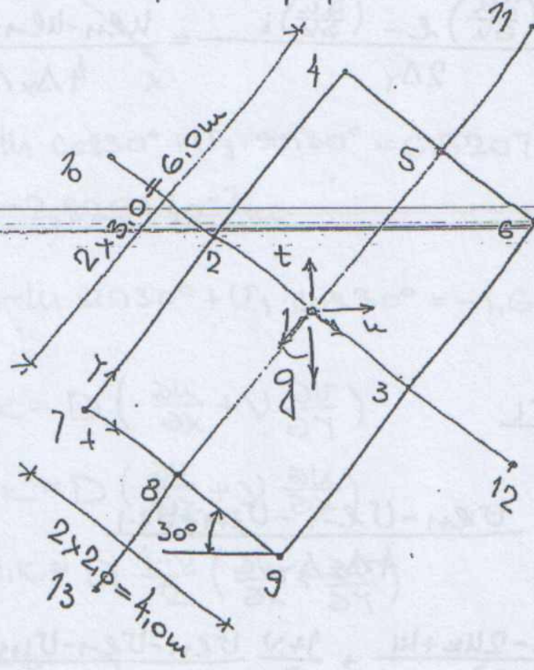
a) komponentalna pomeranja u označenoj tački koristeći j-ne metode deformacije:

$$\frac{\partial^2 u}{\partial x^2} + \frac{1-\nu}{2}\frac{\partial^2 u}{\partial y^2} + \frac{1+\nu}{2}\frac{\partial^2 v}{\partial x \partial y} + \frac{X}{D} = 0$$

$$\frac{1+\nu}{2}\frac{\partial^2 u}{\partial x \partial y} + \frac{1-\nu}{2}\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{Y}{D} = 0 \qquad D = \frac{Eh}{1-\nu^2}$$

b) presečne sile u označenoj tački u pravcu koordinatnih osa nit.

Zidno platno je po konturi slobodno oslonjeno.



$E = 30\,GPa$

$\nu = 0{,}2$

$\gamma = 24\,kN/m^3$

$h = 8\,cm$

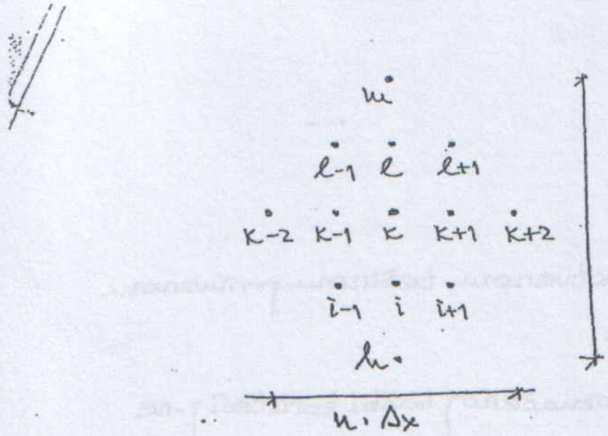
$g = 24 \cdot 0{,}08 = 1{,}92\,kN/m^2$

$g_x = X = g \cdot \sin 30° = 0{,}96\,kN/m^2$

$g_y = Y = g \cdot \cos 30° = 1{,}6628\,kN/m^2$

$\Delta x = 2{,}0m$

$\Delta y = 3{,}0m$



$$\left(\frac{\partial u}{\partial x}\right)_k = \frac{u_{k+1}-u_{k-1}}{2\Delta x}$$

$$\left(\frac{\partial^2 u}{\partial x^2}\right)_k = \frac{\partial}{\partial x}\left(\frac{\partial u}{\partial x}\right)_k = \frac{\frac{u_{k+1}-u_k}{\Delta x}-\frac{u_k-u_{k-1}}{\Delta x}}{\Delta x} = \frac{u_{k+1}-2u_k+u_{k-1}}{\Delta x^2}$$

$$\left(\frac{\partial u}{\partial y}\right)_k = \frac{u_l-u_i}{2\Delta y}$$

$$\left(\frac{\partial^2 u}{\partial y^2}\right)_k = \frac{\frac{u_l-u_k}{\Delta y}-\frac{u_k-u_i}{\Delta y}}{\Delta y} = \frac{u_l-2u_k+u_i}{\Delta y^2}$$

$$\left(\frac{\partial^2 u}{\partial x \partial y}\right)_k = \frac{\partial}{\partial y}\left(\frac{\partial u}{\partial x}\right)_k = \frac{\left(\frac{\partial u}{\partial x}\right)_l-\left(\frac{\partial u}{\partial x}\right)_i}{2\Delta y} = \frac{u_{l+1}-u_{l-1}-u_{i+1}+u_{i-1}}{4\Delta x\Delta y}$$

$$\left(\frac{\partial v}{\partial x}\right)_k = \frac{v_{k+1}-v_{k-1}}{2\Delta x}$$

$$\left(\frac{\partial^2 v}{\partial x^2}\right)_k = \frac{v_{k+1}-2v_k+v_{k-1}}{\Delta x^2}$$

$$\left(\frac{\partial v}{\partial y}\right)_k = \frac{v_l-v_i}{2\Delta y}$$

$$\left(\frac{\partial^2 v}{\partial y^2}\right)_k = \frac{v_l-2v_k+v_i}{\Delta y^2}$$

$$\left(\frac{\partial v}{\partial x \partial y}\right)_k = \frac{\partial}{\partial y}\left(\frac{\partial v}{\partial x}\right)_k = \frac{v_{l+1}-v_{l-1}-v_{i+1}+v_{i-1}}{4\Delta x\Delta y}$$

$$\frac{u_{k+1}-2u_k+u_{k-1}}{\Delta x^2} + \frac{1-\nu}{2}\,\frac{u_l-2u_k+u_i}{\Delta y^2} + \frac{1+\nu}{2}\,\frac{v_{l+1}-v_{l-1}-v_{i+1}+v_{i-1}}{4\Delta x\Delta y} + \frac{X}{D} = 0$$

$$\frac{1+\nu}{2}\,\frac{u_{l+1}-u_{l-1}-u_{i+1}+u_{i-1}}{4\Delta x\Delta y} + \frac{1-\nu}{2}\,\frac{v_{k+1}-2v_k+v_{k-1}}{\Delta x^2} + \frac{v_l-2v_k+v_i}{\Delta y^2} + \frac{Y}{D} = 0$$

<u>K=1</u>

$$\frac{\cancelto{0}{u_3}-2u_1+\cancelto{0}{u_2}}{4} + \frac{0{,}8}{2}\,\frac{\cancelto{0}{u_5}-2u_1+\cancelto{0}{u_8}}{9} + \frac{1{,}2}{2}\,\frac{\cancelto{0}{v_6}-\cancelto{0}{v_4}-\cancelto{0}{v_9}+\cancelto{0}{v_7}}{4\cdot 2\cdot 3} + \frac{0{,}96}{D} = 0$$

$$\frac{1{,}2}{2}\,\frac{\cancelto{0}{u_6}-\cancelto{0}{u_4}-\cancelto{0}{u_9}+\cancelto{0}{u_7}}{4\cdot 2\cdot 3} + \frac{0{,}8}{2}\,\frac{\cancelto{0}{v_3}-2v_1+\cancelto{0}{v_2}}{4} + \frac{\cancelto{0}{v_5}-2v_1+\cancelto{0}{v_8}}{9} - \frac{1{,}6628}{D} = 0$$

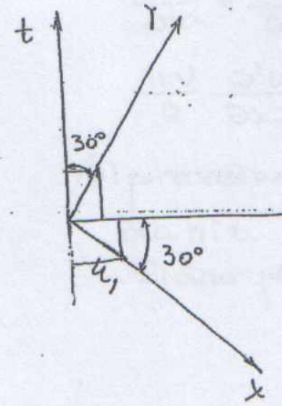
$$-0{,}5u_1 - \frac{0{,}8}{9}u_1 + \frac{0{,}96}{D} = 0 \qquad (1)$$

$$-0{,}2\cdot v_1-\frac{2}{9}v_1-\frac{1{,}6628}{D}=0 \quad (2)$$

$$D=\frac{Eh}{1-\nu^2}=\frac{30\cdot10^6\cdot0{,}08}{1-0{,}2^2}=2{,}5\cdot10^6$$

$$u_1=6{,}5207\cdot10^{-7}\ m$$

$$v_1=-15{,}7525\cdot10^{-7}\ m$$



$$u_n=u_1\cdot\cos30^\circ+v_1\cdot\sin30^\circ=6{,}5207\cdot10^{-7}\cdot\cos30^\circ-15{,}7525\cdot10^{-7}\sin30^\circ$$

$$u_n=-2{,}2291\cdot10^{-7}\ m$$

$$v_t=-u_1\cdot\sin30^\circ+v_1\cdot\cos30^\circ=-1{,}6902\cdot10^{-6}\ m$$

$$N_{x,k}=D\left(\frac{\partial u}{\partial x}+\nu\frac{\partial v}{\partial y}\right)$$

$$N_{y,k}=D\left(\frac{\partial v}{\partial x}+\nu\frac{\partial u}{\partial x}\right)$$

$$N_{xy,k}=D\,\frac{1-\nu}{2}\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)$$

$$\left.\begin{aligned}N_x&=0\\N_y&=0\\N_{xy}&=0\end{aligned}\right\}\text{gruba mreža}$$