Тестови

Ⓘ 18. 2. 2011.

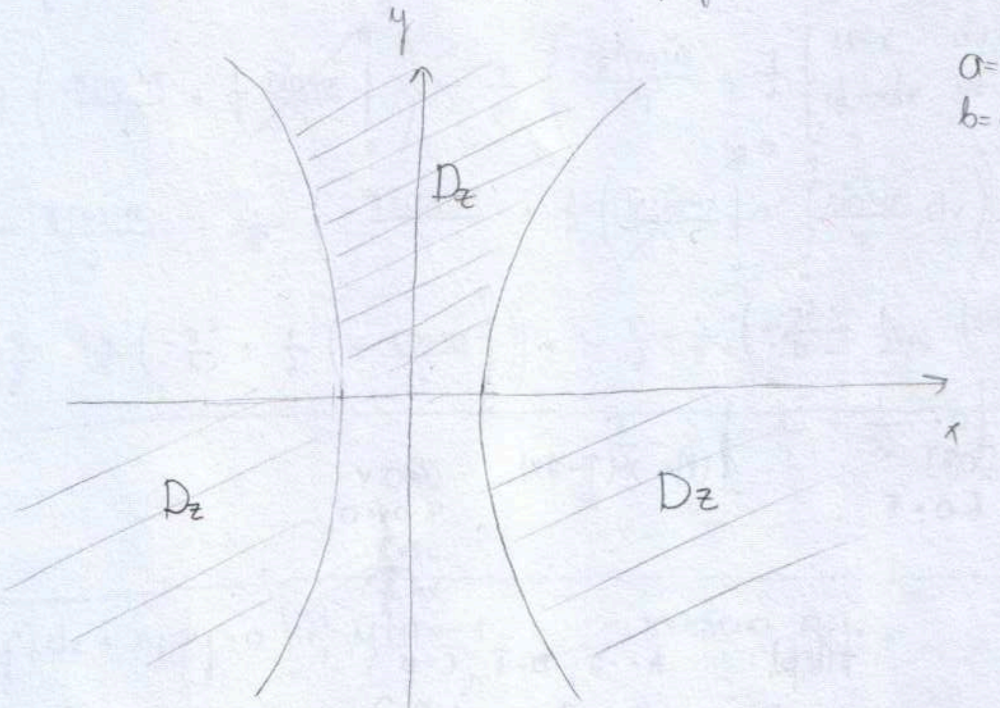
① $z(x,y)=\sqrt{y(1-x^2+y^2)}$

$D_z=\{(x,y)\in\mathbb{R}^2 \mid y(1-x^2+y^2)\geq 0\}$

$(y\geq 0 \wedge 1-x^2+y^2\geq 0) \vee (y<0 \wedge 1-x^2+y^2<0)$

$x^2-y^2\leq 1$ — хипербола — унија — $x^2-y^2>1$ — хипербола

$a=1$
$b=1$



② $z=\varphi\left(\frac{xz}{y}\right)$, $\varphi(u)$ – је диференцијабилна ф-ја $\wedge$ $z=z(x,y)$

$\frac{\partial z}{\partial x}+\frac{\partial z}{\partial y}=z_x+z_y=?$

Смена: $\frac{xz}{y}=u$

$u_x=\frac{(z+xz_x)y-\cancel{xz\cdot 0}}{y^2}$

$u_y=\frac{xz_y y-xz}{y^2}$

рад: $z=\varphi\left(\frac{xz}{y}\right)\Big|'_x\Big|'_y$

$z_x=\varphi'(u)\cdot u_x=\varphi'_x(u)\cdot\frac{z+xz_x}{y}$

$z_y=\varphi'(u)\cdot u_y=\varphi'(u)\cdot\frac{xz_y y-xz}{y^2}$

$$\boxed{z_x+z_y=\varphi'_x(u)\left[\frac{z+xz_x}{y}+\frac{xz_y y-xz}{y^2}\right]}$$

③ $f(x) = \frac{x}{\sqrt{1-x^2}}$

④ $f(x) = \pi x - 2x^2$, $[0,\pi]$

$b-a=\pi$

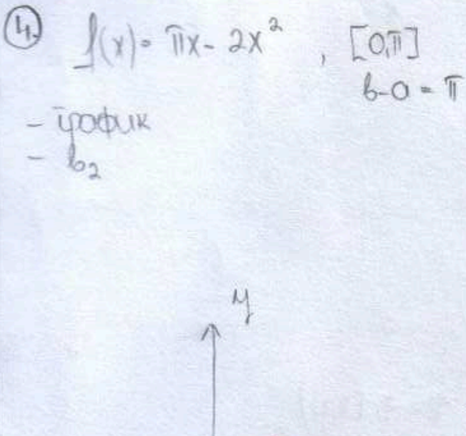
- график
- b_2

$f(x) = x(\pi - 2x)$

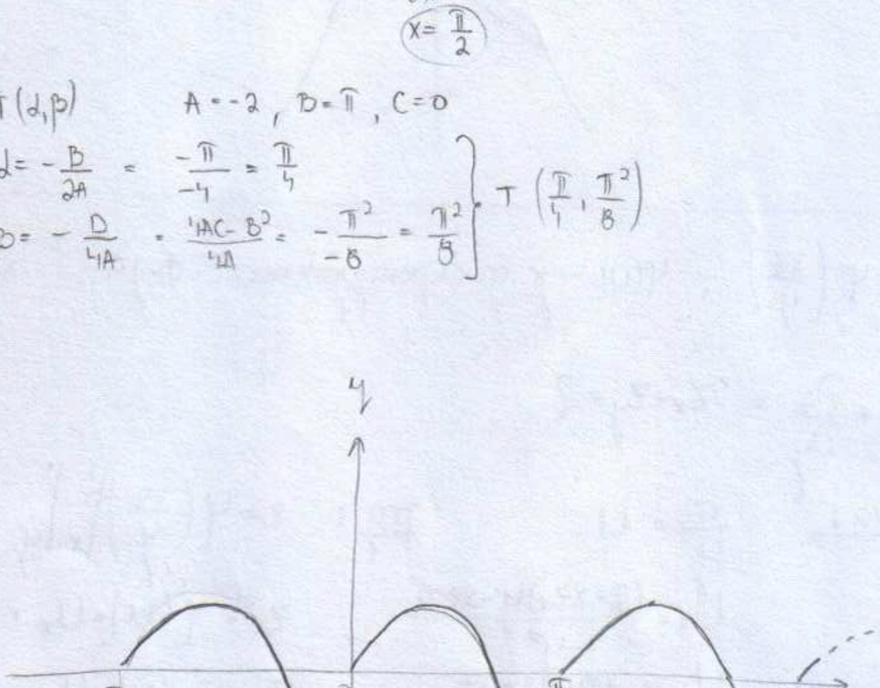
$x=0 \lor$
$\pi - 2x = 0$
$2x = \pi$
$x = \frac{\pi}{2}$

$T(\alpha,\beta)$ $A=-2$, $B=\pi$, $C=0$

$$\left.\begin{aligned} \alpha &= -\frac{B}{2A} = \frac{-\pi}{-4} = \frac{\pi}{4} \\ \beta &= -\frac{D}{4A} = \frac{4AC-B^2}{4A} = -\frac{\pi^2}{-8} = \frac{\pi^2}{8} \end{aligned}\right\} T\left(\frac{\pi}{4}, \frac{\pi^2}{8}\right)$$



$f(x)$



$F(x)$

$$b_n = \frac{2}{b-a}\int_0^b f(x)\sin\frac{2n\pi x}{b-a}\,dx$$

$$b_2 = \frac{2}{\pi}\int_0^\pi (\pi x - 2x^2)\sin\frac{2\cdot 2\pi x}{\pi}\,dx = \frac{2}{\pi}\int_0^\pi(\pi x-2x^2)\sin 4x\,dx =$$

$$= \frac{2}{\pi}\int_0^\pi \pi x\sin 4x\,dx - \frac{2}{\pi}\int_0^\pi 2x^2\sin 4x\,dx = 2\int_0^\pi x\sin 4x\,dx - \frac{4}{\pi}\int_0^\pi x^2 \sin 4x\,dx =$$

$$= \begin{bmatrix} u=x & dv=\sin 4x\,dx \\ du=dx & v=\int\sin 4x\,dx = -\frac{\cos 4x}{4}\end{bmatrix}\begin{bmatrix} u=x^2 & dv=\sin 4x\,dx\\ du=2x\,dx & v=\int \sin 4x\,dx = -\frac{\cos 4x}{4}\end{bmatrix} =$$

$$= 2\left[-\frac{x\cos 4x}{4}\Big|_0^\pi + \frac14\int_0^\pi \cos 4x\,dx\right] - \frac4\pi\left[-\frac{x^2\cos 4x}{4}\Big|_0^\pi + \frac24\int_0^\pi x\cos 4x\,dx\right] =$$

$$= 2\left(-\frac{\pi\cos 4\pi}{4} + \frac14 \cancel{\frac{\sin 4x}{4}\Big|_0^\pi}^{\,0}\right) - \frac4\pi\left(-\frac{\pi^2\cos 4\pi}{4} + \frac12\begin{bmatrix}u=x & dv=\cos 4x\,dx\\ du=dx & v=\frac{\sin 4x}{4}\end{bmatrix}\right)=$$

$$= -\frac{\pi\cos 4\pi}{2} - \frac4\pi\left(-\frac{\pi^2\cos 4\pi}{4} + \frac12\left(\cancel{\frac{x\sin 4x}{4}\Big|_0^\pi}^{\,0} - \int_0^\pi \frac{\sin 4x}{4}\,dx\right)\right) =$$

$$= -\frac\pi2 - \frac4\pi\left(-\frac{\pi^2}{4} + \frac12\left(+\frac{\cos 4x}{16}\Big|_0^\pi\right)\right) = -\frac\pi2 - \frac4\pi\left(-\frac{\pi^2}{4}+\frac{1}{2\cdot 16}\left(\cancel{\cos 4\pi - \cos 0}^{\,0}\right)\right)=$$

$$= -\frac\pi2 - \frac4\pi\left(-\frac{\pi^2}{4} - \frac{1}{32}\cdot 0\right) = -\frac\pi2 + \frac{\cancel{4}\,\pi^{\cancel{2}}}{\cancel{\pi}\,\cancel{4}} =$$

$$= -\frac\pi2 + \pi = \frac\pi2$$

(5) $(x^2+y^2)\,dx + xy\,dy = 0 \;|:x^2,\quad y(1)=-1$ — хомогена д.ј.

— наћи партикуларно решење д.ј.

$\left(1+\frac{y^2}{x^2}\right)dx + \frac yx\,dy = 0 \;|:dx \qquad \boxed{\left(1+\frac{y^2}{x^2}\right) + \frac yx\,y' = 0}$

смена: $\frac yx = z \qquad y = z\cdot x$

$z=z(x) \qquad y' = z'x + z$

$(1+z^2) + z(z'\cdot x + z) = 0$

$1+z^2 + z\frac{dz}{dx}\cdot x + z^2 = 0$

$2z^2 + zx\frac{dz}{dx} + 1 = 0$

$2z^2+1 = -xz\frac{dz}{dx} \;\Big|:z$

$2z + \frac1z = -x\frac{dz}{dx}$

$\left(2z+\frac1z\right)dx = -x\,dz$

$\frac{\left(2z+\frac1z\right)}{dz} = -\frac{x}{dx}$

$\frac{dz}{\left(2z+\frac1z\right)} = -\frac{dx}{x}$

$\frac{dz}{\frac{2z^2+1}{z}} = -\frac{dx}{x}$

$\frac{z\,dz}{2z^2+1} = -\frac{dx}{x} \;\Big|\int$

$\int \frac{z\,dz}{2z^2+1} = -\int\frac{dx}{x} \qquad \begin{bmatrix}2z^2+1=t\\ 4z\,dz=dt\\ z\,dz=\frac{dt}{4}\end{bmatrix}$

$\int\frac{dt/4}{t} = -\int\frac{dx}{x}$

$\frac14\ln|2z^2+1| = -\ln(x)+\ln C$

$\ln(2z^2+1)^{\frac14} = \ln\frac1x\cdot C$

$\sqrt[4]{2z^2+1} = \frac Cx \qquad \sqrt[4]{2\frac{y^2}{x^2}+1} = \frac Cx \;\Big|^4$

$2\frac{y^2}{x^2}+1 = \left(\frac Cx\right)^4 \qquad y^2 = \left[\left(\frac Cx\right)^4 - 1\right]\frac{x^2}{2} \qquad y(1)=-1 \Rightarrow \left(C^4-1\right)\frac12 = 1 \qquad C^4-1=2 \qquad C^4=3 \qquad \boxed{C=\sqrt[4]{3}}$

$$\boxed{y = \pm\sqrt{\left(\frac{3}{x^4} - 1\right)\frac{x^2}{2}}}$$

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6. $y''' - y = e^x$

- нађи опште решење д.ј.

$y_N = y_H + y_P$

y_H: $r^3 - 1 = 0$

$(r-1)(r^2+r+1) = 0$

$r - 1 = 0 \lor r^2 + r + 1 = 0$

$r_1 = 1 \qquad r_{2,3} = \frac{-1 \pm \sqrt{1-4}}{2} = \frac{-1 \pm i\sqrt{3}}{2} \qquad \alpha = -\frac{1}{2} \quad \beta = \frac{\sqrt{3}}{2}$

$e^x \qquad e^{-\frac{1}{2}x}\cos\frac{\sqrt{3}}{2}x$

$\qquad e^{-\frac{1}{2}x}\sin\frac{\sqrt{3}}{2}x$

$$\boxed{y_H = C_1 e^x + C_2 e^{-\frac{x}{2}}\cos\frac{\sqrt{3}}{2}x + C_3 e^{-\frac{x}{2}}\sin\frac{\sqrt{3}}{2}x}$$

y_P: $\left.\begin{array}{l} F(x) = e^x \\ 1 \text{ је корен} \end{array}\right\}$ $y_P = Axe^x$

$y_P' = Ae^x + Axe^x = Ae^x(1+x)$

$y_P'' = Ae^x(1+x) + Ae^x = Ae^x(2+x)$

$y_P''' = Ae^x(2+x) + Ae^x = Ae^x(3+x)$

$Ae^x(3+x) - Axe^x = e^x$

$3Ae^x + \cancel{Axe^x} - \cancel{Axe^x} = e^x$

$3A = 1$

$A = \frac{1}{3}$

$$\boxed{y_P = \frac{1}{3}xe^x}$$

$$\boxed{y_N = C_1 e^x + C_2 e^{-\frac{x}{2}}\cos\frac{\sqrt{3}}{2}x + C_3 e^{-\frac{x}{2}}\sin\frac{\sqrt{3}}{2}x + \frac{1}{3}xe^x}$$

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7. $x - 1 = \rho\cos\varphi$

$y = \rho\sin\varphi$

$D = \left\{(x,y) \in \mathbb{R}^2 \mid x^2 + y^2 \leq 2x,\ x \leq 2 - |y|\right\}$

$|y| \leq -x + 2$ $\begin{cases} y \leq -x + 2,\ y \geq 0 \\ -y \leq -x + 2,\ y < 0 \\ y \geq x - 2 \end{cases}$

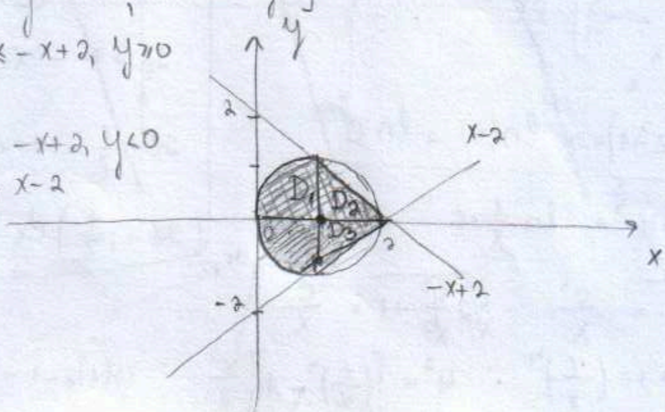
$x^2 + y^2 - 2x \leq 0$

$(x^2 - 2x + 1) - 1 + y^2 \leq 0$

$(x-1)^2 + y^2 \leq 1$

$y^2 = 1 - (x-1)^2$

$y_{1,2} = \pm\sqrt{1 - (x-1)^2}$



$$\boxed{D_1:\ \frac{\pi}{2} \le \varphi \le \frac{3\pi}{4},\quad 0 \le \rho \le 1}$$

$$\boxed{D_2:\ 0 \le \varphi \le \frac{\pi}{2},\quad 0 \le \rho \le \frac{1}{\sin\varphi+\cos\varphi}}$$

кр. О

$$\rho^2\cos^2\varphi + \rho^2\sin^2\varphi = 1$$
$$\rho^2 = 1$$
$$\rho = 1$$

$$\rho\sin\varphi = -\rho\cos\varphi - 1 + 2$$
$$\rho(\sin\varphi+\cos\varphi) = 1$$
$$\rho = \frac{1}{\sin\varphi+\cos\varphi}$$

$$\rho\sin\varphi = \rho\cos\varphi + 1 - 2$$
$$\rho(\sin\varphi-\cos\varphi) = -1$$
$$\rho = \frac{-1}{\sin\varphi-\cos\varphi}$$

$$\boxed{D_3:\ \frac{\pi}{2} \le \varphi \le 0,\quad 0 \le \rho \le -\frac{1}{\sin\varphi-\cos\varphi}}$$

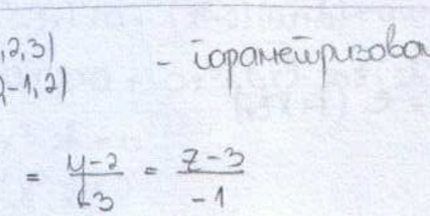
8. $A(1,2,3)$, $B(0,-1,2)$ – параметризовати дуж AB

$$\frac{x-1}{-1} = \frac{y-2}{-3} = \frac{z-3}{-1}$$

$$\begin{cases} x-1 = -t \\ y-2 = -3t \\ z-3 = -t \end{cases} \quad \begin{aligned} x &= -t+1 \\ y &= -3t+2 \\ z &= -t+3 \end{aligned}, \quad 0 \le t \le 1$$

9. $$\iint_S dS = \iint_S \sqrt{1+4x^2+4y^2}\,dxdy = \int_0^{2\pi} d\varphi \int_0^1 \sqrt{1+4(\rho^2)}\,\rho\, d\rho = \left[\begin{matrix} 1+4\rho^2 = t^2 & \rho\, d\rho = \frac{t\,dt}{4} \\ 8\rho\, d\rho = 2t\,dt & \end{matrix}\right] = (*)$$

– S је део површи параболоида $z = x^2+y^2$, који исеца цилиндар $x^2+y^2 = 1$.



$$dS = \sqrt{1+p^2+q^2} = \sqrt{1+4x^2+4y^2}$$
$$p = z_x = 2x$$
$$q = z_y = 2y$$

$x = \rho\cos\varphi$, $y = \rho\sin\varphi$, $|J| = \rho$

$$(*) = \int_0^{2\pi} d\varphi \int_0^1 \frac{t^2\,dt}{4} = \int_0^{2\pi} d\varphi\, \frac{1}{4}\,\frac{t^3}{3}\Big|_0^1 = \frac{1}{12}\,2\pi = \boxed{\frac{\pi}{6}}$$

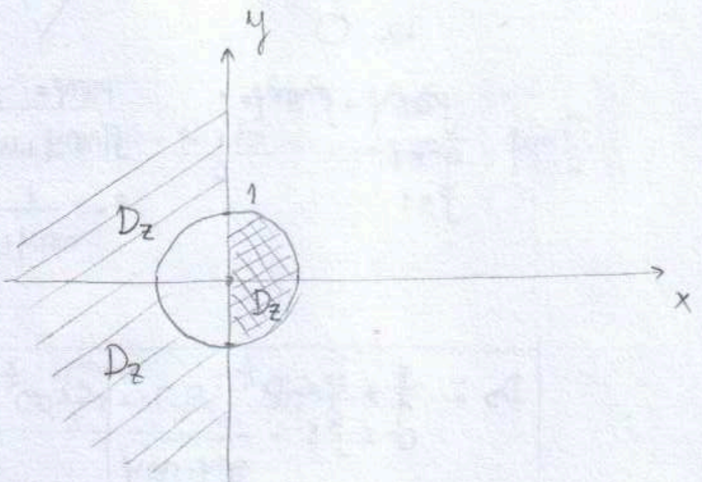
Ⅱ 28. 1. 2011.

① $z(x,y)=\sqrt{x(1-x^2-y^2)}$

- наћи област дефинисаности ф-је. Нацртати слику.

$$D_z=\left\{(x,y)\in\mathbb{R}^2 \,\middle|\, x(1-x^2-y^2)\geq 0\right\}$$

$$(x\geq 0 \wedge \underbrace{1-x^2-y^2\geq 0}_{x^2+y^2\leq 1}) \vee (x<0 \wedge \underbrace{1-x^2-y^2<0}_{x^2+y^2>1})$$



② $F(xe^z, x^2+yz)=0$, $\frac{\partial z}{\partial x}=z_x=?$

$F_u u_x+F_v v_x=0$ $\quad u=xe^z \quad u_x=e^z+xe^z\cdot z_x=e^z(1+xz_x)$

$\quad v=x^2+yz \quad v_x=2x+yz_x$

$F_u\cdot e^z(1+xz_x)+F_v(2x+yz_x)=0$

$F_u\cdot e^z+\underline{F_u e^z x z_x}+2xF_v+\underline{F_v y z_x}=0$

$z_x(xe^zF_u+yF_v)=-2xF_v-e^zF_u$

$$\boxed{z_x=\frac{-2xF_v-e^zF_u}{xe^zF_u+yF_v}}$$

③ $f(x)=\frac{x^2}{2x+1}$

- развити у степени ред
- одредити обл. конв.

4) $f(x)= \begin{cases} 0, & 0 \le x \le 1 \\ 1, & 1 < x \le 3 \end{cases}$ $[0,3]$ – развити у Ф. ред

$$F(x)= \frac{a_0}{2}+\sum_{n=1}^{\infty}\left(a_n\cos\frac{2n\pi x}{b-a}+b_n\sin\frac{2n\pi x}{b-a}\right) \qquad b-a=3$$

$$a_0 = \frac{2}{b-a}\int_a^b f(x)\,dx = \frac{2}{3}\left(\int_0^1 0\cdot dx+\int_1^3 dx\right) = \frac{2}{3}x\Big|_1^3 = \frac{2}{3}(3-1) = \frac{4}{3}$$

$$a_n = \frac{2}{b-a}\int_a^b f(x)\cos\frac{2n\pi x}{b-a}\,dx = \frac{2}{3}\int_1^3 \cos\frac{2n\pi x}{3} = \frac{2}{3}\,\frac{\sin\frac{2n\pi x}{3}}{\frac{2n\pi}{3}}\Bigg|_1^3 = \frac{1}{n\pi}\left(\cancel{\sin 2n\pi}^{\,0}-\sin\frac{2n\pi}{3}\right) = -\frac{1}{n\pi}\sin\frac{2n\pi}{3}$$

$$b_n = \frac{2}{b-a}\int_a^b f(x)\sin\frac{2n\pi x}{b-a}\,dx = \frac{2}{3}\int_1^3 \sin\frac{2n\pi x}{3}\,dx = -\frac{2}{3}\,\frac{\cos\frac{2n\pi x}{3}}{\frac{2n\pi}{3}}\Bigg|_1^3 = -\frac{1}{n\pi}\left(\cos 2n\pi-\cos\frac{2n\pi}{3}\right) =$$

$$= -\frac{1}{n\pi}\left(1-\cos\frac{2n\pi}{3}\right)$$

$$\boxed{F(x)= \frac{2}{3}+\sum_{n=1}^{\infty}\left(-\frac{1}{n\pi}\sin\frac{2n\pi}{3}\cos\frac{2n\pi x}{3}-\frac{1}{n\pi}\left(1-\cos\frac{2n\pi}{3}\right)\sin\frac{2n\pi x}{3}\right)}$$

5.) $(x^4-x^2)y''-(x^3-2x)y'+(x^2-2)y=0$ $\qquad y_1=ax+b \quad y'=a \quad y''=0$

$-(x^3-2x)\cdot a+(x^2-2)(ax+b)=0$

$-\cancel{x^3a}+\cancel{2xa}+\cancel{ax^3}+bx^2-\cancel{2ax}-2b=0$ $\qquad \boxed{y_1=x}$

$bx^2-2b=0$

$b(x^2-2)=0$

$\boxed{b=0}$ v $x^2-2=0$

$x^2=2$

нпр $\boxed{a=1}$ $\qquad x_{1,2}=\pm\sqrt{2}$

6.) $y'''+y=\cos x$ $\qquad \sqrt{-3}=\sqrt{-1}\sqrt{3}$

$y_N = y_H+y_P$

$$\boxed{y_N= C_1e^{-x}+C_2e^{\frac{x}{2}}\cos\frac{\sqrt3}{2}x+C_3e^{\frac{x}{2}}\sin\frac{\sqrt3}{2}x+\frac{1}{2}\cos x-\frac{1}{2}\sin x}$$

y_H: $r^3+1=0$

$(r+1)(r^2-r+1)=0$

$r_1=-1 \quad r_{2,3}=\frac{1\pm\sqrt{1-4}}{2}=\frac{1\pm\sqrt{3}i}{2} \qquad \alpha=\frac{1}{2} \quad \beta=\frac{\sqrt3}{2}$

$e^{-x} \qquad e^{\frac{1}{2}x}\cos\frac{\sqrt3}{2}x \qquad e^{\frac{1}{2}x}\sin\frac{\sqrt3}{2}x$

$y_H = C_1e^{-x}+C_2e^{\frac{x}{2}}\cos\frac{\sqrt3}{2}x+C_3e^{\frac{x}{2}}\sin\frac{\sqrt3}{2}x$ $\qquad y_P=\frac{1}{2}\cos x-\frac{1}{2}\sin x$

y_P: $F(x)=\cos x$, i није корен

$y_P = A\cos x+B\sin x$

$y_P' = -A\sin x+B\cos x$

$y_P'' = -A\cos x-B\sin x$

$y_P''' = A\sin x-B\cos x$

$A\sin x-B\cos x+A\cos x+B\sin x=\cos x$

$\sin x(A+B)+\cos x(A-B)=\cos x$ $\qquad \boxed{A=\frac{1}{2}}$

$A+B=0 \qquad A-B=1 \qquad 1+B=-B$

$A=-B \qquad A=1+B \qquad -2B=1$

$\boxed{B=-\frac{1}{2}}$

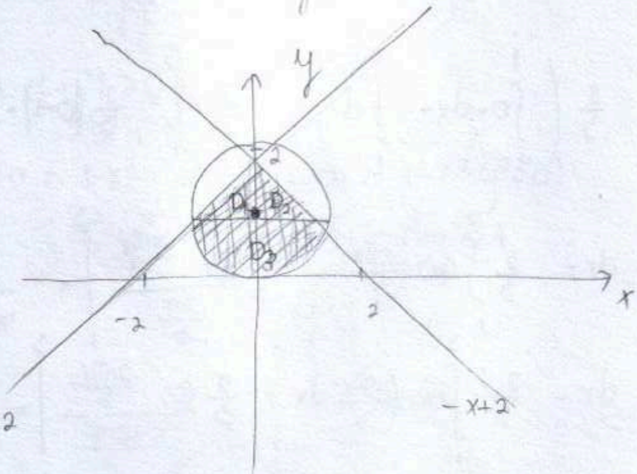
7. $x = \rho\cos\varphi$
$y = \rho\sin\varphi$

$D = \{(x,y) \in \mathbb{R}^2 \mid x^2+y^2 \leq 2y,\ y \leq 2-|x|\}$

$x^2+(y^2-2y+1)-1 \leq 0$

$x^2+(y-1)^2 \leq 1$

$y \leq \begin{cases} 2-x, & x \geq 0 \\ 2+x, & x < 0 \end{cases}$



$x = \rho\cos\varphi$
$y-1 = \rho\sin\varphi$

$\rho = 1$

$\rho\sin\varphi+1 = \rho\cos\varphi+2$
$\rho(\sin\varphi-\cos\varphi) = 1$
$\rho = \dfrac{1}{\sin\varphi-\cos\varphi}$

$\rho\sin\varphi+1 = 2-\rho\cos\varphi$
$\rho(\sin\varphi+\cos\varphi) = 1$
$\rho = \dfrac{1}{\sin\varphi+\cos\varphi}$

D_3: $-\pi \leq \varphi < 0$, $0 \leq \rho < 1$

D_2: $0 \leq \varphi \leq \frac{\pi}{2}$, $0 \leq \rho \leq \dfrac{1}{\sin\varphi+\cos\varphi}$

D_3: $\frac{\pi}{2} < \varphi \leq \pi$, $0 \leq \rho \leq \dfrac{1}{\sin\varphi-\cos\varphi}$

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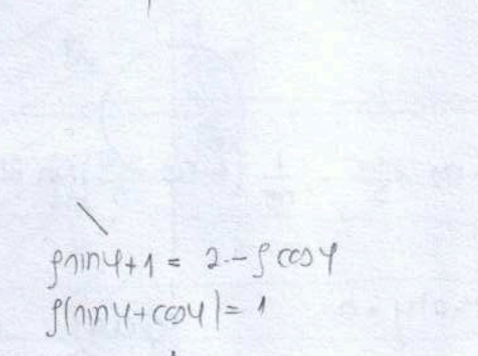
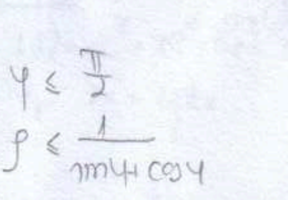
8. C: $x^2+y^2+z^2=4$, $z \leq 0$
$x^2+y^2 = 2x$
$(x^2-2x+1)+y^2 = 1$
$(x-1)^2+y^2 = 1$



$x-1 = \cos t$ $\quad -\pi \leq t \leq \pi$
$y = \sin t$
$z = \sqrt{-(x^2+y^2)+4} = \dots$

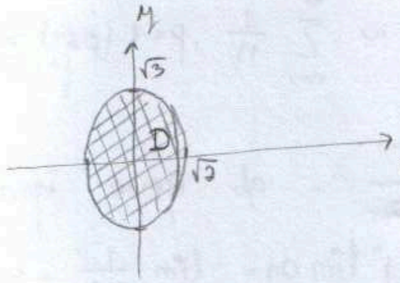


(9) $3x^2+2y^2=6 \mid :6$

- Р елипсе применом двојног интеграла

$\frac{x^2}{2}+\frac{y^2}{3}=1$

$a=\sqrt{2}$ $b=\sqrt{3}$



$\frac{y^2}{3}=1-\frac{x^2}{2}$

$y^2=3\left(1-\frac{x^2}{2}\right)$

$y_{1/2}=\pm\sqrt{3\left(1-\frac{x^2}{2}\right)}$

$\pi ab=\boxed{\pi\sqrt{6}}$ W

$$P=\iint_D dx\,dy=\int_0^{2\pi}d\varphi\int_0^1\sqrt{6}\rho\,d\rho=2\pi\cdot\sqrt{6}\cdot\frac{1}{2}=\boxed{\sqrt{6}\pi}$$

$$\left[\begin{matrix}x=\sqrt{2}\rho\cos\varphi\\ y=\sqrt{3}\rho\sin\varphi\end{matrix}\quad |J|=\sqrt{6}\rho\right]$$

$3\cdot2\rho^2\cos^2\varphi+2\cdot3\cdot\rho^2\sin^2\varphi=6$

$6\rho^2=6$

$\rho^2=1$

$\rho=1$

(III) 25.1.2011.

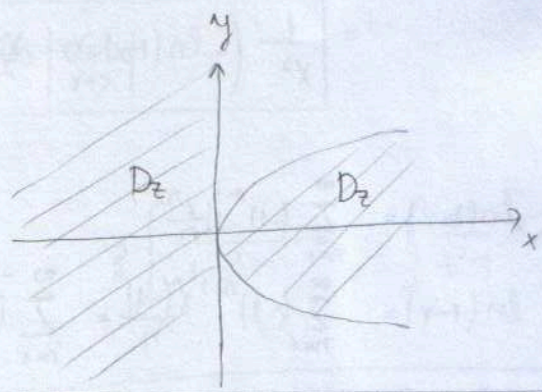
(1) $z(x,y)=\sqrt{x^2-xy^2}$

$D_z=\left\{(x,y)\in\mathbb{R}^2 \mid x^2-xy^2\geq0\right\}$

$x(x-y^2)\geq0$

$(x\geq0\wedge x-y^2\geq0)\vee(x<0\wedge x-y^2<0)$

$y^2\leq x$ $y^2>x$



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(2) $F(x^2yz,\ xy^2-yz^2)=0$, $\frac{\partial z}{\partial y}=?$ $z_y=\dots$

$U=x^2yz$

$V=xy^2-yz^2$

$U_x=2xyz+x^2y\cdot z_x=xy(2z+xz_x)$

$V_x=y^2-y2z\cdot z_x=y(y-2zz_x)$

$U_y=x^2z+x^2yz_y$

$V_y=x2y-(z^2+y2zz_y)$

$F_UU_x+F_VV_x=0$

$F_U\cdot xy(2z+xz_x)+F_V\cdot y(y-2zz_x)=0$

$F_Uxy2z+\underline{F_Uxyxz_x}+F_Vy^2-\underline{F_V2zz_x}=0$

$z_x(F_Ux^2y-2F_Vz)=-F_Uxy2z-F_Vy^2$

$$\boxed{z_x=-\frac{y(F_Ux2z+F_Vy)}{F_Ux^2y-2F_Vz}}$$

$F_UU_y+F_VV_y=0$

$F_U(x^2z+x^2yz_y)+F_V(xy2-z^2-2yzz_y)=0$

$F_Ux^2z+\underline{F_Ux^2yz_y}+2xyF_V-z^2F_V-\underline{2yzz_yF_V}=0$

$z_y(F_Ux^2y-2yzF_V)=-F_Ux^2z-2xyF_V+z^2F_V$

$$\boxed{z_y=-\frac{F_Ux^2z-2xyF_V+z^2F_V}{F_Ux^2y-2yzF_V}}$$

---

(3) $\sum_{n=1}^{\infty}\frac{x^{n+1}}{n+3}$ - област конв.
- сума

$\sum_{n=1}^{\infty}(-1)^{n-1}\frac{x^n}{n}=\ln(1+x)$

$\sum_{n=1}^{\infty}\frac{x^n}{n}=-\ln(1-x)$

$$R=\lim_{n\to\infty}\left|\frac{a_n}{a_{n+1}}\right|=\lim_{n\to\infty}\frac{\frac{1}{n+3}}{\frac{1}{n+4}}=\lim_{n\to\infty}\frac{n+4}{n+3}=\lim_{n\to\infty}\frac{n(1+\frac{4}{n})}{n(1+\frac{3}{n})}=1$$

(О.К.) $x\in(-1,1)$ крајеви?

$x=1$: $\sum_{n=1}^{\infty} \frac{1}{n+3} \sim \sum_{n=1}^{\infty} \frac{1}{n}$, $p=1$ $(p\leq 1) \Rightarrow$ дв

$x=-1$: $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n+3}$ – дв ћемо претходни $\left\langle \begin{matrix} \text{ув} \\ \text{дв} \end{matrix} \right.$?

$1^\circ \lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{1}{n+3} = 0$

$2^\circ f(x) \downarrow \quad f(x) = \frac{1}{x+3} \quad f'(x) = \frac{-1}{(x+3)^2} \downarrow$ ув

О.К. $[-1,1)$

$$\sum_{n=1}^{\infty} \frac{x^{n+1}}{n+3} = \frac{1}{x^2} \sum_{n=1}^{\infty} \frac{x^{n+3}}{n+3} = \frac{1}{x^2} \sum_{n=4}^{\infty} \frac{x^n}{n} = \frac{1}{x^2}\left(\sum_{n=1}^{\infty} \frac{x^n}{n} - \frac{x^3}{3} - \frac{x^2}{2} - \frac{x}{1}\right) =$$

$g(x) \Rightarrow g'(x)$ или $\uparrow$

$$= \boxed{\frac{1}{x^2}\left(-\ln(1-x) - x - \frac{x^2}{2} - \frac{x^3}{3}\right)}$$

$$\ln(1+x) = \sum_{n=1}^{\infty} (-1)^{n-1} \frac{x^n}{n}$$

$$\ln(1-x) = \sum_{n=1}^{\infty} (-1)^{n-1} \frac{x^n(-1)^n}{n} = \sum_{n=1}^{\infty} (-1)^n(-1)(-1)^n \frac{x^n}{n} = -\sum_{n=1}^{\infty} \frac{x^n}{n}$$

---

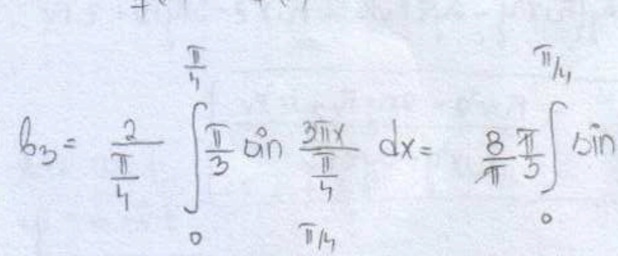
4. $f(x) = \frac{\pi}{3}$, $x \in [0, \frac{\pi}{4}]$

- график $\Phi(x)$
- b_3

$$\Phi(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left(a_n \cos\frac{2n\pi x}{b-a} + b_n \sin\frac{2n\pi x}{b-a}\right)$$

$$\Phi(x) = \sum_{n=1}^{\infty} b_n \sin\frac{n\pi x}{\ell}$$

$$b_n = \frac{2}{\ell}\int_0^{\ell} f(x) \sin\frac{n\pi x}{\ell}\,dx$$



$f(x) \wedge \Phi(x)$

$$b_3 = \frac{2}{\frac{\pi}{4}} \int_0^{\frac{\pi}{4}} \frac{\pi}{3} \sin\frac{3\pi x}{\frac{\pi}{4}}\,dx = \frac{8}{\pi}\frac{\pi}{3}\int_0^{\pi/4} \sin\frac{12\pi x}{\pi}\,dx = \frac{8}{3}\int_0^{\frac{\pi}{4}} \sin 12x\,dx = -\frac{8}{3\cdot 12}\cos 12x \Big|_0^{\frac{\pi}{4}} =$$

$$= -\frac{2}{9}\cos 12x \Big|_0^{\pi/4} = -\frac{2}{9}(\cos 3\pi - \cos 0) = -\frac{2}{9}(-1-1) = \boxed{\frac{4}{9}}$$

⑤ $(x^2-4)y' = \cos y$

$(x^2-4)\dfrac{dy}{dx} = \cos y$

$\dfrac{x^2-4}{dx} = \dfrac{\cos y}{dy}$

$\dfrac{dx}{x^2-4} = \dfrac{dy}{\cos y} \quad \Big|\Big| \int$

$$\int \frac{dx}{x^2-4} = \int \frac{dy}{\cos y}$$

$$\frac{1}{(x-2)(x+2)} = \frac{A}{x-2} + \frac{B}{x+2} = \frac{A(x+2)+B(x-2)}{x^2-4} = \frac{Ax+2A+Bx-2B}{x^2-4}$$

$\left.\begin{array}{l} 2A+2B=0 \\ 2A-2B=1 \end{array}\right\} +$

$4A=1$

$A=\frac{1}{4} \qquad 2B=-2\frac{1}{4}$

$2B=-\frac{1}{2}$

$B=-\frac{1}{4}$

$$= \frac{\frac{1}{4}}{x-2} - \frac{\frac{1}{4}}{x+2}$$

$$\underline{\int \frac{dx}{x^2-4}} = \frac{1}{4}\int \frac{dx}{\frac{x^2}{4}-1} = \frac{1}{4}\int \frac{dx}{\left(\frac{x}{2}\right)^2-1} = \left[\begin{array}{ll} \frac{x}{2}=t & dx=2dt \\ \frac{1}{2}dx=dt & \end{array}\right] = \frac{1}{4}\int \frac{2dt}{t^2-1} =$$

$$= \frac{1}{2}\int \frac{dt}{t^2-1} = \frac{1}{2}\,\frac{1}{2}\ln\left|\frac{t-1}{t+1}\right| = \frac{1}{4}\ln\left|\frac{\frac{x}{2}-1}{\frac{x}{2}+1}\right| = \frac{1}{4}\ln\left|\frac{x-2}{x+2}\right|$$

$$\underline{\int \frac{dy}{\cos y}} = \left[\begin{array}{ll} \operatorname{tg}\frac{y}{2}=t & dy=\frac{2dt}{1+t^2} \\ \cos y=\frac{1-t^2}{1+t^2} & \end{array}\right] = \int \frac{\frac{2dt}{\cancel{1+t^2}}}{\frac{1-t^2}{\cancel{1+t^2}}} = 2\int \frac{dt}{1-t^2} = -2\int \frac{dt}{t^2-1} =$$

$$= -2\,\frac{1}{2}\ln\left|\frac{t-1}{t+1}\right| = -\ln\left|\frac{\operatorname{tg}\frac{y}{2}-1}{\operatorname{tg}\frac{y}{2}+1}\right|$$

$$\boxed{\frac{1}{4}\ln\left|\frac{x-2}{x+2}\right| = -\ln\left|\frac{\operatorname{tg}\frac{y}{2}-1}{\operatorname{tg}\frac{y}{2}+1}\right| + C} \ \ldots$$

---

⑥ $y''' + y'' = x^2$

$$\boxed{y_N = C_1 + C_2x + C_3e^{-x} + x^2\left(\tfrac{1}{12}x^2 - \tfrac{1}{3}x + 1\right)}$$

$y_N = y_H + y_p$

(y_H) $r^3 + r^2 = 0$

$r^2(r+1)=0$

$r=0$ - двострука нула

$r=-1$

$1 \quad x \quad e^{-x}$

$\underline{y_H = C_1 + C_2x + C_3e^{-x}}$

$\underline{y_p = x^2\left(\frac{1}{12}x^2 + \frac{1}{3}x - 1\right)}$

(y_p) $F(x) = x^2$

0 је решење реда 2 $\Big\}$ $y_p = x^2(Ax^2+Bx+C)$

$y_p' = 2x(Ax^2+Bx+C) + x^2(2Ax+B) = 2Ax^3 + 2Bx^2 + 2Cx + 2Ax^3 + Bx^2 = 4Ax^3 + 3Bx^2 + 2Cx$

$y_p'' = 6Ax^2 + 4Bx + 2C + 6Ax^2 + 2Bx = 12Ax^2 + 6Bx + 2C$

$y_p''' = 12Ax + 4B + 12Ax + 2B = 24Ax + 6B$

$24Ax + 6B + 12Ax^2 + 6Bx + 2C = x^2$

$12Ax^2 + x(24A+6B) + 6B + 2C = x^2$

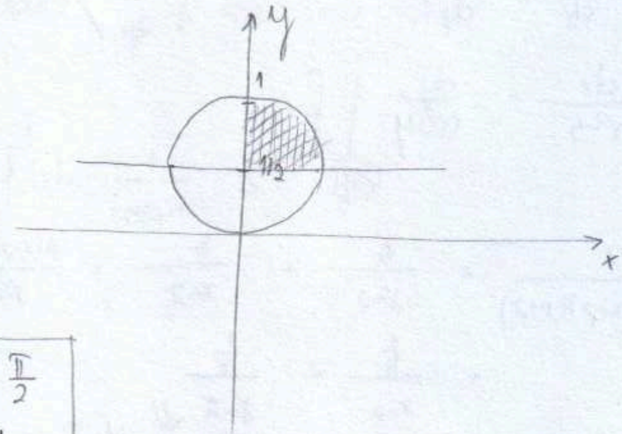
$12A = 1$	$24A + 6B = 0$	$6B + 2C = 0$
$A = \frac{1}{12}$	$6B = -2$	$2C = +2$
	$B = -\frac{1}{3}$	$C = +1$

7. $D = \left\{ (x,y) \in \mathbb{R}^2 \mid x^2+y^2 \le y,\ x \le 0,\ y \ge \frac{1}{2} \right\}$

$x^2+y^2-y \le 0 \quad , x \le 0,\ y \ge \frac{1}{2}$

$x^2+y^2-2\cdot\frac{1}{2}y \le 0$

$x^2+\left(y-\frac{1}{2}\right)^2 = \left(\frac{1}{2}\right)^2$



$\begin{cases} x = \rho\cos\varphi \\ y-\frac{1}{2} = \rho\sin\varphi \end{cases} \quad |J| = \rho$

$$\boxed{\begin{matrix} 0 \le \varphi \le \frac{\pi}{2} \\ 0 \le \rho \le \frac{1}{2} \end{matrix}}$$

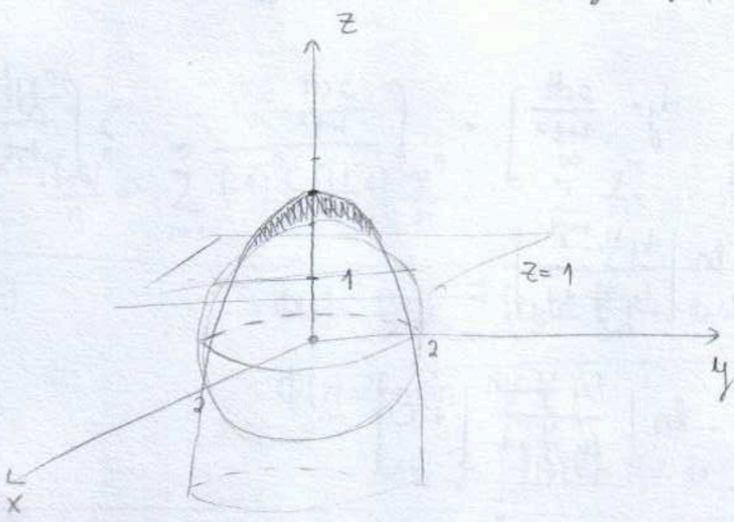
8. $x^2+y^2+z^2 \ge 4$ — нацртати слику

$2z \le 5-x^2-y^2$

$z \ge 1$

$2z-5 = -(x^2+y^2)$

$z-\frac{5}{2} = -\frac{1}{2}(x^2+y^2)$



$-2z+5+z^2 = 4$

$z^2-2z+1 = 0$

$(z-1)^2 = 0$

$z = 1$

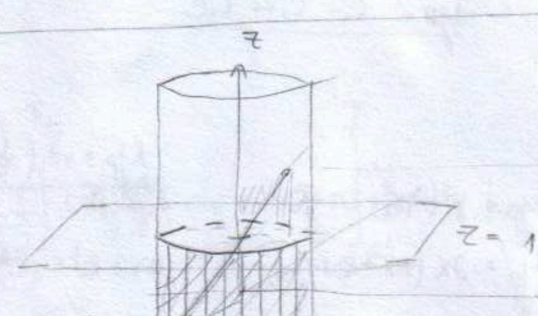
9. $x^2+y^2 = 4$

$z = 1$

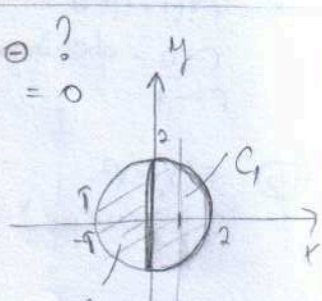
$z = 1-x$

I н.

- површину дела цилиндра ограничена површима



$\ominus \overset{?}{=} 0$



$x = 2\cos t \Rightarrow x'_t = -2\sin t$

$y = 2\sin t \Rightarrow y'_t = 2\cos t$

$$2\int_{C_1}(z_1-z_2)\,ds = 2\int_{C_1}(1-1+x)\cdot ds = 2\int_{C_1} x\,ds = 2\int_{-\pi/2}^{\pi/2} 2\cos t\cdot 2\,dt = 8\int_{-\pi/2}^{\pi/2}\cos t\,dt = 8\sin t\Big|_{-\pi/2}^{\pi/2} = 8(1+1) = \boxed{16}$$

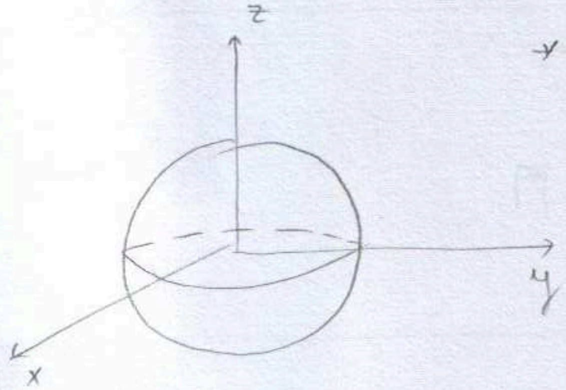
$$ds = \sqrt{x_t'^2 + y_t'^2}\,dt = \sqrt{4}\,dt = 2\,dt$$

II н. $\iint_D \sqrt{1+p^2+q^2}\,dx\,dy = \dots$

(10.) $x^2+y^2+z^2 \le R^2$

– израчунати запремину сфере

→ може и $2\iint_D (z_1 - z_2)\,dx\,dy$



$$T: \begin{cases} x = \rho\cos\varphi\cos\theta \\ y = \rho\sin\varphi\cos\theta \\ z = \rho\sin\theta \end{cases} \qquad |J| = \rho^2\cos\theta$$

$0 \le \varphi \le 2\pi$

$0 \le \rho \le R$

$-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$

$$V(T) = \iiint_T dx\,dy\,dz = \int_0^{2\pi} d\varphi \int_{-\pi/2}^{\pi/2} d\theta \int_0^R \rho^2\cos\theta\,d\rho = \frac{R^3}{3}\,2\,2\pi = \boxed{\frac{4\pi R^3}{3}}$$

(IV) 16. 10. 2010.

(1.) $z(x,y) = \ln(y-|x|) + \sqrt{4y-y^2}$

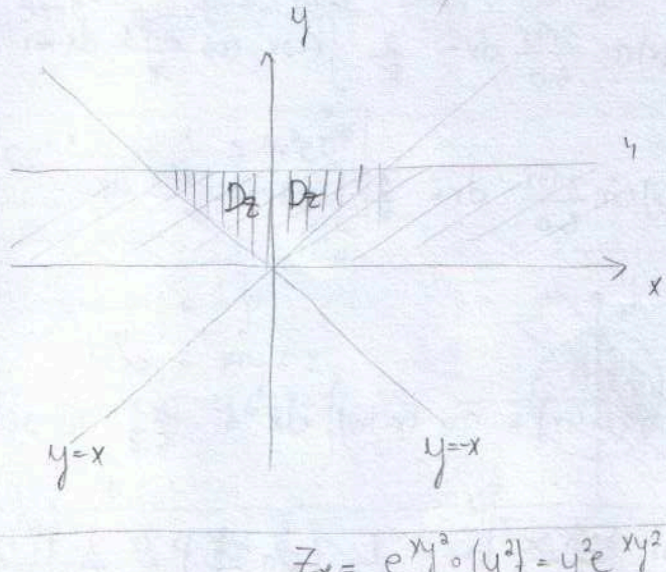
$$D_z = \{(x,y)\in\mathbb{R}^2 \mid y-|x|>0 \ \wedge\ y(4-y)\ge 0\}$$

$$y>|x| \ \wedge\ y(4-y)\ge 0$$

$$y > \begin{cases} x, & x\ge 0 \\ -x, & x<0 \end{cases} \ \wedge\ (y\ge 0 \wedge 4-y\ge 0) \vee (y<0 \wedge 4-y<0)$$

$y \le 4$ $\qquad y>4$

$\emptyset$



(2.) $d^2z(1,2)$, $z(x,y) = e^{xy^2}$

$$\boxed{d^2z = z_{xx}\,dx^2 + 2z_{xy}\,dx\,dy + z_{yy}\,dy^2}$$

$z_{xx}(1,2) = 16e^4 \qquad z_{yy} = 2e^4(1+2\cdot 4) = 18e^4$

$z_{xy} = 2\cdot 2e^4(1+4) = 20e^4$

$z_x = e^{xy^2}\cdot(y^2) = y^2e^{xy^2}$

$z_y = e^{xy^2}\cdot(x2y) = 2xye^{xy^2}$

$z_{xx} = y^2\cdot e^{xy^2}\cdot y^2 = y^4e^{xy^2}$

$z_{yy} = 2xe^{xy^2} + 2xye^{xy^2}\cdot 2xy = 2xe^{xy^2}(1+2xy^2)$

$z_{xy} = 2ye^{xy^2} + y^2e^{xy^2}\cdot(x2y) = 2ye^{xy^2}(1+xy^2)$

$$\boxed{d^2z(1,2) = 16e^4 dx^2 + 40e^4 dx dy + 18e^4 dy^2}$$

③ $f(x) = \dfrac{x}{\sqrt[3]{1-2x^2}}$

- развити у степени ред
- О.К.
- коеф. уз x^5

④

$$f(x) = \begin{cases} \cos x, & 0 \le x \le \frac{\pi}{2} \\ 0, & \frac{\pi}{2} < x \le \pi \end{cases} \qquad b-a=\pi \qquad \begin{matrix} a_0, a_n, b_n \\ a_2 \end{matrix}$$

$$a_0 = \frac{2}{b-a}\int_a^b f(x)\,dx = \frac{2}{\pi}\int_0^{\pi/2}\cos x\,dx$$

$$a_n = \frac{2}{b-a}\int_a^b f(x)\cos\frac{2n\pi x}{b-a}\,dx = \frac{2}{\pi}\int_0^{\pi/2}\cos x\cdot\cos\frac{2n\pi x}{\pi}\,dx \Rightarrow a_2 = \frac{2}{\pi}\int_0^{\pi/2}\cos x\cos 4x\,dx = ⊛$$

$$b_n = \frac{2}{b-a}\int_a^b f(x)\sin\frac{2n\pi x}{b-a}\,dx = \frac{2}{\pi}\int_0^{\pi/2}\cos x\sin\frac{2n\pi x}{\pi}\,dx$$

$$⊛ = \frac{2}{\pi}\int_0^{\pi/2}\frac{1}{2}\Big(\cos(x+4x)+\cos(x-4x)\Big)dx = \frac{\cancel{2}}{\pi}\frac{1}{\cancel{2}}\int_0^{\pi/2}\cos 5x\,dx + \frac{1}{\pi}\int_0^{\pi/2}\cos 3x\,dx =$$

$$= \frac{\sin 5x}{\pi\cdot 5}\Big|_0^{\pi/2} + \frac{\sin 3x}{\pi\cdot 3}\Big|_0^{\pi/2} = \frac{1}{\pi\cdot 5}\left(\sin\frac{5\pi}{2}\right) + \frac{1}{\pi\cdot 3}\left(\sin\frac{3\pi}{2}\right) = \frac{1}{5\pi} + \frac{1}{\pi\cdot 3}(-1) =$$

$$= \frac{1}{5\pi} - \frac{1}{3\pi} = \frac{3\pi - 5\pi}{15\pi^2} =$$

$$= -\frac{2\cancel{\pi}}{15\pi^{\cancel{2}}} = \boxed{-\frac{2}{15\pi}}$$

⑤ $xy'' - (x+1)y' + y = 0$ $y_p = \alpha e^x$ $\alpha = ?$

$y_p' = \alpha e^x$

$y_p'' = \alpha e^x$

$\boxed{y_p = e^x}$

$x\alpha e^x - (x+1)\alpha e^x + \alpha e^x = 0$

$\alpha e^x(\cancel{x} - \cancel{x} - \cancel{1} + \cancel{1}) = 0$

$0 = 0$, npr $\boxed{\alpha = 1}$

---

⑥ $y''' + 2y'' + y' = 5e^{-x}$

$y_H = y_H + y_p$

y_H: $r^3 + 2r^2 + r = 0$

$r(r^2 + 2r + 1) = 0$

$r(r+1)^2 = 0$

$r_1 = 0$ $r_2 = -1$ - реда 2

$e^{0 \cdot x}$ e^{-x} xe^{-x}

$y_H = C_1 + C_2e^{-x} + C_3xe^{-x}$

y_p: $\left.\begin{array}{l} F(x) = 5e^{-x} \\ -1 \text{ je } 0 \text{ реда } 2 \end{array}\right\}$ $y_p = Ax^2e^{-x}$ $y_p = -\frac{5}{2}x^2e^{-x}$

$y_p' = 2Axe^{-x} - Ax^2e^{-x}$

$y_p'' = 2Ae^{-x} - 2Axe^{-x} - (2Axe^{-x} - Ax^2e^{-x}) = 2Ae^{-x} - 2Axe^{-x} - 2Axe^{-x} + Ax^2e^{-x} =$
$= 2Ae^{-x} - 4Axe^{-x} + Ax^2e^{-x}$

$y_p''' = -2Ae^{-x} - (4Ae^{-x} - 4Axe^{-x}) + 2Axe^{-x} - Ax^2e^{-x} = -2Ae^{-x} - 4Ae^{-x} + 4Axe^{-x} + 2Axe^{-x} - Ax^2e^{-x}$
$= -6Ae^{-x} + 6Axe^{-x} - Ax^2e^{-x}$

$-6Ae^{-x} + 6Axe^{-x} - Ax^2e^{-x} + 2(2Ae^{-x} - 4Axe^{-x} + Ax^2e^{-x}) + 2Axe^{-x} - Ax^2e^{-x} = 5e^{-x}$

$-6Ae^{-x} + 6Axe^{-x} - Ax^2e^{-x} + 4Ae^{-x} - 8Axe^{-x} + 2Ax^2e^{-x} + 2Axe^{-x} - Ax^2e^{-x} = 5e^{-x}$

$-2A\cancel{e^{-x}} = 5\cancel{e^{-x}}$

$-2A = 5$

$A = -\frac{5}{2}$

$\boxed{y_H = C_1 + C_2e^{-x} + C_3xe^{-x} - \frac{5}{2}x^2e^{-x}}$

---

⑦ $D = \left\{(x,y) \in \mathbb{R}^2 \mid x^2 + y^2 \le 4,\ 0 \le x+y \le 2\right\}$

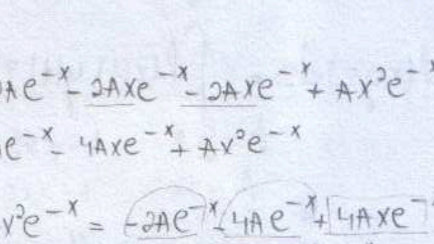
$x+y \ge 0 \wedge x+y \le 2$

$y \ge -x$ $y \le -x+2$

$x = \rho\cos\varphi$

$y = \rho\sin\varphi$

$\pi/2 + \pi/4$



$\boxed{\begin{array}{l} D_1: \frac{\pi}{2} \le \varphi \le \frac{3\pi}{4} \\ 0 \le \rho \le 2 \end{array}}$ $\boxed{\begin{array}{l} D_2: 0 \le \varphi \le \frac{\pi}{2} \\ 0 \le \rho \le \frac{2}{\sin\varphi + \cos\varphi} \end{array}}$ $\boxed{\begin{array}{l} D_3: -\frac{\pi}{4} \le \varphi \le 0 \\ 0 \le \rho \le 2 \end{array}}$

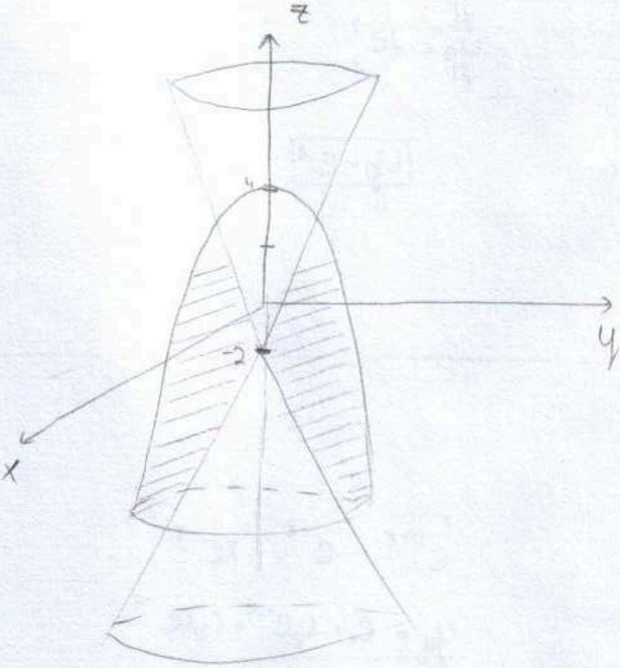
$\rho\sin\varphi = -\rho\cos\varphi + 2$

$\rho(\sin\varphi + \cos\varphi) = 2$

(8) $z \leq 4 - x^2 - y^2 \qquad z-4 \leq -(x^2+y^2)$ - паработоид

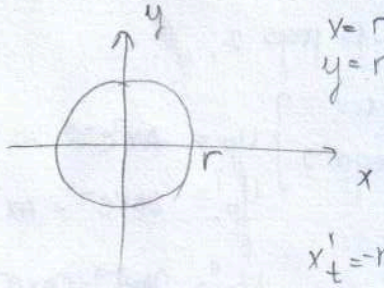
$(z+2)^2 \leq x^2+y^2$ - конус

- слика



(9) $x^2+y^2 = r^2$

- обим круга (крив. I врсте)



$x = r\cos t$
$y = r\sin t$

$$\int_c f(x,y,z)\,ds = \int_a^b f(x(t),y(t),z(t))\sqrt{x_t'^2+y_t'^2+z_t'^2}\,dt$$

$x_t' = -r\sin t$
$y_t' = r\cos t$

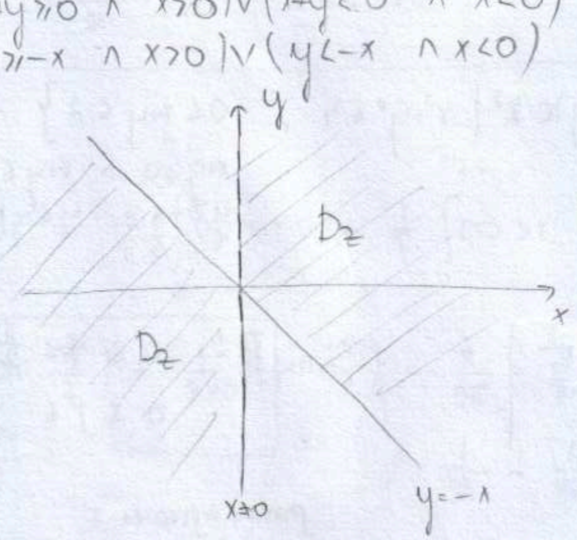
$$\int_c ds = \int_{-\pi}^{\pi}\sqrt{r^2}\,dt = \int_{-\pi}^{\pi} r\,dt = rt\Big|_{-\pi}^{\pi} = r(\pi+\pi) = \boxed{2r\pi}$$

(V) 17. 9. 2010.

(1.) $z(x,y) = \sqrt{\frac{x+y}{x}}$

$D_z = \left\{ (x,y) \in \mathbb{R}^2 \;\middle|\; \frac{x+y}{x} \geq 0 \right\}$

$(x+y \geq 0 \wedge x>0) \vee (x+y<0 \wedge x<0)$
$(y \geq -x \wedge x>0) \vee (y<-x \wedge x<0)$



② $z(x,y) = x^{xy}$; $dz(1,1) = ?$

$dz = z_x dx + z_y dy$

$[x^n]\ z_x = xy \cdot x^{xy-1} \cdot y = xy^2 x^{xy-1} = y^2 x^{xy-1+1} = y^2 x^{xy} = 1$

$[a^x]\ z_y = x^{xy} \cdot \ln x \cdot [x] = x^{xy+1} \ln x = 0$

$\boxed{dz(1,1) = dx}$

---

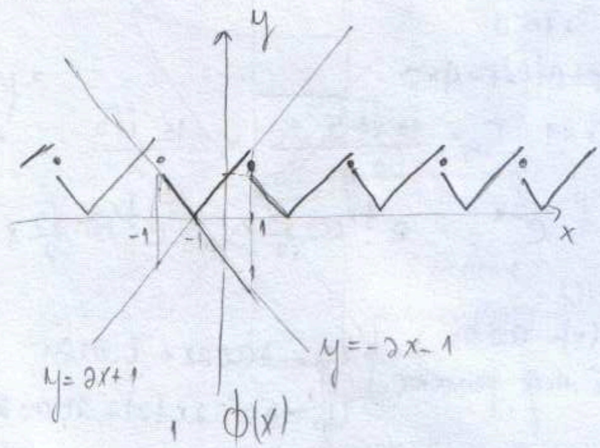
③ $f(x) = x\sqrt[3]{1-2x}$

---

④ $f(x) = |2x+1|$, $x \in [-1,1]$

$y = \begin{cases} 2x+1, & 2x+1 \geq 0 \quad 2x \geq -1 \quad x \geq -\frac{1}{2} \\ -2x-1, & 2x+1 < 0 \quad 2x < -1 \quad x < -\frac{1}{2} \end{cases}$

- график $\Phi(x)$
- a_0



$$a_0 = \frac{2}{b-a}\int_a^b f(x)dx = \frac{2}{2}\int_{-1}^{1}|2x+1|\,dx = \int_{-1}^{-1/2}(-2x-1)\,dx + \int_{-1/2}^{1}(2x+1)\,dx =$$

$$= -2\int_{-1}^{-1/2}x\,dx - \int_{-1}^{-1/2}dx + 2\int_{-1/2}^{1}x\,dx + \int_{-1/2}^{1}dx = -\cancel{2}\frac{x^2}{\cancel{2}}\Big|_{-1}^{-1/2} - x\Big|_{-1}^{-1/2} + \cancel{2}\frac{x^2}{\cancel{2}}\Big|_{-1/2}^{1} + x\Big|_{-1/2}^{1} =$$

$$= -\left(\frac{1}{4}-1\right) - \left(-\frac{1}{2}+1\right) + \left(1-\frac{1}{4}\right) + \left(1+\frac{1}{2}\right) = +\frac{3}{4} - \frac{1}{2} + \frac{3}{4} + \frac{3}{2} = 1 + \frac{6}{4} = \frac{10}{4} = \boxed{\frac{5}{2}}$$

⑤ $y' = \frac{x+y}{x-y} =$

$x\,dy - y\,dy = x\,dx + y\,dx \quad \Big| : x$

$dy - \frac{y}{x}dy = dx + \frac{y}{x}dx$

- опште решење

$\frac{dy}{dx} = \frac{x+y}{x-y}$

$dy(x-y) = dx(x+y)$

смена: $\frac{y}{x} = z$, $z = z(x)$

$y = xz$

$y' = z + xz'$

$z + xz' = \frac{1+z}{1-z}$

$xz' = \frac{1+z}{1-z} - z$

$x\frac{dz}{dx} = \frac{1+\cancel{z}-\cancel{z}+z^2}{1-z}$

$\frac{x}{dx} = \frac{1+z^2}{(1-z)dz}$

$\frac{dx}{x} = \frac{(1-z)dz}{1+z^2} \Big| \int$

$\int\frac{dx}{x} = \int\frac{dz}{1+z^2} - \int\frac{z\,dz}{1+z^2}$

$\ln x = \operatorname{arctg}\frac{y}{x} - \left[\begin{matrix}1+z^2 = t\\ 2z\,dz = dt\end{matrix}\right]$

$\ln x = \operatorname{arctg}\frac{y}{x} - \frac{1}{2}\int\frac{dt}{t}$

$\boxed{\ln x = \operatorname{arctg}\frac{y}{x} - \frac{1}{2}\ln\left(1+\frac{y^2}{x^2}\right) + C}$

---

⑥ $y''' + y = \cos 2x$

$y_N = y_H + y_p$

(y_H) $r^3 + 1 = 0$

$(r+1)(r-r+1) = 0$

$r_1 = -1 \quad r_{2,3} = \frac{1 \pm \sqrt{1-4}}{2} = \frac{1 \pm i\sqrt{3}}{2} \quad \alpha = \frac{1}{2} \quad \beta = \frac{\sqrt{3}}{2}$

$e^{-x} \quad e^{\frac{1}{2}x}\cos\frac{\sqrt{3}}{2}x \quad e^{\frac{1}{2}x}\sin\frac{\sqrt{3}}{2}x$

$y_H = C_1e^{-x} + C_2e^{\frac{x}{2}}\cos\frac{\sqrt{3}}{2}x + C_3e^{\frac{x}{2}}\sin\frac{\sqrt{3}}{2}x$

(y_N) $F(x) = \cos 2x$
2i није решење

$y_p = A\cos 2x + B\sin 2x$

$y_p' = -A\sin 2x\,(2) + 2B\cos 2x = -2A\sin 2x + 2B\cos 2x$

$y_p'' = -2A\cos 2x \cdot 2 + 2B\sin 2x\,(-1)2 = -4A\cos 2x - 4B\sin 2x$

$y_p''' = +4A\sin 2x \cdot 2 - 4B\cos 2x \cdot 2 = 8A\sin 2x - 8B\cos 2x$

$y_p = \frac{1}{65}\cos 2x - \frac{8}{65}\sin 2x$

$8A\sin 2x - \underline{8B\cos 2x + A\cos 2x} + B\sin 2x = \cos 2x$

$\cos 2x(-8B + A) + \sin 2x\,(8A + B) = \cos 2x$

$-8B + A = 1$
$8A + B = 0$

$+8B + 64A = 0$

$65A = 1$

$\boxed{A = \frac{1}{65}}$

$\boxed{-\frac{8}{65} = B}$

$\boxed{y_N = C_1e^{-x} + C_2e^{\frac{x}{2}}\cos\frac{\sqrt{3}}{2}x + C_3e^{\frac{x}{2}}\sin\frac{\sqrt{3}}{2}x + \frac{1}{65}\cos 2x - \frac{8}{65}\sin 2x}$

7. $D=\left\{(x,y)\in\mathbb{R}^2 \mid x\le x^2+y^2\le 2x,\ y\ge 0\right\}$

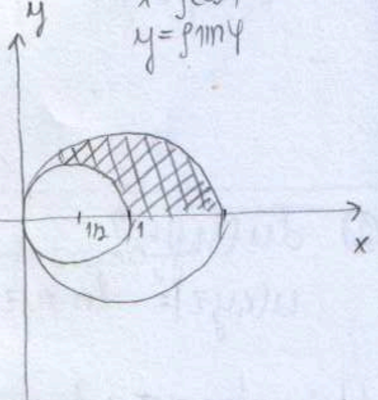
$x^2+y^2-x\ge 0 \ \wedge\ x^2+y^2-2x\le 0 \ \wedge\ y\ge 0$

$\left(x^2-2\cdot\frac{1}{2}x+\frac{1}{4}\right)+y^2\ge\left(\frac{1}{2}\right)^2$

$\left(x-\frac{1}{2}\right)^2+y^2\ge\left(\frac{1}{2}\right)^2 \ \wedge\ (x-1)^2+y^2\le 1 \ \wedge\ y\ge 0$

$x=\rho\cos\varphi$

$y=\rho\sin\varphi$



$$\boxed{\begin{array}{c} 0\le\varphi\le\frac{\pi}{2} \\ \cos\varphi\le\rho\le 2\cos\varphi \end{array}}$$

$\left(\rho\cos\varphi-\frac{1}{2}\right)^2+\rho^2\sin^2\varphi=\frac{1}{4}$

$\rho^2\cos^2\varphi-\rho\cos\varphi+\cancel{\frac{1}{4}}+\rho^2\sin^2\varphi=\cancel{\frac{1}{4}}$

$\rho^2-\rho\cos\varphi=0$

$\rho(\rho-\cos\varphi)=0$

$\rho=\cos\varphi$ – мала

$(\rho\cos\varphi-1)^2+\rho^2\sin^2\varphi=1$

$\rho^2\cos^2\varphi-2\rho\cos\varphi+\cancel{1}+\rho^2\sin^2\varphi=\cancel{1}$

$\rho^2-2\rho\cos\varphi=0$

$\rho(\rho-2\cos\varphi)=0$

$\rho=2\cos\varphi$ – велика

---

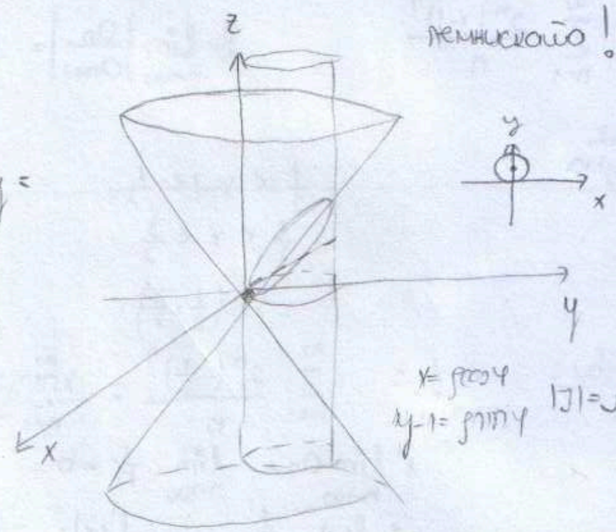
8. $z=\sqrt{x^2+y^2}$

$x^2+y^2=y \qquad x^2+y^2-y+\frac{1}{4}=\frac{1}{4} \qquad x^2+\left(y-\frac{1}{2}\right)^2=\left(\frac{1}{2}\right)^2$

$p=z_x=\frac{2x}{2\sqrt{x^2+y^2}} \qquad q=z_y=\frac{2y}{2\sqrt{x^2+y^2}}$

- израчунати површину дела конусне површи који исеца цилиндар

лемниската !?



$P=\iint_D\sqrt{1+p^2+q^2}\,dxdy=\iint_D\sqrt{1+\frac{x^2+y^2}{x^2+y^2}}\,dxdy=\sqrt{2}\iint_D dxdy=$

$=\sqrt{2}\int_0^{2\pi}d\varphi\int_0^1\rho\,d\rho=\frac{1}{2}\cdot2\pi\cdot\sqrt{2}=\boxed{\sqrt{2}\pi}$

$x=\rho\cos\varphi$

$y-1=\rho\sin\varphi \qquad |J|=\rho$

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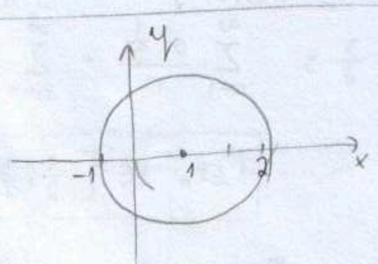
9. $\left.\begin{array}{l} z=x^2+y^2 \\ z=2x+3 \end{array}\right\}$ c: параметризовати криву c



$2x+3=x^2+y^2$

$(x^2-2x+1)-1+y^2=3$

$(x-1)^2+y^2=2^2$

$\left.\begin{array}{l} x-1=2\cos t \\ y=2\sin t \end{array}\right\} \quad -\pi\le t\le\pi$

$z=2(2\cos t+1)+3=4\cos t+5$

$4\cos^2 t+4\sin^2 t=4$

$4=4$

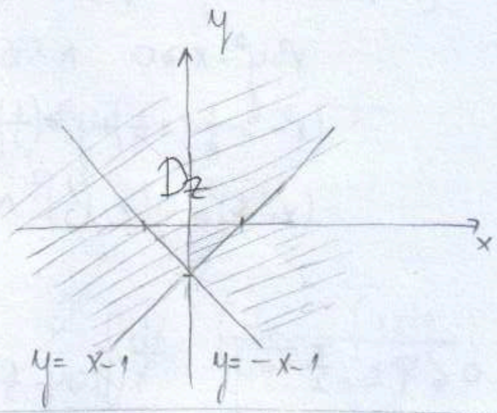
⊤

(VI) 25.6.2010.

1. $z(x,y)=\sqrt{1+|x|+y}$

$D_z=\left\{(x,y)\in\mathbb{R}^2 \;\middle|\; 1+|x|+y\geq 0\right\}$

$y\geq -|x|-1 \left\langle \begin{array}{l} -x-1,\ x\geq 0 \\ x-1,\ x<0 \end{array}\right.$



2. $d^2u(1,1,1)=?$ $\qquad d^2u=u_{xx}dx^2+u_{yy}dy^2+u_{zz}dz^2+2u_{xy}dxdy+2u_{xz}dxdz+2u_{yz}dydz$

$u(x,y,z)=\ln xyz+\frac{x}{y}+y+z$

$u_x=\frac{1}{x\cancel{yz}}\cdot\cancel{yz}+\frac{1}{y}=\frac{1}{x}+\frac{1}{y}$ $\qquad u_{xx}=-\frac{1}{x^2}=-1$ $\quad u_{xy}=-\frac{1}{y^2}=-1$

$u_y=\frac{1}{\cancel{x}y\cancel{z}}\cancel{xz}+(-1)\frac{x}{y^2}+1=\frac{1}{y}-\frac{x}{y^2}+1$ $\qquad u_{yy}=-\frac{1}{y^2}+\frac{2x}{y^3}=1$ $\quad u_{xz}=0$

$u_z=\frac{1}{\cancel{xy}z}\cancel{xy}+1=\frac{1}{z}+1$ $\qquad u_{zz}=-\frac{1}{z^2}=-1$ $\quad u_{yz}=0$

$$\boxed{d^2u(1,1,1)=-dx^2+dy^2-dz^2-2dxdy}$$

3. $\sum\limits_{n=1}^{\infty}\frac{2^n(x-1)^n}{n}$ $\qquad R=\lim\limits_{n\to\infty}\left|\frac{a_n}{a_{n+1}}\right|=\lim\limits_{n\to\infty}\frac{\frac{2^n}{n}}{\frac{2^{n+1}}{n+1}}=\lim\limits_{n\to\infty}\frac{\cancel{2^n}(n+1)}{\cancel{2^n}\cdot 2\cdot n}=\lim\limits_{n\to\infty}\frac{\cancel{n}(1+\frac{1}{n})}{2\cdot\cancel{n}}=\frac{1}{2}$

- о.к.
- сума

$-\frac{1}{2}<x-1<\frac{1}{2}$

$\frac{1}{2}<x<\frac{3}{2}$

$x\in\left(\frac{1}{2},\frac{3}{2}\right)$

крајеви: $x=\frac{1}{2}$: $\sum\limits_{n=1}^{\infty}\frac{2^n\left(-\frac{1}{2}\right)^n}{n}=\sum\limits_{n=1}^{\infty}\frac{\frac{2^n(-1)^n}{2^n}}{n}=\sum\limits_{n=1}^{\infty}\frac{(-1)^n}{n}$ дв према доњем доказу $\left\langle \begin{array}{l} \text{укв} \\ \text{дв?} \end{array}\right.$

1° $\lim\limits_{n\to\infty}a_n=\lim\limits_{n\to\infty}\frac{1}{n}=0$ $\qquad$ укв

2° $f(x)=\frac{1}{x}$ $\qquad f'(x)=-\frac{1}{x^2}\downarrow$

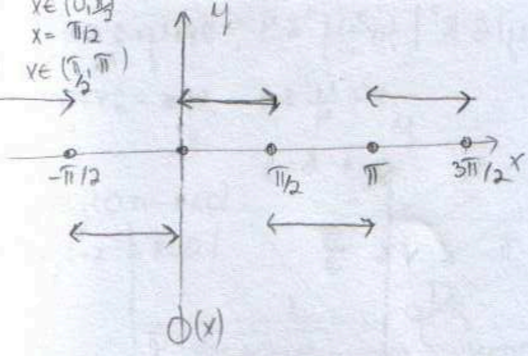
$x=\frac{3}{2}$: $\sum\limits_{n=1}^{\infty}\frac{2^n\frac{1}{2^n}}{n}=\sum\limits_{n=1}^{\infty}\frac{1}{n}$, $p=1$ т.к. дв

$$\boxed{\text{О.к. } x\in\left[-\frac{1}{2},\frac{3}{2}\right)}$$

$\sum\limits_{n=1}^{\infty}\frac{2^n(x-1)^n}{n}=\sum\limits_{n=1}^{\infty}\frac{(2x-2)^n}{n}=-\ln(1-2x+2)=\boxed{-\ln(-2x+3)}$? $\qquad \sum\limits_{n=1}^{\infty}\frac{x^n}{n}=-\ln(1-x)$

4. $f(x) = sgn(\cos x)$, $x \in (0,\pi)$ $\begin{cases} 1, & \cos x > 0 & x \in (0, \frac{\pi}{2}) \\ 0, & \cos x = 0 & x = \frac{\pi}{2} \\ -1, & \cos x < 0 & x \in (\frac{\pi}{2}, \pi) \end{cases}$

- график
- a_0



$$a_0 = \frac{2}{b-a}\int_a^b f(x)dx = \frac{2}{\pi}\int_0^{\pi/2} dx - \frac{2}{\pi}\int_{\pi/2}^{\pi} dx = \frac{2}{\pi}\frac{\pi}{2} - \frac{2}{\pi}\frac{\pi}{2} = \boxed{0}$$

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5. $\sqrt{2-y^2}\; y' = 1$
- опште решење

$$\int \sqrt{2-y^2}\, dy = \int dx \qquad \begin{bmatrix} y = \sqrt{2}\sin t \\ dy = \sqrt{2}\cos t\, dt \end{bmatrix} \quad t = \arcsin \frac{y}{\sqrt{2}}$$

$$\sqrt{2-y^2}\,\frac{dy}{dx} = 1 \qquad \int \sqrt{2-2\sin^2 t}\;\sqrt{2}\cos t\, dt = \int dx$$

$$\sqrt{2-y^2}\, dy = dx \qquad \sqrt{2}\int \sqrt{1-\sin^2 t}\;\sqrt{2}\cos t\, dt = \int dx$$

$$2\int \cos^2 t\, dt = \int dx$$

$$2\int \frac{1+\cos 2t}{2}\, dt = \int dx \qquad \boxed{x = \arcsin\frac{y}{\sqrt{2}} + \frac{1}{2}\sin 2\left(\arcsin\frac{y}{\sqrt{2}}\right) + C}$$

$$x + C = \int dt + \int \cos 2t\, dt$$

$$x + C = t + \frac{\sin 2t}{2}$$

$$x = t + \frac{\sin 2t}{2} + C$$

---

6. $y''' + y' = x^2$

$y_N = y_H + y_P$

(y_H) $r^3 + r = 0$ $\qquad \underline{y_H = C_1 + C_2\cos x + C_3\sin x}$

$r(r^2+1) = 0$

$r_1 = 0 \quad r_{2,3} = \pm i \quad \alpha = 0 \quad \beta = 1$

$1 \quad \cos x \quad \sin x$

(y_P) $\left.\begin{matrix} F(x) = x^2 \\ 0 \text{ је решење} \end{matrix}\right\}$ $y_p = x(Ax^2 + Bx + C) = Ax^3 + Bx^2 + Cx$

$y_p' = 3Ax^2 + 2Bx + C$

$y_p'' = 6Ax + 2B$

$y_p''' = 6A$ $\qquad \underline{y_p = x\left(\frac{1}{3}x^2 - 2\right)}$

$6A + 3Ax^2 + 2Bx + C = x^2$ $\qquad \boxed{y_N = C_1 + C_2\cos x + C_3\sin x + \frac{1}{3}x^3 - 2x}$

$3A = 1 \qquad 2B = 0 \qquad 6A + C = 0$

$A = \frac{1}{3} \qquad B = 0 \qquad 2 + C = 0$

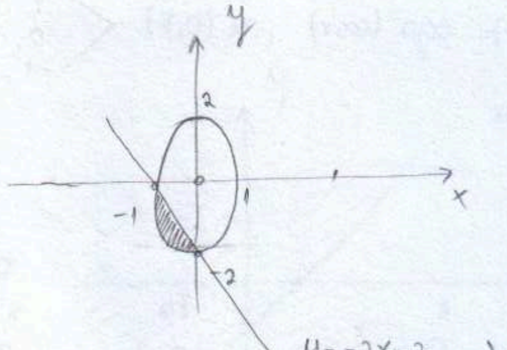
$C = -2$

(7.) $D = \left\{ (x,y) \in \mathbb{R}^2 \mid 4x^2+y^2 \le 4,\ 2x+y+2 \le 0 \right\}$

$x^2 + \frac{y^2}{4} \le 1 \quad y \le -2x-2$

$a=1 \quad b=2$

$$\boxed{\begin{gathered} \pi \le \varphi \le \frac{3\pi}{2} \\ -\frac{2}{\sin\varphi + 2\cos\varphi} \le \rho \le \frac{1}{\sqrt{4\cos^2\varphi + \sin^2\varphi}} \end{gathered}}$$



$y = -2x-2$

$x = \rho\cos\varphi$
$y = \rho\sin\varphi$

$\rho\sin\varphi = -2\rho\cos\varphi - 2$

$\rho(\sin\varphi + 2\cos\varphi) = -2$

$\rho = -\frac{2}{\sin\varphi + 2\cos\varphi}$

0

$\frac{4\rho^2\cos^2\varphi + \rho^2\sin^2\varphi}{4} = 1$

$\rho^2(4\cos^2\varphi + \sin^2\varphi) = 1$

$\rho = \frac{1}{\sqrt{4\cos^2\varphi + \sin^2\varphi}}$

V

$x = \rho\cos\varphi$
$y = 2\rho\sin\varphi$

$y = -2x-2$

$y = -2\rho\cos\varphi - 2$

$\cancel{2}\rho\sin\varphi = -\cancel{2}\rho\cos\varphi - \cancel{2}$

$\rho(\sin\varphi + \cos\varphi) = -1$

$\rho = \frac{-1}{\sin\varphi + \cos\varphi}$

$\rho^2\cos^2\varphi + \rho^2\sin^2\varphi = 1$

$\rho = 1$

$$\boxed{-\frac{1}{\sin\varphi + \cos\varphi} \le \rho \le 1}$$
$+ 4ucito$

(8.) $\int_c (x-y+z)\,ds = \int_c (t+1+\cancel{3t}-\cancel{3t}+2)\sqrt{1+9+9}\,dt = \sqrt{19}\int_0^1 (t+3)\,dt = \sqrt{19}\left(\frac{t^2}{2}\Big|_0^1 + 3t\Big|_0^1\right)$

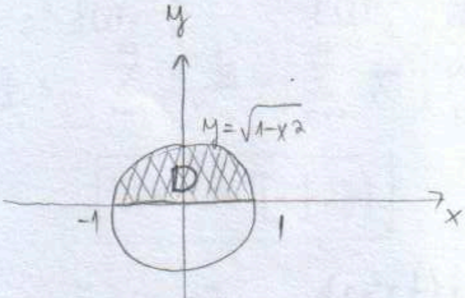
$= \sqrt{19}\left(\frac{1}{2}+3\right) = \boxed{\frac{7\sqrt{19}}{2}}$

AB: $A(1,0,2)$
(c) $B(2,-3,-1)$

$\frac{x-1}{1} = \frac{y-0}{-3} = \frac{z-2}{-3}$

$\begin{cases} x-1=t & x=t+1 \\ y=-3t & y=-3t \\ z-2=-3t & z=-3t+2 \end{cases} \quad 0 \le t \le 1 \quad \begin{aligned} x'_t &= 1 \\ y'_t &= -3 \\ z'_t &= -3 \end{aligned}$

(9.) $\iint_D \sin\sqrt{x^2+y^2}\,dx\,dy, \quad D = \left\{ (x,y) \mid 0 \le y \le +\sqrt{1-x^2} \right\}$



$y=0 \ \wedge\ y = \sqrt{1-x^2} \ \Big|^2$

$y^2 = 1-x^2$

$y^2+x^2 = 1$

$x^2+y^2 = 1$

$0 \le \varphi \le \pi$

$0 \le \rho \le 1$

$x = \rho\cos\varphi$
$y = \rho\sin\varphi$ $\quad |J| = \rho$

$\iint_D \sin\sqrt{x^2+y^2}\,dx\,dy = \int_0^\pi d\varphi \int_0^1 \rho\sin\rho\,d\rho = \boxed{(\sin 1 - \cos 1)\pi}$

$\int_0^1 \rho\sin\rho\,d\rho = \begin{bmatrix} u=\rho & dv=\sin\rho\,d\rho \\ du=d\rho & v=-\cos\rho \end{bmatrix} = -\rho\cos\rho\Big|_0^1 + \int_0^1 \cos\rho\,d\rho = -\rho\cos\rho\Big|_0^1 + \sin\rho\Big|_0^1 = -(\cos 1) + \sin 1 = \sin 1 - \cos 1$

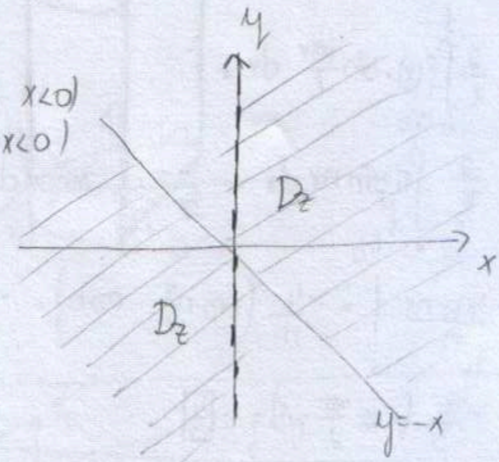
(VII) 23. 6. 2010.

1. $z(x,y) = \ln\left(1+\frac{y}{x}\right)$

$1+\frac{y}{x}>0 \quad \frac{x+y}{x}>0 \iff (x+y>0 \wedge x>0) \vee (x+y<0 \wedge x<0)$

$(y>-x \wedge x>0) \vee (y<-x \wedge x<0)$

$D_z = \left\{(x,y)\in\mathbb{R}^2 \mid 1+\frac{y}{x}>0\right\}$



2. $F(yz, x-yz^2)=0$, $z_x=?$ $\quad z=z(x,y)$

$F_u u_x + F_v v_x = 0 \qquad u=yz \qquad u_x = yz_x$

$v=x-yz^2 \qquad v_x = 1-(+y2zz_x) = 1-2yzz_x$

$F_u\cdot yz_x + F_v\cdot(1-2yzz_x)=0$

$F_u y z_x + F_v - 2yzz_x F_v = 0$

$z_x(yF_u - 2yzF_v) = -F_v$

$$\boxed{z_x = \frac{-F_v}{yF_u - 2yzF_v}}$$

3. $\sum_{n=1}^{\infty} \frac{x^n}{n+1}$ $\qquad \sum_{n=1}^{\infty} \frac{x^n}{n} = -\ln(1-x)$

- о.к.
- сума

$R = \lim_{n\to\infty}\left|\frac{a_n}{a_{n+1}}\right| = \lim_{n\to\infty}\frac{\frac{1}{n+1}}{\frac{1}{n+2}} = \lim_{n\to\infty}\frac{n+2}{n+1} = \lim_{n\to\infty}\frac{n(1+\frac{2}{n})}{n(1+\frac{1}{n})} = 1 \qquad x\in(-1,1)$? крајеви

крајеви: $x=1$: $\sum_{n=1}^{\infty}\frac{1}{n+1} \overset{\text{у.к.}}{\sim} \sum_{n=1}^{\infty}\frac{1}{n}$, $p=1$ (дв)

$x=-1$: $\sum_{n=1}^{\infty}\frac{(-1)^n}{n+1}$ = (дв) (због претходног) < укв / дв ?

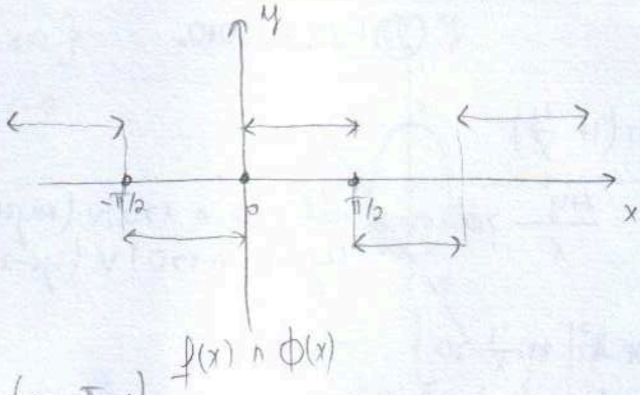
1° $\lim_{n\to\infty} a_n = \lim_{n\to\infty}\frac{1}{n+1} = 0$

2° $f(x)\downarrow \qquad f(x)=\frac{1}{x+1} \qquad f'(x) = \frac{-1}{(x+1)^2}<0 \downarrow$ (укв)

$\boxed{\text{О.К. } x\in[-1,1)}$

$\sum_{n=1}^{\infty} \frac{x^n}{n+1} = \frac{1}{x}\sum_{n=1}^{\infty} \frac{x^{n+1}}{n+1} = \frac{1}{x}\sum_{n=2}^{\infty} \frac{x^n}{n} = \frac{1}{x}\left(\sum_{n=1}^{\infty}\frac{x^n}{n}-1\right) = \boxed{\frac{1}{x}\left(-\ln(1-x)-1\right)}$

4. $f(x)=\pi \quad , x\in\left[0,\frac{\pi}{2}\right]$ — графиk — b_3



$$b_n=\frac{2}{l}\int_0^l f(x)\cdot\sin\frac{n\pi x}{l}\,dx=$$

$$=\frac{2}{\pi/2}\int_0^{\pi/2}\pi\sin nx\,dx=\frac{4\pi}{\cancel{\pi}}\int_0^{\pi/2}\sin nx\,dx$$

$$=-\frac{4\cos nx}{n}\Big|_0^{\pi/2}=-\frac{4}{n}\left(\cos n\frac{\pi}{2}-\cos 0\right)=-\frac{4}{n}\left(\cos n\frac{\pi}{2}-1\right)$$

$$b_3=-\frac{4}{3}\left(\cos\frac{3\pi}{2}-1\right)=\boxed{\frac{4}{3}}$$

5. $y'=\sqrt{2-y^2}$

$\frac{dy}{dx}=\sqrt{2-y^2}$

$\frac{dy}{\sqrt{2-y^2}}=dx \ \Big/\int$

$$\int\frac{dy}{\sqrt{2-y^2}}=\int dx \qquad \frac{1}{\sqrt2}\int\frac{dy}{\sqrt{1-\left(\frac{y}{\sqrt2}\right)^2}}=\int dx \qquad \left[\begin{array}{l}\frac{y}{\sqrt2}=t\\ \frac{1}{\sqrt2}dy=dt\end{array}\right]$$

$$\frac{1}{\sqrt2}\int\frac{\sqrt2\,dt}{\sqrt{1-t^2}}=\int dx \qquad \int\frac{dt}{\sqrt{1-t^2}}=\int dx \qquad \arcsin t=x+C$$

$$\boxed{\arcsin\frac{y}{\sqrt2}=x+C}$$

6. $y'''+y=e^{-x}$

$y_N=y_H+y_P$

y_H: $r^3+1=0$

$(r+1)(r^2-r+1)=0$

$r_1=-1 \quad r_{2,3}=\frac{1\pm\sqrt{1-4}}{2}=\frac{1\pm i\sqrt3}{2} \qquad \alpha=\frac12 \quad \beta=\frac{\sqrt3}{2} \qquad y_H=C_1e^{-x}+C_2e^{\frac{x}{2}}\cos\frac{\sqrt3}{2}+C_3e^{\frac{x}{2}}\sin\frac{\sqrt3}{2}x$

$e^{-x} \quad e^{\frac12 x}\cos\frac{\sqrt3}{2}x \quad e^{\frac12 x}\sin\frac{\sqrt3}{2}x$

$Ae^{-x}(3-x)+Axe^{-x}=e^{-x}$

$3A-Ax+Ax=1$

$A=\frac13 \qquad y_P=\frac13xe^{-x}$

y_P: $F(x)=e^{-x}$, -1 je rešenje $\Big\}\ y_P=Axe^{-x}$

$y_P'=A\left(e^{-x}+xe^{-x}(-1)\right)=Ae^{-x}(1-x)$

$y_P''=A\left(-e^{-x}(1-x)+e^{-x}(-1)\right)=Ae^{-x}(-1+x-1)=Ae^{-x}(-2+x)$

$y_P'''=A\left(-e^{-x}(x-2)+e^{-x}\right)=Ae^{-x}(-x+2+1)=Ae^{-x}(3-x)$

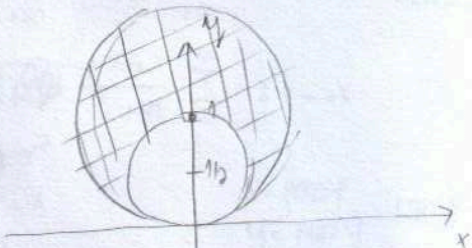
$$\boxed{y_N=C_1e^{-x}+C_2e^{\frac{x}{2}}\cos\frac{\sqrt3}{2}+C_3e^{\frac{x}{2}}\sin\frac{\sqrt3}{2}x+\frac13xe^{-x}}$$

7. $D=\{(x,y)\in\mathbb{R}^2 \mid y\le x^2+y^2\le 2y\}$

$x^2+y^2-y\ge0 \quad \wedge \quad x^2+y^2-2y\le0$

$x^2+\left(y^2-\frac12y\cdot2+\frac14\right)\ge\left(\frac12\right)^2 \qquad x^2+(y^2-2y+1)\le1$

$x^2+\left(y-\frac12\right)^2\ge\left(\frac12\right)^2 \qquad x^2+(y-1)^2\le1^2$



$x=\rho\cos\varphi$

$y=\rho\sin\varphi$

$\rho^2\cos^2\varphi+\left(\rho\sin\varphi-\frac12\right)^2=\left(\frac12\right)^2$

$\rho^2\cos^2\varphi+\rho^2\sin^2\varphi-\rho\sin\varphi+\cancel{\frac14}=\cancel{\frac14}$

$\rho^2-\rho\sin\varphi=0$

$\rho(\rho-\sin\varphi)=0$

$\rho=\sin\varphi$

$\rho^2\cos^2\varphi+(\rho\sin\varphi-1)^2=1$

$\rho^2\cos^2\varphi+\rho^2\sin^2\varphi-2\rho\sin\varphi+\cancel{1}=\cancel{1}$

$\rho^2-2\rho\sin\varphi=0$

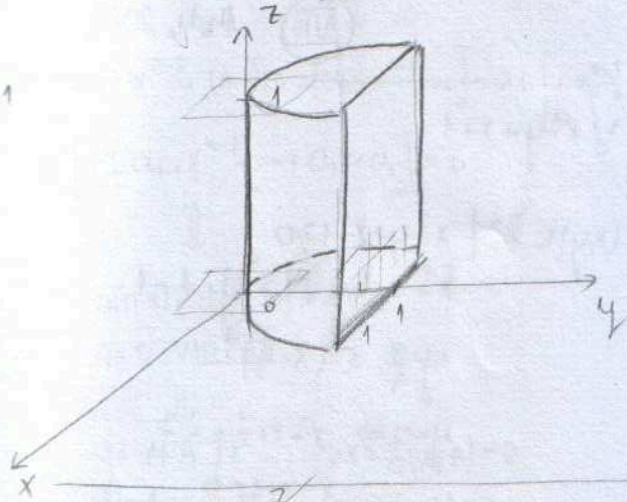
$\rho(\rho-2\sin\varphi)=0$

$\rho=2\sin\varphi$

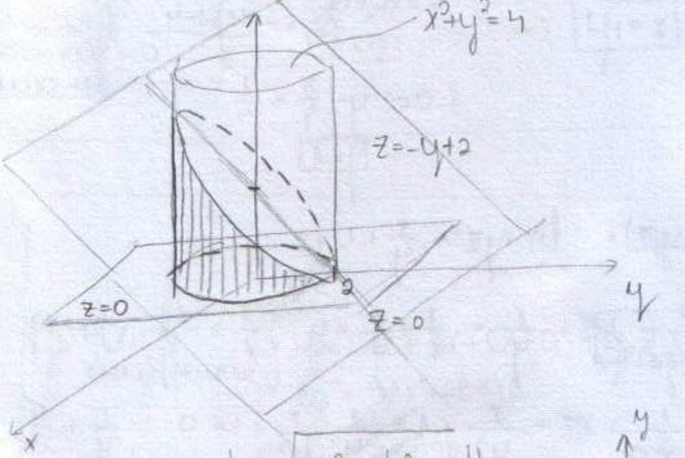
$$\boxed{\begin{array}{c}0\le\varphi\le\pi\\ \sin\varphi\le\rho\le2\sin\varphi\end{array}}$$

(8) $T = \{(x,y,z) \in \mathbb{R}^3 \mid 0 \le z \le 1,\ x^2 \le y \le 1\}$

$y = x^2 \quad y = 1$



(9) $x^2+y^2=4$, $z=0$, $z=2-y$



I начин

$\int_S (z_1 - z_2)\, ds$ — криволинијски инт. прве врсте!

$\int_S (-y+2-0)\, ds = \int_S 2(-y+2)\, dt =$

$= -2\int_S 2\sin t\, dt + 4\int_S dt =$

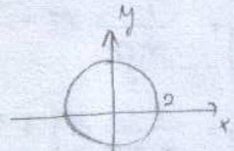
$= -4\int_{-\pi}^{\pi} \sin t\, dt + 4\int_{-\pi}^{\pi} dt = +4\cos t \Big|_{-\pi}^{\pi} + 4\cdot t \Big|_{-\pi}^{\pi} =$

$= +4(\cos\pi - \cancel{\cos\pi}) + 4(\pi+\pi) = \boxed{8\pi}$

$ds = \sqrt{x_t'^2 + y_t'^2}\, dt$

$ds = \sqrt{4\sin^2 t + 4\cos^2 t}\, dt$

$ds = 2\, dt$



$x = 2\cos t$

$y = 2\sin t$

$x_t' = -2\sin t$

$y_t' = 2\cos t$

II начин

$\cancel{P(S) = \iint_D \sqrt{1+p^2+q^2}\, dx\,dy}$ $\quad p = z_x' \quad q = z_y' \qquad z_x' = 0 \quad z_y' = -1$

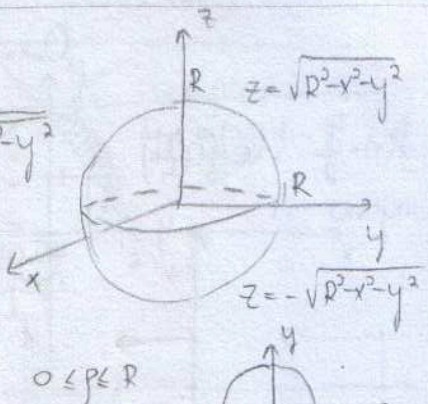
$-\pi \le \varphi \le \pi \quad |J| = \rho$

$0 \le \rho \le 2$

$\cancel{P(S) = \iint_D \sqrt{1+1}\, dx\,dy = \sqrt{2}\int_{-\pi}^{\pi} d\varphi \int_0^2 \rho\, d\rho = \sqrt{2}\int_{-\pi}^{\pi} d\varphi \cdot \frac{4}{2} = 2\sqrt{2}\cdot(\pi+\pi) = 2\sqrt{2}\cdot 2\pi = \boxed{4\sqrt{2}\,\pi}}$?

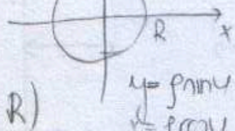
(10) $x^2+y^2+z^2 = R^2 \qquad z^2 = R^2 - x^2 - y^2 \qquad z_{1,2} = \pm\sqrt{R^2-x^2-y^2}$

– израчунати површину сфере $\qquad p = z_x = \frac{-\cancel{2}x}{\cancel{2}\sqrt{R^2-x^2-y^2}} \qquad q = \frac{-y}{\sqrt{R^2-x^2-y^2}}$



$\frac{P}{2} = \iint_D \sqrt{1+p^2+q^2}\, dx\,dy = \iint_D \sqrt{1 + \frac{x^2+y^2}{R^2-x^2-y^2}}\, dx\,dy =$

$= \iint_D \sqrt{\frac{R^2}{R^2-x^2-y^2}}\, dx\,dy = \iint_D \frac{R}{\sqrt{R^2-(x^2+y^2)}}\, dx\,dy = R\int_{-\pi}^{\pi} d\varphi \int_0^R \frac{\rho\, d\rho}{\sqrt{R^2-\rho^2}} =$ $\quad 0 \le \rho \le R$, $\ -\pi \le \varphi \le \pi$



$= \left[\begin{matrix} R^2-\rho^2 = t^2 & \rho\, d\rho = t\,dt \\ -2\rho\, d\rho = 2t\,dt & \end{matrix}\right] = -R\int_{\pi}^{\pi} d\varphi \int_0^R \frac{\cancel{t}\,dt}{\cancel{t}} = -R\int_{-\pi}^{\pi} d\varphi \sqrt{R^2-\rho^2}\Big|_0^R = -R2\pi\cdot(\sqrt{R^2-R^2} - R)$

$= 2R^2\pi \Rightarrow \boxed{P = 4R^2\pi}$

$x = \rho\cos\varphi$, $y = \rho\sin\varphi$ $\quad |J| = \rho$

(VIII) 14.4.2010.

1. $z(x,y)=\sqrt{x^2-y+x+1}$

$D_z=\left\{(x,y)\in\mathbb{R}^2 \mid x^2-y+x+1\geq 0\right\}$

$\left(x^2+2\cdot\frac{1}{2}x+\frac{1}{4}\right)\geq y+\frac{1}{4}-1$

$y-\frac{3}{4}\leq\left(x+\frac{1}{2}\right)^2$

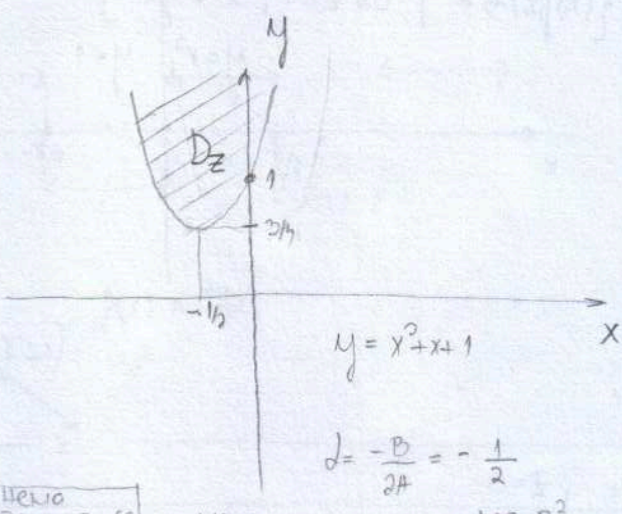
$y=0 \Rightarrow x^2+x+\frac{1}{4}=-\frac{3}{4}$

$x^2+x+1=0$

$x_{1/2}=-\frac{1\pm\sqrt{1-4}}{2}$ — $\boxed{\text{нема пресек со } x\text{-оска}}$

$x=0 \Rightarrow y-\frac{3}{4}=\frac{1}{4}$

$\boxed{y=1}$



$y=x^2+x+1$

$\alpha=-\frac{B}{2A}=-\frac{1}{2}$

или $\beta=-\frac{D}{4A}=\frac{4AC-B^2}{4A}=\frac{4-1}{4}=\frac{3}{4}$

2. $u(x,y,z)=\ln xyz+\frac{x}{y}+y+z$

$u_x=\frac{1}{x\cancel{yz}}\cancel{yz}+\frac{1}{y}=\frac{1}{x}+\frac{1}{y}=0 \quad \frac{1}{x}=-\frac{1}{y} \quad x=-y \quad \boxed{x=2}$

$u_y=\frac{1}{\cancel{x}y\cancel{z}}\cancel{xz}-\frac{x}{y^2}+1=\frac{1}{y}-\frac{x}{y^2}+1=0 \quad \frac{1}{y}+\frac{\cancel{y}}{y^{\cancel{2}}}+1=0 \quad \frac{2}{y}=-1 \quad \boxed{y=-2}$

$u_z=\frac{1}{\cancel{xy}z}\cancel{xy}+1=\frac{1}{z}+1=0 \quad \frac{1}{z}=-1 \quad \boxed{z=-1}$

$\boxed{M(2,-2,-1)}$

3. $\sum_{n=1}^{\infty}3^{\ln n}(x+1)^n$

$R=\lim_{n\to\infty}\left|\frac{a_n}{a_{n+1}}\right|=\lim_{n\to\infty}\frac{3^{\ln n}}{3^{\ln(n+1)}}=\lim_{n\to\infty}\frac{3^{\ln n}}{3^{\ln n(1+\frac{1}{n})}}\to 0=1$

$-1<x+1<1$

$-2<x<0$

$x\in(-2,0)$

краеви: $x=0: \sum_{n=1}^{\infty}3^{\ln n} \Rightarrow \lim_{n\to\infty}a_n=\lim_{n\to\infty}3^{\ln n}=+\infty$ дв

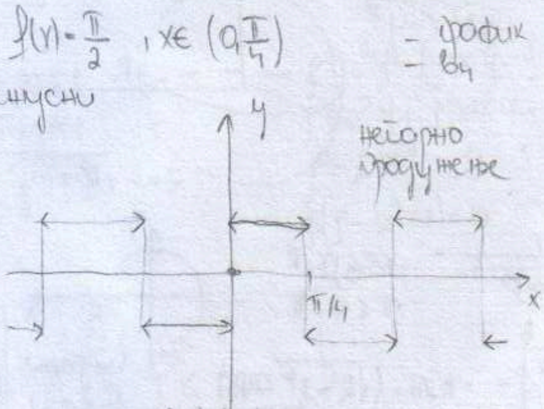
$x=-2: \sum_{n=1}^{\infty}3^{\ln n}(-1)^n \Rightarrow$ дв према претходнаи $<$ цив / дв $(\lim_{n\to\infty}a_n\neq 0)$

О.к. $\boxed{x\in(-2,0)}$

4. $f(x)=\frac{\pi}{2}, \; x\in\left(0,\frac{\pi}{4}\right)$ — график — b_4

- синуси

непарно продолжение



$f(x)\wedge\Phi(x)$

$b_n=\frac{2}{l}\int_0^l f(x)\sin\frac{n\pi x}{l}dx=\frac{2}{\pi/4}\int_0^{\pi/4}\frac{\pi}{2}\sin\frac{4\pi x}{\pi/4}dx$

$=\frac{8}{\pi}\frac{\pi}{2}\int_0^{\pi/4}\sin 16x\,dx=4\int_0^{\pi/4}\sin 16x\,dx=-\frac{4\cos 16x}{16}\Big|_0^{\pi/4}=$

$=-\frac{1}{4}\left(\cancel{\cos 4\pi}^{\,0}-\cos 0\right)=\boxed{0}$

(5.) $x^2(x^2+1)y'' - 2xy' + 2y = 0 \quad | : x^2(x^2+1)$

$y'' - \frac{2y'}{x(x^2+1)} + \frac{2y}{x^2(x^2+1)} = 0$

$y_1 = P_n(x),\ n = ?$

$y_1 = x^n + a_{n-1}x^{n-1} + \dots + a_1 x + a_0$

$y_1' = nx^{n-1} + (n-1)a_{n-1}x^{n-2} + \dots$

$y_1'' = n(n-1)x^{n-2} + (n-1)(n-2)a_{n-1}x^{n-3} + \dots$

$x^2(x^2+1)\left[n(n-1)x^{n-2} + (n-1)(n-2)a_{n-1}x^{n-3} + \dots\right] - 2x\left[nx^{n-1} + (n-1)a_{n-1}x^{n-2} + \dots\right] + 2\left[x^n + a_{n-1}x^{n-1} + \dots + a_1x + a_0\right] = 0$

$\Updownarrow$

$n(n-1) = 0$

$n = 0 \lor \boxed{n = 1}$

$\Downarrow$

$\left.\begin{array}{l} y = x + A \\ y' = 1 \\ y'' = 0 \end{array}\right\} \Rightarrow \begin{array}{l} -2x + 2(x+A) = 0 \\ -\cancel{2x} + \cancel{2x} + 2A = 0 \\ \boxed{A = 0} \end{array} \Rightarrow \boxed{y_1 = x}$

$y_2 = y_1 \int \frac{1}{y_1^2} e^{-\int p(x)dx} dx = \left[p(x) = \frac{-2}{x(x^2+1)}\right] =$

$= \boxed{x \int \frac{1}{x^2} e^{+2\int \frac{dx}{x(x^2+1)}} dx} = y_2$

(6.) $y''' + y = x^3 + \cos x$

$y_N = y_H + y_p$

(y_H) $r^3 + 1 = 0$

$(r+1)(r^2 - r + 1) = 0$

$r_1 = 1 \quad r_{2,3} = \frac{-1 \pm \sqrt{1-4}}{2} = \frac{-1 \pm i\sqrt{3}}{2}$

$\alpha = -\frac{1}{2} \quad \beta = \frac{\sqrt{3}}{2}$

$e^{0\cdot x} \qquad e^{-\frac{1}{2}x}\cos\frac{\sqrt{3}}{2}x \qquad e^{-\frac{1}{2}x}\sin\frac{\sqrt{3}}{2}x$

$\boxed{y_H = C_1 + C_2 e^{-\frac{x}{2}}\cos\frac{\sqrt{3}}{2}x + C_3 e^{-\frac{x}{2}}\sin\frac{\sqrt{3}}{2}x}$

(y_p) $F(x) = x^3$

0 nije rešenje

$y_p = Ax^3 + Bx^2 + Cx + D$

$y_p' = 3Ax^2 + 2Bx + C$

$y_p'' = 6Ax + 2B$

$y_p''' = 6A$

$\boxed{y_{p_1} = x^3 - 6}$

$6A + Ax^3 + Bx^2 + Cx + D = x^3$

$A = 1 \quad 6A + D = 0$

$B = 0 \quad D = -6$

$C = 0$

$F(x) = \cos x \qquad \alpha = 0 \ \beta = 1 \quad \pm i$

$\pm i$ nije rešenje

$y_p = A\cos x + B\sin x$

$y_p' = -A\sin x + B\cos x$

$y_p'' = -A\cos x - B\sin x$

$y_p''' = +A\sin x - B\cos x$

$A\sin x - B\cos x + A\cos x + B\sin x = \cos x$

$\sin x(A+B) + \cos x(A-B) = \cos x$

$\left.\begin{array}{l} A + B = 0 \\ A - B = 1 \end{array}\right\} +$

$2A = 1$

$A = \frac{1}{2} \quad B = -\frac{1}{2}$

$\boxed{y_{p_2} = \frac{1}{2}\cos x + \frac{1}{2}\sin x}$

$\boxed{y_N = C_1 + C_2 e^{-\frac{x}{2}}\cos\frac{\sqrt{3}}{2}x + C_3 e^{-\frac{x}{2}}\sin\frac{\sqrt{3}}{2}x + x^3 - 6 + \frac{1}{2}\cos x + \frac{1}{2}\sin x}$

(7.) $D = \{(x,y) \in \mathbb{R}^2 \mid x^2 + y^2 \le 2y,\ y \ge 2 - x\}$

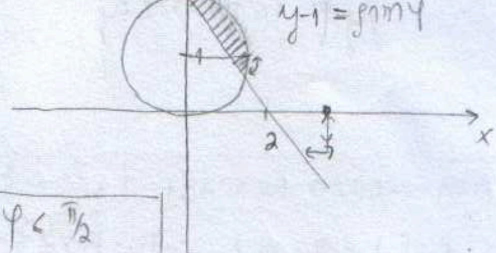
$x^2 + y^2 - 2y + 1 \le 1 \quad y \ge -x + 2$

$x^2 + (y-1)^2 \le 1$

$\rho = 1 \qquad \rho\sin\varphi + 1 = -\rho\cos\varphi + 2$

$\rho\sin\varphi + \rho\cos\varphi = 1$

$\rho = \frac{1}{\sin\varphi + \cos\varphi}$



$x = \rho\cos\varphi$

$y - 1 = \rho\sin\varphi$

$$\boxed{\begin{array}{c} 0 \le \varphi \le \frac{\pi}{2} \\ \frac{1}{\sin\varphi + \cos\varphi} \le \rho \le 1 \end{array}}$$

⑧ $\int\limits_{C}(x-y)\,ds = \int\limits_{C}(4t-3t)\sqrt{x_t'^2+y_t'^2}\,dt = \int\limits_0^1 t\sqrt{16+9}\,dt = 5\int\limits_0^1 t\,dt = 5\frac{t^2}{2} = \boxed{\frac{5}{2}}$

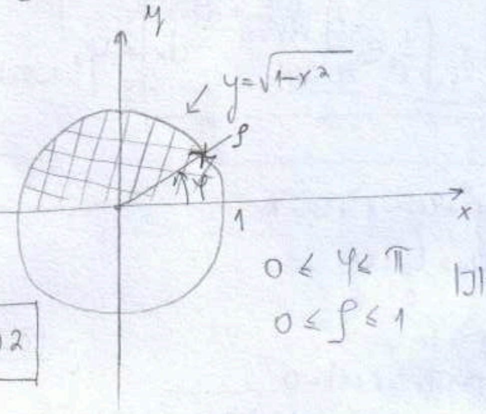
$AB:\ A(0,0)$
$\quad\ \ B(4,3)$

$\overline{AB}:\quad \frac{x}{4} = \frac{y}{3} \qquad \begin{cases} x = 4t \\ y = 3t \end{cases} \quad 0 \le t \le 1 \qquad x_t' = 4,\ \ y_t' = 3$

---

⑨ $\iint\limits_D \frac{dx\,dy}{1+x^2+y^2} = (*)$, $\quad D = \left\{(x,y)\in\mathbb{R}^2 \mid y \ge 0,\ y \le +\sqrt{1-x^2}\right\}$

$y^2 = 1-x^2$
$y^2+x^2 = 1$



$0 \le \varphi \le \pi \qquad |J| = \rho$
$0 \le \rho \le 1$

$(*) = \int\limits_0^{\pi} d\varphi \int\limits_0^1 \frac{\rho\,d\rho}{1+\rho^2} = \begin{bmatrix} 1+\rho^2 = t \\ 2\rho\,d\rho = dt \\ \rho\,d\rho = dt/2 \end{bmatrix} =$

$= \int\limits_0^{\pi} d\varphi \int\limits_0^1 \frac{dt/2}{t} = \frac{1}{2}\int\limits_0^{\pi} d\varphi \int\limits_0^1 \frac{dt}{t} = \frac{1}{2}\varphi\Big|_0^{\pi} \ln(1+\rho^2)\Big|_0^1 = \boxed{\frac{1}{2}\pi \cdot \ln 2}$

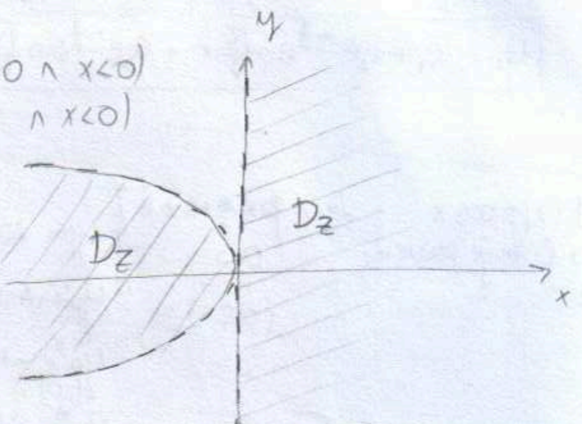
Ⓘⓧ 27.1.2010.

① $z(x,y) = \ln\left(1+\frac{y^2}{x}\right)$

$D_z = \left\{(x,y)\in\mathbb{R}^2 \mid 1+\frac{y^2}{x} > 0\right\}$

$\frac{x+y^2}{x} > 0$

$(x+y^2>0 \wedge x>0) \vee (x+y^2<0 \wedge x<0)$
$(y^2 > -x \wedge x>0) \vee (y^2 < -x \wedge x<0)$



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② $F\left(\frac{y}{xz},\ xyz^2\right) = 0,\quad z_y = ?\qquad z = z(x,y)$

$U = \frac{y}{xz} \qquad U_y = \frac{xz - y(xz_y)}{x^2z^2}$

$V = xyz^2 \qquad V_y = xz^2 + xy\,2z\,z_y$

$F_U U_y + F_V V_y = 0$

$F_U \cdot \left(\frac{xz - y(xz_y)}{x^2z^2}\right) + F_V\left(xz^2 + xy\,2z\,z_y\right) = 0$

$F_U \cdot \frac{1}{xz} - F_U\frac{yz_y}{xz^2} + F_V xz^2 + F_V\,2xyz\,z_y = 0$

$z_y\left(2xyzF_V - \frac{yF_U}{xz^2}\right) = -\frac{F_U}{xz} - F_V xz^2$

$z_y = \frac{-\frac{F_U}{xz} - F_V xz^2}{2xyzF_V - \frac{yF_U}{xz^2}} = \frac{\frac{-F_U - F_V x^2z^3}{xz}}{\frac{2x^2yz^3 - yF_U}{xz^2}} = \boxed{\frac{(-F_U - F_V x^2z^3)z}{2x^2yz^3 - yF_U} = z_y}$

③ $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{2^n}(x+1)^n$ — О.к. — сума

$R = \lim_{n\to\infty}\left|\frac{a_n}{a_{n+1}}\right| = \lim_{n\to\infty}\left|\frac{\frac{1}{2^n}}{\frac{1}{2^{n+1}}}\right| = \lim_{n\to\infty}\frac{\cancel{2^n}\cdot 2}{\cancel{2^n}} = 2$

$-2 < x+1 < 2$

$-3 < x < 1 \qquad x\in(-3,1)$ крајеви?

крајеви: $x=1$: $\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{\cancel{2^n}}\cancel{2^n} = -\sum_{n=1}^{\infty}(-1)^n$ дв ($\lim_{n\to\infty} a_n$ не постоји)

$x=-3$: $\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{\cancel{2^n}}\cdot(-1)^n\cdot\cancel{2^n} = \sum_{n=1}^{\infty}(-1)^{n-1+n} = \sum_{n=1}^{\infty}(-1)^{2n-1}$ дв $\quad \boxed{\text{О.к. } x\in(-3,1)}$

$\sum_{n=1}^{\infty}\frac{(x+1)^n(-1)^{n-1}}{2^n} = -\sum_{n=1}^{\infty}\left[\frac{(x+1)}{2}\right]^n(-1)^n = -\sum_{n=0}^{\infty}\left[\frac{-x-1}{2}\right]^{n*} + 1 = \boxed{-\frac{1}{1-\frac{-x-1}{2}}+1} = \sum_{n=0}^{\infty}x^n = \frac{1}{1-x}$

$= -\frac{1}{\frac{2+x+1}{2}} + 1 = -\frac{2}{x+3} + 1 = \frac{-2+x+3}{x+3} = \boxed{\frac{x+1}{x+3}}$?

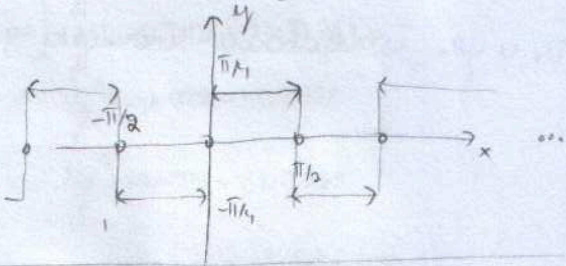
*нзнм да ли сме овако

④ $f(x) = \frac{\pi}{4},\ x\in\left[0,\frac{\pi}{2}\right]$ — график — b_3

— синусни

$b_n = \frac{2}{l}\int f(x)\sin\frac{n\pi x}{l}\,dx \Rightarrow b_3 = \frac{4}{\pi}\int_0^{\pi/2}\frac{\pi}{4}\sin\frac{3\pi x}{\frac{\pi}{2}}\,dx = \frac{\pi}{4}\int_0^{\pi/2}\sin 6x\,dx = -\frac{\pi}{4\cdot 6}\cos 6x\Big|_0^{\pi/2} = -\frac{1}{6}(\cos 3\pi - \cos 0)$

$= -\frac{1}{6}(-1-1) = \boxed{\frac{1}{3}}$



⑤ $(x^2+x)y' = \operatorname{tg} y$

$(x^2+x)\frac{dy}{dx} = \operatorname{tg} y$

$\frac{x^2+x}{dx} = \frac{\operatorname{tg} y}{dy}\ \Big|\int$

$\frac{dx}{x^2+x} = \frac{dy}{\operatorname{tg} y}$

$\int\frac{dx}{x^2+x} = \int\frac{dy}{\operatorname{tg} y}$

$\int\frac{dx}{x(x+1)} = \int\frac{dy}{\operatorname{tg} y}$

$\int\frac{1}{x}dx - \int\frac{1}{x+1}dx = \int\frac{\cos y}{\sin y}dy \quad \left[\begin{array}{l}\sin y = t\\ \cos y\,dy = dt\end{array}\right]$

$\ln x - \ln(x+1) = \int\frac{dt}{t}$

$\ln x - \ln(x+1) + \ln c = \ln\sin y$

$\boxed{\ln\frac{x}{x+1}\cdot c = \ln\sin y}$

$\frac{1}{x(x+1)} = \frac{A}{x} + \frac{B}{x+1} = \frac{A(x+1)+Bx}{x(x+1)} = \frac{Ax+Bx+A}{x(x+1)} = \frac{x(A+B)+A}{x(x+1)} = \frac{1}{x} - \frac{1}{x+1}$

$A+B=0 \quad \boxed{A=1}$

$\boxed{B=-1}$

⑥ $y''' - y = \sin 2x \qquad \alpha = -\frac{1}{2}$

$y_N = y_H + y_P \qquad \beta = \frac{\sqrt{3}}{2}$

y_H: $r^3 - 1 = 0$

$(r-1)(r^2+r+1) = 0$

$r_1 = 1 \quad r_{2,3} = \frac{-1\pm\sqrt{1-4}}{2} = \frac{-1\pm i\sqrt{3}}{2}$

$y_H = C_1e^x + C_2e^{-\frac{x}{2}}\cos\frac{\sqrt{3}}{2}x + C_3e^{-\frac{x}{2}}\sin\frac{\sqrt{3}}{2}x$

y_P: $F(x) = \sin 2x$, $\pm 2i$ није нула; $\alpha=0\ \beta=2$

$y_p = A\cos 2x + B\sin 2x$

$y_p' = -2A\sin 2x + 2B\cos 2x$

$y_p'' = -2\cdot 2A\cos 2x - 2\cdot 2B\sin 2x$

$y_p''' = +2\cdot 4A\sin 2x - 8B\cos 2x$

$+8A\sin 2x - 8B\cos 2x - A\cos 2x - B\sin 2x = \sin 2x$

$\left.\begin{array}{l}8A - B = 1\\ -8A - 64B = 0\end{array}\right\}+$

$-65B = 1$

$B = \frac{-1}{65} \quad A = \frac{8}{65}$

$y_p = \frac{8}{65}\cos 2x - \frac{1}{65}\sin 2x$

$$\boxed{y_N = C_1e^{x} + C_2 e^{-\frac{x}{2}}\cos\frac{\sqrt{3}}{2}x + C_3 e^{-\frac{x}{2}}\sin\frac{\sqrt{3}}{2}x - \frac{1}{65}\sin 2x + \frac{8}{65}\cos 2x}$$

(7.) $D = \{(x,y)\in\mathbb{R}^2 \mid x^2+2y^2 \le 2y\}$

$a = \frac{\sqrt{2}}{2}$ $\qquad x = \frac{\sqrt{2}}{2}\rho\cos\varphi \qquad |J| = \sqrt{2}\rho$

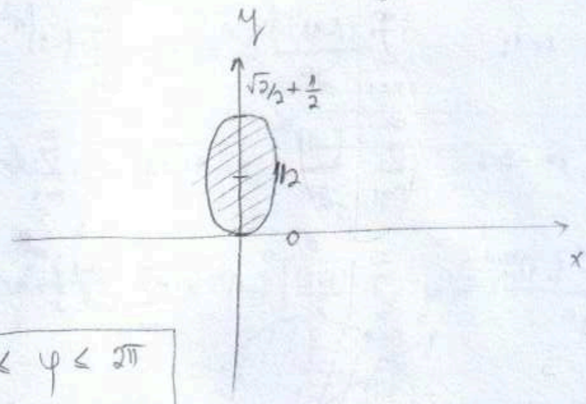
$b = \frac{1}{2}$ $\qquad y - \frac{1}{2} = \frac{1}{2}\rho\sin\varphi$

$$x^2 + 2\left(y^2 - y + \frac{1}{4}\right) - \frac{1}{2} = 0$$

$$x^2 + 2\left(y-\frac{1}{2}\right)^2 = \frac{1}{2} \quad |\cdot 2$$

$$2x^2 + 4\left(y-\frac{1}{2}\right)^2 = 1$$

$$\frac{x^2}{\frac{1}{2}} + \frac{\left(y-\frac{1}{2}\right)^2}{\frac{1}{4}} = 1$$



$$\frac{\frac{1}{2}\rho^2\cos^2\varphi}{\frac{1}{2}} + \frac{\frac{1}{4}\rho^2\sin^2\varphi}{\frac{1}{4}} = 1$$

$\boxed{\rho = 1}$

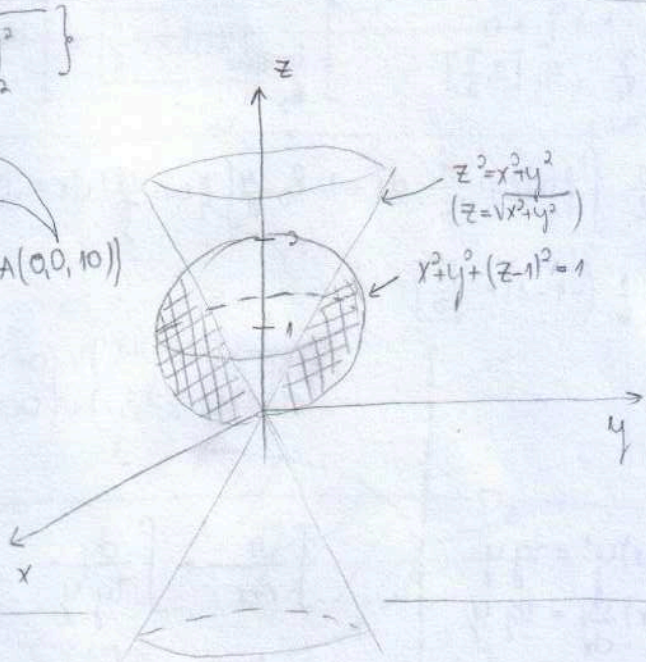
$$\boxed{\begin{array}{l} 0 \le \varphi \le 2\pi \\ 0 \le \rho \le 1 \end{array}}$$

(8.) $T = \left\{(x,y,z)\in\mathbb{R}^3 \mid x^2+y^2+z^2 \le 2z,\ z \le \sqrt{x^2+y^2}\right\}$

$x^2+y^2+z^2-2z+1 \le 1 \qquad z^2 = x^2+y^2$

$x^2+y^2+(z-1)^2 \le 1$

- ст. и ун. проверавати тачкама (нпр $A(0,0,10)$)



(9.) $\iint\limits_D xy^2\,dx\,dy = (*)$

(I н.)

$A(1,0),\ B(0,1);\ C(-1,0) \qquad x = \rho\cos\varphi \quad |J| = \rho$

$\qquad\qquad\qquad\qquad\qquad\quad y = \rho\sin\varphi$

~~$y - y_1 = \frac{y_2-y_1}{x_2-x_1}(x-x_1)$~~

~~$y - 0 = \frac{1-0}{0-1}(x-1)$~~

$y = -x+1 \quad 2)\ -x = y-1 \quad x = -y+1$

$y = x+1 \quad 1)\ x = y-1$

$\rho\sin\varphi = -\rho\cos\varphi + 1$

$\rho(\sin\varphi + \cos\varphi) = 1$

$\Rightarrow \rho = \frac{1}{\sin\varphi+\cos\varphi}$

D_1: $0 \le \rho \le \frac{1}{\sin\varphi + \cos\varphi}$

$0 \le \varphi \le \frac{\pi}{2}$

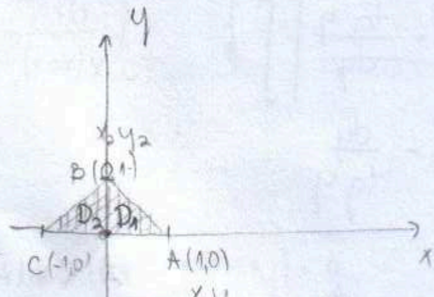
$\rho\sin\varphi = \rho\cos\varphi + 1$

$\rho(\sin\varphi - \cos\varphi) = 1$

$\Rightarrow \rho = \frac{1}{\sin\varphi - \cos\varphi}$

D_2: $0 \le \rho \le \frac{1}{\sin\varphi - \cos\varphi}$

$\frac{\pi}{2} \le \varphi \le \pi$



$= ? = 2\iint\limits_{D_1} xy^2\,dx\,dy = 2\int_0^{\pi/2} d\varphi \int_0^{1/(\sin\varphi+\cos\varphi)} d\rho\cdot\rho\cdot\rho\cos\varphi\cdot\rho^2\sin^2\varphi = 2\int_0^{\pi/2} d\varphi \int_0^{\frac{1}{\sin\varphi+\cos\varphi}} \rho^4\sin^2\varphi\cos\varphi\,d\rho = \begin{bmatrix}\sin\varphi=t\\ \cos\varphi\,d\varphi=dt\end{bmatrix} =$

$= 2\int_0^{\pi/2}\sin^2\varphi\cos\varphi\,d\varphi\cdot\frac{\rho^5}{5}\Big|_0^{\frac{1}{\sin\varphi+\cos\varphi}} = \frac{2}{5}\int_0^{\pi/2}\frac{\sin^2\varphi\cos\varphi\,d\varphi}{(\sin\varphi+\cos\varphi)^5} =$ не може со поларни координати. (роди се со обични!)

II н.) $\int_0^1 dy\int_{+y-1}^{-y+1} dx\,xy^2 = \int_0^1 dy\int_{y-1}^{-y+1} xy^2\,dx = \int_0^1 y^2dy\int_{y-1}^{-y+1} x\,dx = \int_0^1 y^2dy\,\frac{x^2}{2}\Big|_{y-1}^{-y+1} = \int_0^1 y^2dy\,\frac12\left[(-y+1)^2-(y-1)^2\right] =$

$= \frac12\int_0^1 y^2dy\,(y^2-2y+1-(y^2-2y+1)) = \frac12\int_0^1 y^2dy\,(\cancel{y^2}\cancel{-2y+1}\cancel{-y^2}\cancel{+2y-1}) = \frac12\int_0^1 y^2dy\,(\cdot 0) = \boxed{0}$

X 16. 1. 2010.

1. 2. 3. 4. 5. - исти кои из IX тесто!

6. $y''' - y = \cos 2x$

$y_N = y_H + y_P$

y_H: $r^3 - 1 = 0$

$(r-1)(r^2+r+1) = 0$

$r_1 = 1 \quad r_{2,3} = -\frac{1\pm\sqrt{1-4}}{2} = -\frac{1\pm i\sqrt3}{2}$

$\alpha = -\frac12 \quad \beta = \frac{\sqrt3}{2}$

$y_H = C_1e^x + C_2e^{-\frac{x}{2}}\cos\frac{\sqrt3}{2}x + C_3e^{-\frac{x}{2}}\sin\frac{\sqrt3}{2}x$

y_P: $F(x) = \cos 2x$, $\alpha = 0 \quad \beta = 2$, $\pm 2i$ не је нула $\Big\}$ $y_P = A\cos 2x + B\sin 2x$

$y_P' = -2A\sin 2x + 2B\cos 2x$

$y_P'' = -4A\cos 2x - 4B\sin 2x$

$y_P''' = +8A\sin 2x - 8B\cos 2x$

$8A\sin 2x - 8B\cos 2x - A\cos 2x - B\sin 2x = \cos 2x$

$(8A-B)\sin 2x - (8B+A)\cos 2x = \cos 2x$

$\left.\begin{aligned} 8\cdot 8A - 8B &= 0\\ +A + 8B &= -1\end{aligned}\right\}+$

$65A = -1$

$A = -\frac{1}{65}$

$-\frac{1}{65} + 8B = -1$

$8B = -\frac{65}{65} + \frac{1}{65} = -\frac{64}{65}$

$B = -\frac{8}{65}$

$y_P = -\frac{1}{65}\cos 2x - \frac{8}{65}\sin 2x$

$$\boxed{y_N = C_1e^x + C_2e^{-\frac{x}{2}}\cos\frac{\sqrt3}{2}x + C_3e^{-\frac{x}{2}}\sin\frac{\sqrt3}{2}x - \frac{1}{65}\cos 2x - \frac{8}{65}\sin 2x}$$

---

7. $D = \{(x,y)\in\mathbb{R}^2 \mid x \le x^2+y^2 \le 4x\}$

$x^2+y^2 \ge x \;\cap\; x^2+y^2 \le 4x$

$\left(x^2 - x + \frac14\right) + y^2 - \frac14 = 0 \qquad (x^2-4x+4)+y^2 = 4$

$\left(x-\frac12\right)^2 + y^2 \ge \left(\frac12\right)^2 \qquad (x-2)^2+y^2 \le 2^2$

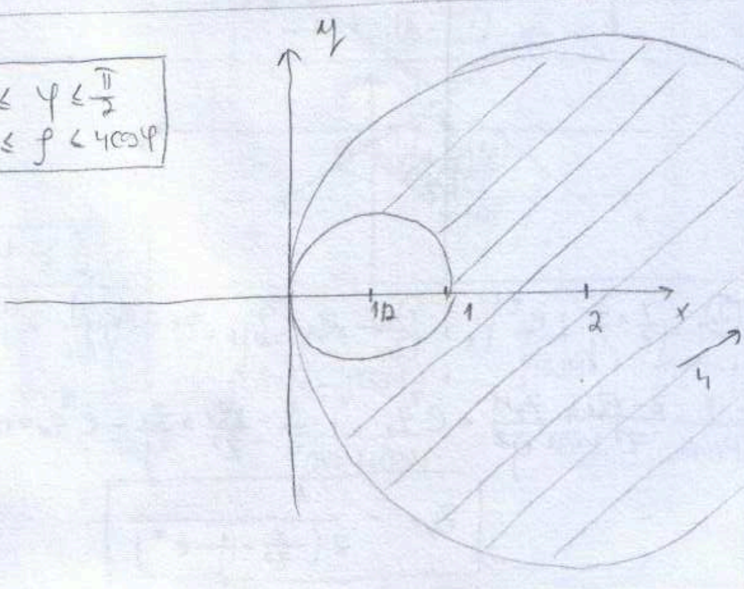
$x = \rho\cos\varphi$
$y = \rho\sin\varphi$

$\rho^2 - \rho\cos\varphi = 0 \qquad \rho^2 - 4\rho\cos\varphi = 0$

$\rho(\rho-\cos\varphi) = 0 \qquad \rho(\rho - 4\cos\varphi) = 0$

$\rho = \cos\varphi$ (мала) $\qquad \rho = 4\cos\varphi$ (велика)

$$\boxed{\begin{aligned} -\frac{\pi}{2} &\le \varphi \le \frac{\pi}{2}\\ \cos\varphi &\le \rho \le 4\cos\varphi\end{aligned}}$$



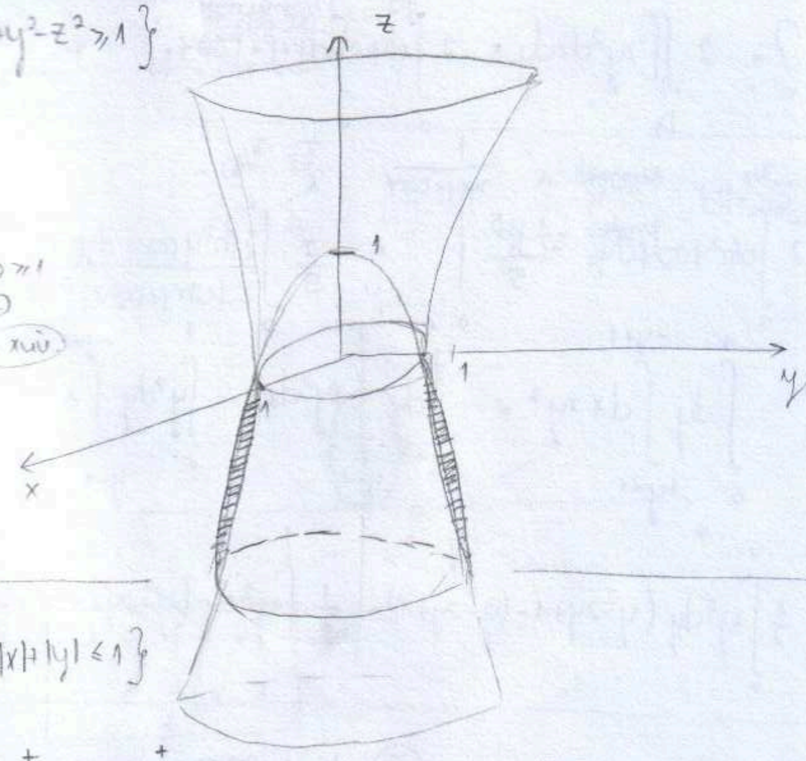
⑧ $T = \left\{ (x,y,z) \in \mathbb{R}^3 \mid x^2+y^2+z \le 1 \ \wedge \ x^2+y^2-z^2 \ge 1 \right\}$

$z - 1 \le -(x^2+y^2)$

$-1 \le -(25)$

$0+0-10 \le 1$ (T) $\qquad -100 \ge 1$ (⊥)

(rot. par.) $\qquad$ (j. hip.)



⑨ $\iint_D x^2 y \, dx dy = (*)$ $\qquad D = \left\{ (x,y) \in \mathbb{R}^2 \mid |x|+|y| \le 1 \right\}$

$(*) = \int_{-1}^{0} dx\, x^2 \int_{-x-1}^{x+1} dy\, y + \int_{0}^{1} dx\, x^2 \int_{x-1}^{-x+1} dy\, y =$

$= \int_{-1}^{0} x^2 dx \left. \frac{y^2}{2} \right|_{-x-1}^{x+1} + \int_{0}^{1} x^2 dx \left. \frac{y^2}{2} \right|_{x-1}^{-x+1} =$

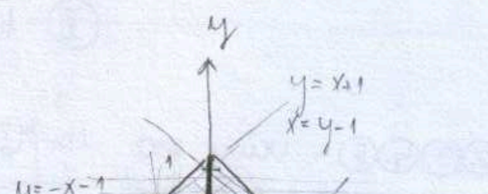
$= \int_{-1}^{0} x^2 dx \frac{1}{2}\left[(x+1)^2 - (-x-1)^2 \right] + \int_{0}^{1} x^2 dx \frac{1}{2}\left[(-x+1)^2 - (x-1)^2 \right] = \boxed{0}$

$\overset{+}{x} \ge 0 \ \wedge \ \overset{+}{y} \ge 0$:
$x+y \le 1 \qquad y \le -x+1$

$\overset{+}{x} \ge 0 \qquad \overset{-}{y} < 0$
$x-y \le 1 \qquad y \ge x-1$

$\overset{-}{x} < 0 \qquad \overset{+}{y} \ge 0$
$-x+y \le 1 \qquad y \le x+1$

$\overset{-}{x} < 0 \qquad \overset{-}{y} < 0$
$-x-y \le 1 \qquad y \ge -x-1$

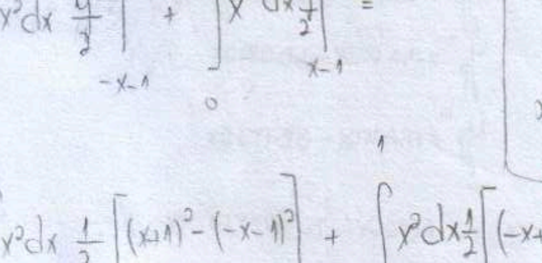


Ⓧ 26. 8. 2008.

① $z = \sqrt{1-|x|-y}$ $\qquad D_z = \left\{ (x,y) \in \mathbb{R}^2 \mid 1-|x|-y \ge 0 \right\}$ $\quad y \le -|x|+1$

$1-x-y \ge 0, \ x \ge 0 \Rightarrow y \le -x+1, \ x \ge 0$

$1+x-y \ge 0, \ x < 0 \Rightarrow y \le x+1, \ x < 0$



② $\frac{x}{z} + \frac{z}{y} = e^z \Big|'_x, \ \frac{\partial z}{\partial x} = z_x = ?, \ z = z(x,y)$

$\frac{z - x z_x}{z^2} + \frac{z_x y}{y^2} = e^z z_x \qquad \frac{1}{z} - \frac{x z_x}{z^2} + \frac{z_x}{y} - e^z z_x = 0 \qquad z_x\left(\frac{-x}{z^2} + \frac{1}{y} - e^z \right) = -\frac{1}{z}$

$$\boxed{z_x = -\frac{1}{z\left(-\frac{x}{z^2} + \frac{1}{y} - e^z \right)}}$$

③ $\sum_{n=1}^{\infty}(n+3^n)x^n$ $\quad \ell=\lim_{n\to\infty}\left|\frac{a_n}{a_{n+1}}\right|=\lim_{n\to\infty}\frac{n+3^n}{(n+1+3^{n+1})}=\lim_{n\to\infty}\frac{3^n\left(\frac{n}{3^n}+1\right)}{3^n\left(\frac{n}{3^n}+\frac{1}{3^n}+3\right)}=\frac{1}{3}$ $\quad -\frac{1}{3}<x<\frac{1}{3}$

- о.к.
- суме

крајеви: $x=\frac{1}{3}$: $\sum_{n=1}^{\infty}(n+3^n)\frac{1}{3^n}$ $\quad \lim_{n\to\infty}a_n=\lim_{n\to\infty}\left(\frac{n}{3^n}+1\right)=1$ (див)

$\boxed{\text{О.к. } x\in\left(-\frac{1}{3},\frac{1}{3}\right)}$

$x=\left(-\frac{1}{3}\right)^n$: $\sum_{n=1}^{\infty}(n+3)^n\frac{(-1)^n}{3^n}$ $\quad \lim_{n\to\infty}a_n=(-1)^n\cdot 1$ (не постоји) (див)

$$\sum_{n=1}^{\infty}(n+3^n)x^n=\sum_{n=1}^{\infty}nx^n+\sum_{n=1}^{\infty}(3x)^n=\boxed{\frac{x}{(1-x)^2}+\frac{1}{1-3x}-1}$$

④ $f(x)=1$, $x\in\left[0,\frac{\pi}{2}\right]$, синусни

$b_1=?$

$$b_n=\frac{2}{\ell}\int_0^{\ell}f(x)\cdot\sin\frac{n\pi x}{\ell}dx\Rightarrow b_1=\frac{2}{\pi/2}\int_0^{\pi/2}\sin\frac{\pi x}{\pi/2}dx=\frac{4}{\pi}\int_0^{\pi/2}\sin 2x\,dx=-\frac{4}{\pi\cdot 2}\cos 2x\Big|_0^{\pi/2}=-\frac{2}{\pi}(\cos\pi-\cos 0)=$$

$$=-\frac{2}{\pi}(-1-1)=\boxed{\frac{4}{\pi}}$$

⑤ $y'=1-\frac{1}{y}$ $\quad dx=\frac{dy}{\frac{y-1}{y}}$ $\quad \int dx=\int\frac{y-1+1}{y-1}dy$ $\quad x+C=y+\ln(y-1)$

$\frac{dy}{dx}=1-\frac{1}{y}$ $\qquad \boxed{x=y-\ln(y-1)+C}$

$\frac{1}{dx}=\frac{1-\frac{1}{y}}{dy}$ $\quad dx=\frac{y\,dy}{y-1}\ \Big|\int$ $\quad \int dx=\int dy+\int\frac{dy}{y-1}$

⑥ $y''+y'=1+e^{-x}$

$y_H=y_h+y_p$

(y_h) $r^2+r=0$ $\quad \boxed{y_H=C_1+C_2e^{-x}}$

$r(r+1)=0$

$r_1=0\quad r_2=-1$

$e^{0\cdot x}\quad e^{-x}$

(y_p) $F(x)=1$, нула је корен $\Big\}$ $y_p=Ax$ $\boxed{y_{p_1}=x}$; $y_p'=A$; $y_p''=0$; $\boxed{A=1}$

$F(x)=e^{-x}$, -1 је корен $\Big\}$ $y_p=Axe^{-x}=\boxed{-xe^{-x}}$

$y_p'=A(e^{-x}-xe^{-x})=Ae^{-x}(1-x)$

$y_p''=A(-e^{-x}(1-x)-e^{-x})=Ae^{-x}(-1-1+x)=Ae^{-x}(x-2)$

$\boxed{y_N=C_1+C_2e^{-x}+x-xe^{-x}}$

$Ae^{-x}(x-2)+Ae^{-x}(1-x)=e^{-x}$

$\cancel{Ax}-2A+A-\cancel{Ax}=1$

$-A=1\quad \boxed{A=-1}$

⑦ $A(1,0)\quad x=\rho\cos\varphi$

$B(1,1)\quad y=\rho\sin\varphi$

$C(0,1)$

$\boxed{D_1:\ 0\le\varphi\le\frac{\pi}{4},\quad \frac{1}{\sin\varphi+\cos\varphi}\le\rho\le\frac{1}{\cos\varphi}}$

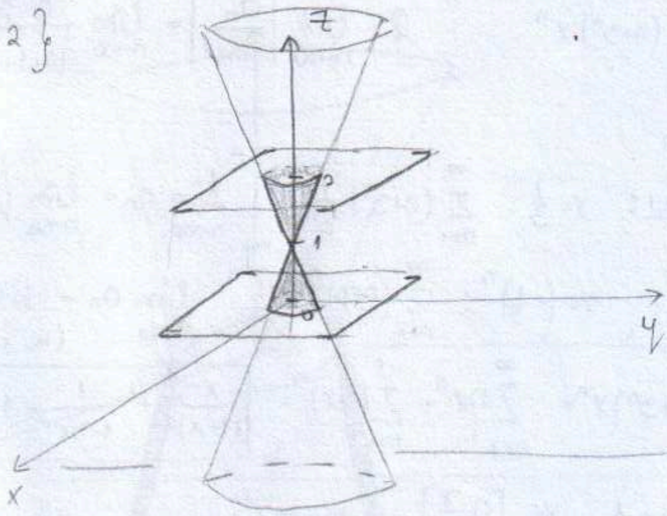
$\boxed{D_2:\ \frac{\pi}{4}\le\varphi\le\frac{\pi}{2},\quad \frac{1}{\sin\varphi+\cos\varphi}\le\rho\le\frac{1}{\sin\varphi}}$

$y=-x+1$ $\qquad x=1\qquad y=1$

$\rho\sin\varphi=-\rho\cos\varphi+1$ $\qquad \rho\cos\varphi=1\qquad \rho\sin\varphi=1$

$\rho=\frac{1}{\sin\varphi+\cos\varphi}$ $\qquad \rho=\frac{1}{\cos\varphi}\qquad \rho=\frac{1}{\sin\varphi}$

8. $T=\{(x,y,z)\in\mathbb{R}^3 \mid (z-1)^2 \geq x^2+y^2,\ 0\leq z\leq 2\}$

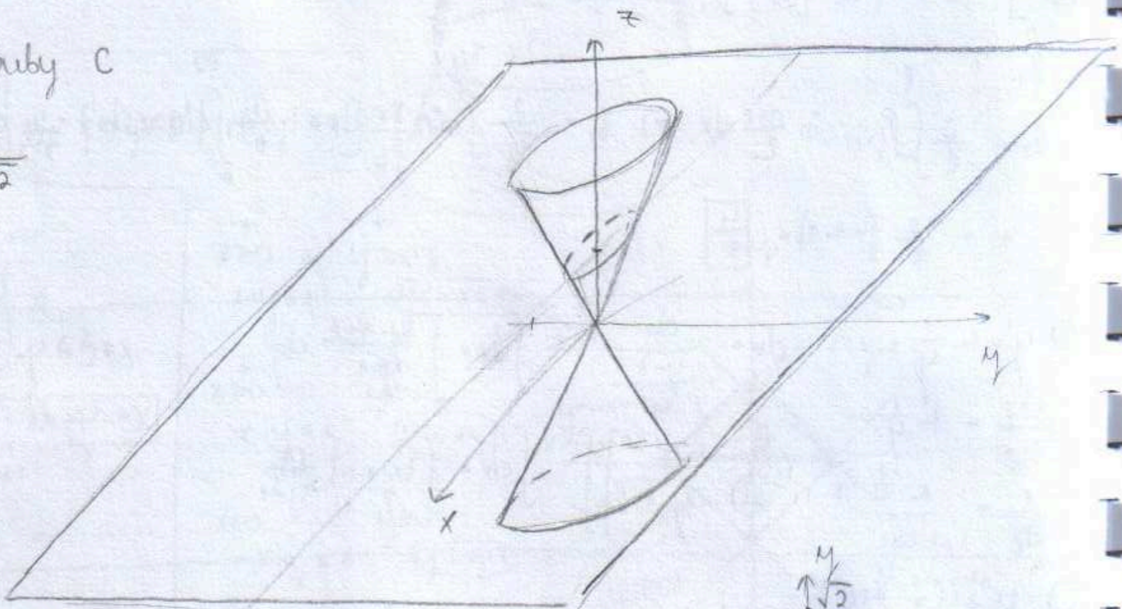


9. $x^2+2y^2=z^2,\ z-y=1$

- параметризовати криву C

$z^2=x^2+\dfrac{y^2}{\left(\frac{1}{\sqrt{2}}\right)^2}$ — $a=1$, $b=\frac{1}{\sqrt{2}}$

$z=y+1$



$x^2+2y^2=(y+1)^2$

$x^2+2y^2=y^2+2y+1$

$x^2+y^2-2y=1$

$x^2+(y-1)^2-1=1$

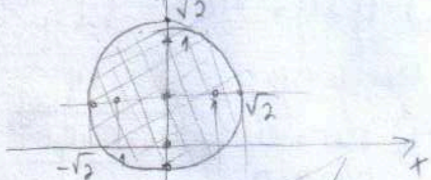
$x^2+(y-1)^2=2$

$\dfrac{x^2}{2}+\dfrac{(y-1)^2}{2}=1$ $\quad a=\sqrt{2}$, $b=\sqrt{2}$

кружница

$$C:\begin{cases} x=\sqrt{2}\cos t \\ y-1=\sqrt{2}\sin t \\ z=\sqrt{2}\sin t+2 \end{cases}$$

$-\pi \leq t < \pi$



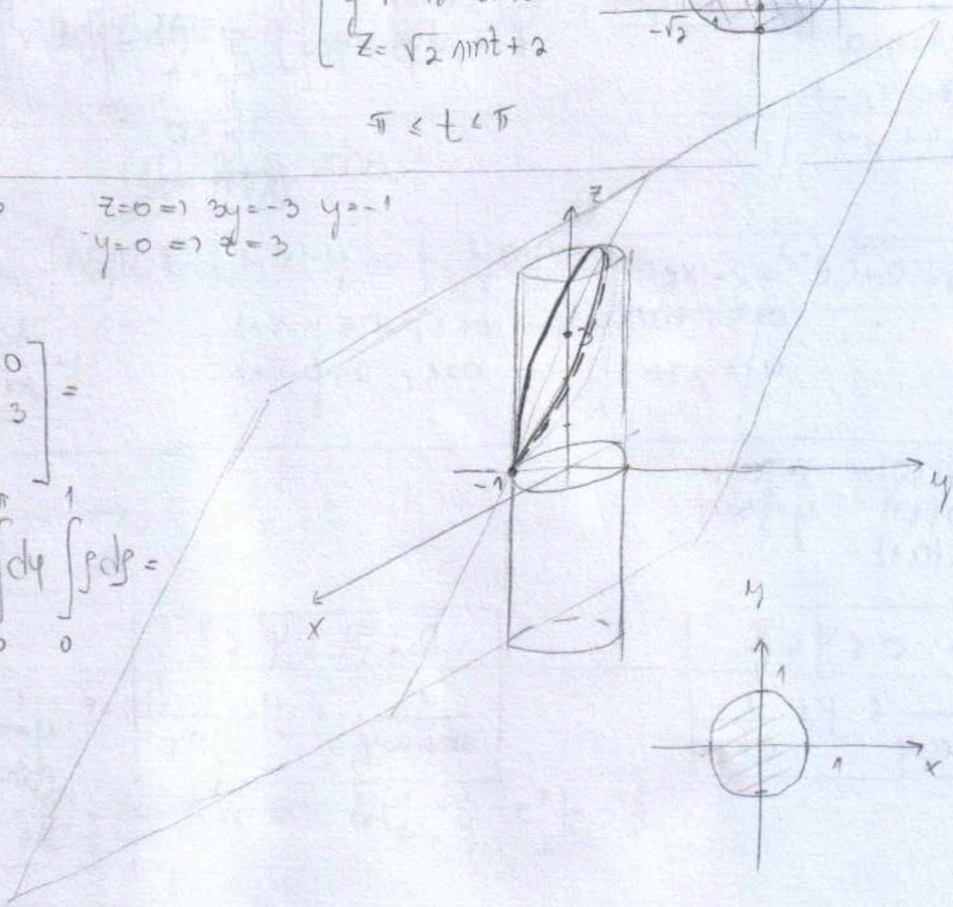
10. Γ: $3y-z=-3$ $\quad z=3y+3$ $\quad z=0 \Rightarrow 3y=-3,\ y=-1$; $y=0 \Rightarrow z=3$

$x^2+y^2=1$

$$\iint_D \sqrt{1+p^2+q^2}\,dx\,dy=\begin{bmatrix} p=z_x=0 \\ q=z_y=3 \end{bmatrix}=$$

$$=\iint_D \sqrt{1+9}\,dx\,dy=\sqrt{10}\int_0^{2\pi}d\varphi\int_0^1\rho\,d\rho=$$

$$=\sqrt{10}\cdot 2\pi\cdot\frac{1}{2}=\boxed{\sqrt{10}\,\pi}$$



(XII) 23.6.2008.

1. $z(x,y)=(x+y^2)e^{\frac{x}{2}}$, $dz(0,1)=?$ $z_x dx+z_y dy$

$z_x=e^{\frac{x}{2}}+(x+y^2)e^{\frac{x}{2}}\cdot\frac{1}{2}=e^{\frac{x}{2}}\left(1+\frac{x}{2}+\frac{y^2}{2}\right)$ $\quad z_x(0,1)=1+\frac{1}{2}=\frac{3}{2}$

$z_y=2ye^{\frac{x}{2}}+(x+y^2)e^{\frac{x}{2}}\cdot\frac{1}{2}=e^{\frac{x}{2}}\left(2y+\frac{x}{2}+\frac{y^2}{2}\right)$ $\quad z_y(0,1)=2+\frac{1}{2}=\frac{5}{2}$

$$\boxed{dz(0,1)=\frac{3}{2}dx+\frac{5}{2}dy}$$

---

2. $y^z+z^y=x\Big|'_x$, $\frac{\partial z}{\partial x}=z_x=?$ $\qquad \boxed{z_x=\frac{1}{y^z\ln y+yz^{y-1}}}$

$a^x \quad x^n$

$z_x y^z\ln y+yz^{y-1}\cdot z_x=1$

$z_x(y^z\ln y+yz^{y-1})=1$

---

3. $\sum_{n=2}^{\infty}\frac{(-1)^{n-1}x^n}{n-1}$

$R=\lim_{n\to\infty}\left|\frac{a_n}{a_{n+1}}\right|=\lim_{n\to\infty}\frac{\frac{1}{n-1}}{\frac{1}{n+1-1}}=\lim_{n\to\infty}\frac{n}{n-1}=\lim_{n\to\infty}\frac{\cancel{n}}{\cancel{n}(1-\frac{1}{n})}=1 \qquad -1<x<1$

крајеви: $x=1$: $\sum_{n=2}^{\infty}\frac{(-1)^{n-1}}{n-1}$ $\overset{\text{о.к.}}{\sim}$ $\sum_{n=2}^{\infty}\frac{1}{n}$, $p=1$ $(p\le 1\Rightarrow$ ap$)$ укв ap ?

$x=-1$: $\sum_{n=2}^{\infty}\frac{(-1)^{n-1+n}}{n-1}\Rightarrow$ ap према претходном

1° $\lim_{n\to\infty}a_n=\lim_{n\to\infty}\frac{1}{n-1}=0$

2° $f(x)\downarrow$ $\quad f(x)=\frac{1}{x-1}$ $\quad f'(x)=-\frac{1}{(x-1)^2}\downarrow$ $\Big\}$ укв $\qquad \boxed{x\in[-1,1]}$

$\sum_{n=1}^{\infty}\frac{x^n}{n}=-\ln(1-x)$

$\sum_{n=2}^{\infty}\frac{(-1)^{n-1}x^n}{n-1}=x\sum_{n=2}^{\infty}\frac{(-1)^{n-1}x^{n-1}}{n-1}=x\sum_{n=2}^{\infty}\frac{(-x)^{n-1}}{n-1}=x\sum_{n=1}^{\infty}\frac{(-x)^n}{n}=\boxed{-x\cdot\ln(1+x)}$

---

4. $f(x)=x+1$, $x\in[0,\pi]$, $b_1=?$ синусни

$b_n=\frac{2}{l}\int_0^l f(x)\cdot\sin\frac{n\pi x}{l}dx=\frac{2}{\pi}\int_0^\pi(x+1)\sin\frac{\pi x}{\pi}dx=\frac{2}{\pi}\int_0^\pi x\sin x\,dx+\frac{2}{\pi}\int_0^\pi\sin x\,dx=\begin{bmatrix}u=x & dv=\sin x\,dx\\ du=dx & v=-\cos x\end{bmatrix}=$

$=\frac{2}{\pi}\left(-x\cos x\Big|_0^\pi+\int_0^\pi\cos x\,dx\right)-\frac{2}{\pi}\cos x\Big|_0^\pi=\frac{2}{\pi}\left(-\pi\cdot\cos\pi+\cancel{\sin x\Big|_0^\pi}\right)-\frac{2}{\pi}(\cos\pi-\cos 0)=$

$=\frac{2}{\pi}(+\pi)-\frac{2}{\pi}(-1-1)=2+\frac{4}{\pi}=\boxed{\frac{2\pi+4}{\pi}}$

5. $y' + x(y^2 + y') = 0$

$1 + \frac{xy^2}{y'} + x = 0$

$\frac{xy^2}{\frac{dy}{dx}} = -x - 1 \quad \Big| : x$

$\frac{y^2 dx}{dy} = -1 - \frac{1}{x}$

$\frac{y^2}{dy} = \frac{\left(-1 - \frac{1}{x}\right)}{dx}$

$\frac{dy}{y^2} = \frac{dx}{\frac{-x-1}{x}}$

$\frac{dy}{y^2} = \frac{x\,dx}{-x-1}$

$\int \frac{dy}{y^2} = \int \frac{-x\,dx}{x+1} \quad \Big| \int$

$\int y^{-2} dy = -\int \frac{x+1-1}{x+1}\, dx$

$-\frac{1}{y} = -\int dx + \int \frac{1}{x+1}\, dx$

$$\boxed{-\frac{1}{y} = -x + \ln(x+1) + C}$$

6. $y''' - y'' = x$

$y_N = y_H + y_p$

(y_H) $r^3 - r^2 = 0$

$r^2(r-1) = 0$

$r_{1,2} = 0 \quad r_3 = 1$

$e^{0 \cdot x} \quad xe^{0 \cdot x} \quad e^x$

$y_H = C_1 + C_2 x + C_3 e^x$

(y_p) $F(x) = x$; 0 je корен реда 2

$y_p = x^2(Ax + B) = Ax^3 + Bx^2 \qquad y_p = x^2\left(-\frac{1}{6}x - \frac{1}{2}\right)$

$y_p' = 3Ax^2 + 2Bx$

$y_p'' = 6Ax + 2B$

$y_p''' = 6A$

$$\boxed{y_N = C_1 + C_2 x + C_3 e^x + x^2\left(-\frac{1}{6}x - \frac{1}{2}\right)}$$

$6A - 6Ax - 2B = x$

$6A - 2B = 0$

$-6A = 1 \qquad \boxed{A = -\frac{1}{6}}$

$-1 - 2B = 0 \qquad -2B = 1 \qquad \boxed{B = -\frac{1}{2}}$

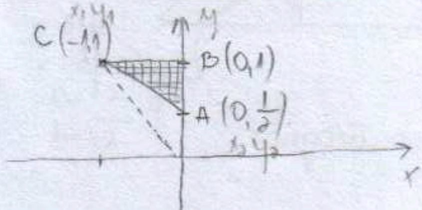
7. $A(0, \frac{1}{2})$

$B(0,1)$

$C(-1,1)$

$x = \rho\cos\varphi$

$y = \rho\sin\varphi$



$$\boxed{\frac{\pi}{2} \le \varphi \le \frac{3\pi}{4} \qquad \frac{1}{2\sin\varphi + \cos\varphi} \le \rho \le \frac{1}{\sin\varphi}}$$

1) $y = 1 \qquad \rho\sin\varphi = 1 \qquad \rho = \frac{1}{\sin\varphi}$

$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$

$y - 1 = \frac{\frac{1}{2} - 1}{0 + 1}(x + 1)$

$y - 1 = -\frac{1}{2}(x+1)$

$y = -\frac{1}{2}x - \frac{1}{2} + 1$

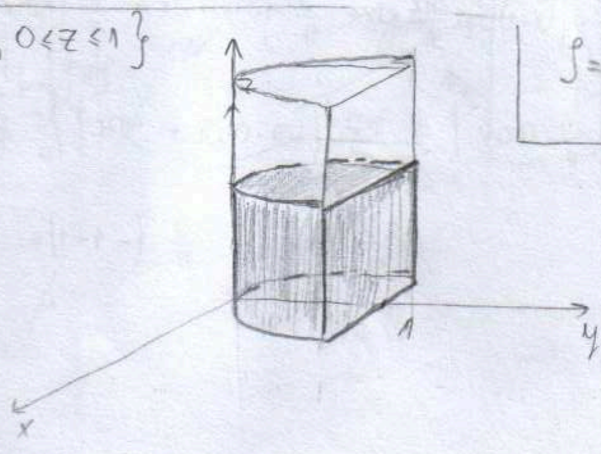
2) $y = -\frac{1}{2}x + \frac{1}{2} \qquad \rho\sin\varphi = -\frac{1}{2}\rho\cos\varphi + \frac{1}{2}$

$\rho\sin\varphi + \frac{1}{2}\rho\cos\varphi = \frac{1}{2}$

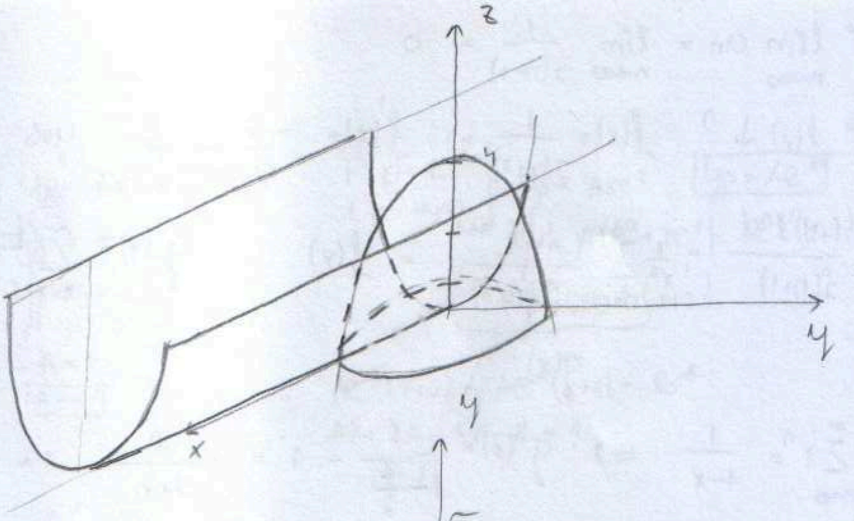
$\rho(\sin\varphi + \frac{1}{2}\cos\varphi) = \frac{1}{2}$

$\rho = \frac{1}{2(\sin\varphi + \frac{1}{2}\cos\varphi)}$

8. $T = \{(x,y,z) \in \mathbb{R}^3 \mid x^2 \le y \le 1,\ 0 \le z \le 1\}$



9. C: $z=4-x^2-y^2$ $\quad z-4=-(x^2+y^2)$
$z=y^2$ $\quad z=y^2$

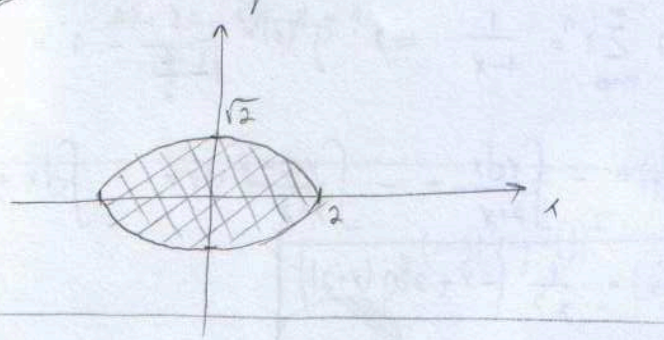


$y^2=4-x^2-y^2$

$x^2+2y^2=4$

$\frac{x^2}{4}+\frac{y^2}{2}=1$ елипса

$a=2$
$b=\sqrt{2}$



$$\boxed{\left.\begin{array}{l} x=2\cos t \\ y=\sqrt{2}\sin t \\ z=2\sin^2 t \end{array}\right\} -\pi\le t\le\pi}$$

---

10. $x^2+y^2+z^2=R^2$ $\qquad \frac{P}{2}=\iint_D\sqrt{1+p^2+q^2}\,dx\,dy$

исти као VII (10.)

XIII 19.4.2008.

1. $u(x,y,z)=\frac{x^z}{y}$, $du(1,1,1)=?$ $\qquad du=U_x dx+U_y dy+U_z dz$

$x^n \quad a^x \quad x^z$

$$U_x=\frac{zx^{z-1}\cdot y-\cancel{x^z\cdot 0}}{y^2}=\frac{zx^{z-1}}{y} \qquad U_x=1$$

$$\boxed{du=dx-dy}$$

$$U_y=-\frac{x^z}{y^2} \qquad U_y=-1$$

$$U_z=\frac{x^z\ln x\cdot y-\cancel{x^z\cdot 0}}{y^2}=\frac{x^z\ln x}{y} \qquad U_z=0$$

---

2. $e^z=x\,\mathrm{arctg}\,\frac{y}{z}\Big|'_y$, $\frac{\partial z}{\partial y}=z_y$, $z=z(x,y)$

$$e^z\cdot z_y=x\cdot\frac{1}{1+\left(\frac{y}{z}\right)^2}\cdot\frac{z-yz_y}{z^2}=\frac{xz^2}{x^2+y^2}\,\frac{z-yz_y}{\cancel{z^2}}=\frac{x(z-yz_y)}{x^2+y^2}=e^z z_y$$

$$\frac{xz}{x^2+y^2}-\frac{xyz_y}{x^2+y^2}=e^z z_y \qquad \frac{xz}{x^2+y^2}=e^z z_y+\frac{xyz_y}{x^2+y^2} \qquad z_y\left(e^z+\frac{xy}{x^2+y^2}\right)=\frac{xz}{x^2+y^2} \qquad z_y=\frac{\frac{xz}{\cancel{x^2+y^2}}}{\frac{e^z(x^2+y^2)+xy}{\cancel{x^2+y^2}}}$$

$$\boxed{z_y=\frac{xz}{e^z(x^2+y^2)+xy}}$$

---

3. $\sum_{n=1}^{\infty}\frac{(-1)^n x^{n-1}}{2^n(n+1)}$ – об.
– сума

$$R=\lim_{n\to\infty}\left|\frac{a_n}{a_{n+1}}\right|=\lim_{n\to\infty}\frac{\frac{1}{2^n(n+1)}}{\frac{1}{2^n\cdot 2(n+2)}}=\lim_{n\to\infty}\frac{\cancel{2^n}\cdot 2(n+2)}{\cancel{2^n}(n+1)}=2 \qquad -2<x<2$$

крајеви: $x=2$: $\sum_{n=1}^{\infty}\frac{(-1)^n\cdot\cancel{2^n}\frac{1}{2}}{\cancel{2^n}(n+1)}=\sum_{n=1}^{\infty}\frac{(-1)^n}{2(n+1)}$ (ок) $\xrightarrow{}$ цкв ок?

$x=-2$: $\sum_{n=1}^{\infty}\frac{\cancel{(-1)^n}\cancel{(-2)^n}\frac{1}{(-2)}}{\cancel{2^n}(n+1)}=\sum_{n=1}^{\infty}\frac{1}{-2(n+1)}\sim\frac{1}{n}$ (дв)

$1^\circ \lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{1}{2(n+1)} = 0$

$2^\circ\ f(x) \downarrow ?\quad f(x) = \frac{1}{2(x+1)} \Rightarrow f'(x) = \frac{-2}{4(x+1)^2} \downarrow$ yx6

$\boxed{\text{O.K. } x\in[-2,2]}$

$$\sum_{n=1}^{\infty} \frac{(-1)^n x^{n-1}}{2^n(n+1)} = \frac{1}{x^2} \underbrace{\sum_{n=1}^{\infty} \frac{(-1)^n x^{n+1}}{2^n(n+1)}}_{g(x)} = f(x) \qquad g'(x) = \sum_{n=1}^{\infty} \frac{(-1)^n x^n}{2^n} = \sum_{n=1}^{\infty} \frac{(-x)^n}{2^n} = \sum_{n=1}^{\infty} \left(-\frac{x}{2}\right)^n$$

$$\Rightarrow \sum_{n=0}^{\infty} x^n = \frac{1}{1-x} \Rightarrow g'(x) = \frac{1}{1+\frac{x}{2}} - 1 = \frac{2}{2+x} - 1 = \frac{\cancel{2}-\cancel{2}-x}{2+x} = -\frac{x}{2+x}$$

$$g(x) = -\int \frac{x\,dx}{2+x} = -\int \frac{x+2-2}{x+2}\,dx = -\int dx + 2\int \frac{dx}{x+2} = -x + 2\ln(x+2) \Rightarrow$$

$$\boxed{f(x) = \frac{1}{x^2}\left(-x + 2\ln(x+2)\right)}$$

---

④ $\Phi(x) = \text{sgn}(x^2-1)$, $x\in[0,2]$, $a_3 = ?$

- косинусни

$$\text{sgn}(x^2-1) = \begin{cases} 1, & x^2-1>0 \quad x\in(-\infty,-1)\cup(1,+\infty) \quad x\in(1,2] \\ 0, & x^2-1=0 \quad x=1 \quad x=-1 \\ -1, & x^2-1<0 \quad x\in(-1,1) \quad x\in[0,1) \end{cases}$$

$$a_n = \frac{2}{l}\int_0^l f(x)\cos\frac{n\pi x}{l}\,dx =$$

$$= \int_1^2 \cos\frac{3\pi x}{2}\,dx - \int_0^1 \cos\frac{3\pi x}{2}\,dx = \left.\frac{\sin\frac{3\pi x}{2}}{\frac{3\pi}{2}}\right|_1^2 - \left.\frac{\sin\frac{3\pi x}{2}}{\frac{3\pi}{2}}\right|_0^1 =$$

$$= \frac{2}{3\pi}\left(\cancel{\sin 3\pi} - \sin\frac{3\pi}{2}\right) - \frac{2}{3\pi}\left(\sin\frac{3\pi}{2} - \cancel{\sin 0}\right) = \frac{2}{3\pi}\left(-2\sin\frac{3\pi}{2}\right) = -\frac{4}{3\pi}\sin\frac{3\pi}{2} = \boxed{\frac{4}{3\pi}}$$

---

⑤ $y' = \sin(x+y)$

$\cancel{\frac{dy}{dx} = \sin(x+y) = \sin x\cos y + \cos x \sin y}$

$z = x+y$, $z = z(x)$

$y = z - x$

$y' = z' - 1$

$z' - 1 = \sin z$

$\frac{dz}{dx} = \sin z + 1$

$\frac{1}{dx} = \frac{\sin z}{dz} + \frac{1}{dz}$

$dx = \frac{dz}{\sin z} + dz \quad \Big|\int$

$\int dx = \int \frac{dz}{\sin z} + \int dz \quad \left[u = \text{tg}\frac{z}{2} \quad dz = \frac{2du}{1+u^2} \quad \sin z = \frac{2u}{1+u^2} \right]$

$x = \int \frac{2du}{2u} + z$

$x = \ln \text{tg}\frac{z}{2} + z$

$\boxed{\cancel{x} = \ln\text{tg}\frac{x+y}{2} + \cancel{x} + y + c} \Rightarrow y = -\ln\text{tg}\frac{x+y}{2} + c$

⑥ $y'' - y' = 1 \cdot e^x$

$y_N = y_H + y_p$

(y_H) $r^2 - r = 0$

$r(r-1) = 0$

$r_1 = 0 \quad r_2 = 1$

$e^{0 \cdot x} \quad e^x$

$y_H = C_1 + C_2 e^x$

(y_p) $\left.\begin{array}{l} F(x) = 1 \\ 0 \text{ je rešenje} \end{array}\right\}$ $y_p = Ax = \boxed{-x}$

$y_p' = A$

$y_p'' = 0$

$-A = 1$

$\boxed{A = -1}$

$\left.\begin{array}{l} F(x) = e^x \\ 1 \text{ je nula} \end{array}\right\}$ $y_p = Axe^x$ $\quad \boxed{y_p = xe^x}$

$y_p' = A(e^x + xe^x) = Ae^x(x+1)$

$y_p'' = Ae^x(x+2)$

$A\cancel{e^x}(x+2) - A\cancel{e^x}(x+1) = \cancel{e^x}$

$\cancel{Ax} + 2A - \cancel{Ax} - A = 1$

$\boxed{A = 1}$

$\boxed{y_N = C_1 + C_2 e^x - x + xe^x}$

---

⑦ $D = \left\{(x,y) \in \mathbb{R}^2 \mid x^2 + y^2 \leq x,\ x + y \geq 1\right\}$

$x^2 - x + y^2 \leq 0 \qquad y \geq -x + 1$

$(x - \frac{1}{2})^2 + y^2 = \frac{1}{4}$

$x = \rho\cos\varphi$

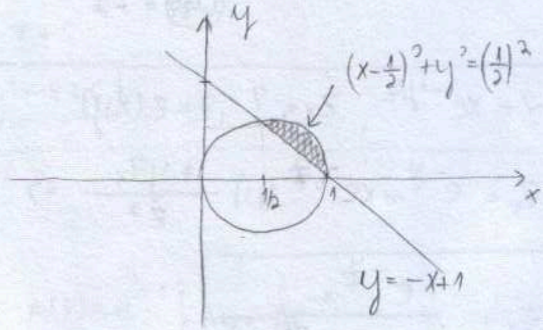
$y = \rho\sin\varphi$

$\Downarrow$ ovako je lakše (mogo može u $x = \rho\cos\varphi$)

$\left.\begin{array}{l} x - \frac{1}{2} = \rho\cos\varphi \\ y = \rho\sin\varphi \end{array}\right\}$ K: $\rho^2 = \frac{1}{4} \quad \rho = \frac{1}{2}$

П: $\rho\sin\varphi = -\rho\cos\varphi - \frac{1}{2} + 1 = -\rho\cos\varphi + \frac{1}{2}$

$\rho(\sin\varphi + \cos\varphi) = \frac{1}{2} \qquad \rho = \dfrac{1}{2(\sin\varphi + \cos\varphi)}$



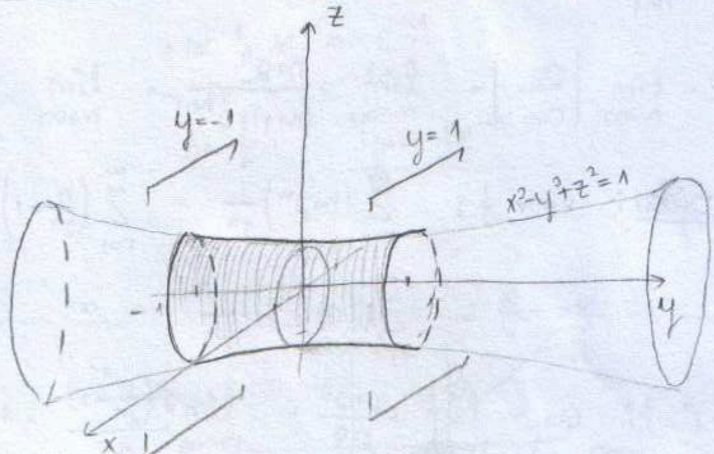
$$\boxed{\begin{array}{c} 0 \leq \varphi \leq \frac{\pi}{2} \\ \dfrac{1}{2(\sin\varphi + \cos\varphi)} \leq \rho \leq \dfrac{1}{2} \end{array}}$$

---

⑧ $T = \left\{(x,y,z) \in \mathbb{R}^3 \mid x^2 + z^2 - 1 \leq y^2 \leq 1\right\}$

$y^2 = x^2 + z^2 - 1 \qquad y^2 - 1 \leq 0$

$x^2 - y^2 + z^2 = 1 \qquad (y-1)(y+1) \leq 0$



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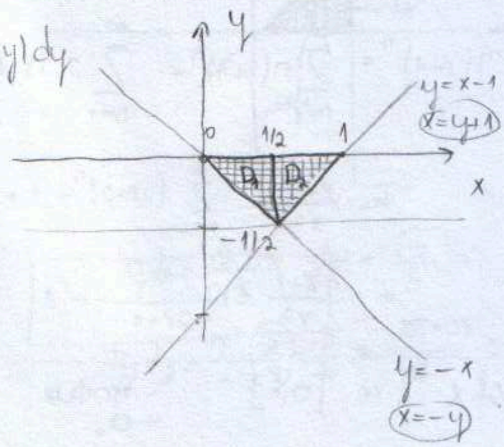
⑨ $\int\limits_{-1/2}^{0} dy \int\limits_{-y}^{y+1} f(x,y)\,dx = \int\limits_{0}^{1/2} dx \int\limits_{-x}^{0} f(x,y)\,dy + \int\limits_{1/2}^{1} dx \int\limits_{x-1}^{0} f(x,y)\,dy$

$0 \leq y \leq -\frac{1}{2}$

$y + 1 \leq x \leq -y$

$y + 1 = x \qquad x = -y$

$y = x - 1 \qquad y = -x$



(XIV) 14.4.2008.

① $u(x,y,z)=\frac{x}{y^2}-\frac{y^2}{z}$, $d^2u(1,1,1)=\ ?\ U_{xx}dx^2+U_{yy}dy^2+U_{zz}dz^2+2U_{xy}dxdy+2U_{xz}dxdz+2U_{yz}dydz$

$U_x=\frac{1}{y^2}$ $\quad U_{xx}=0$ $\quad U_{xy}=-\frac{2}{y^3}$

$U_{xx}=0$ $\quad U_{xy}=-2$

$U_y=-\frac{2x}{y^3}-\frac{1}{z}2y$ $\quad U_{yy}=\frac{+3\cdot 2x}{y^4}-\frac{2}{z}$ $\quad U_{xz}=0$

$U_{yy}=4$ $\quad U_{xz}=0$

$\boxed{d^2u(1,1,1)=4dy^2-2dz^2-2dxdy+2dydz}$

$U_z=+\frac{y^2}{z^2}$ $\quad U_{zz}=\frac{-2y^2}{z^3}$ $\quad U_{yz}=\frac{+2y}{z^2}$

$U_{zz}=-2$ $\quad U_{yz}=2$

② $z=xe^{-y/z}$, $z_x=\ ?$, $z=z(x,y)$

$z_x=e^{-\frac{y}{z}}+xe^{-\frac{y}{z}}(+1)\frac{+yz_x}{z^2}\ \Rightarrow\ z_x=e^{-\frac{y}{z}}+\frac{xe^{-\frac{y}{z}}yz_x}{z^2}$ $\quad z_x\left(1-\frac{xye^{-\frac{y}{z}}}{z^2}\right)=e^{-\frac{y}{z}}$

$$\boxed{z_x=\frac{e^{-\frac{y}{z}}}{1-\frac{xye^{-\frac{y}{z}}}{z^2}}}$$

③ $\sum_{n=1}^{\infty}(n+2^n)(x+1)^n$ $\quad$ – О.к.
– сума

$R=\lim_{n\to\infty}\left|\frac{a_n}{a_{n+1}}\right|=\lim_{n\to\infty}\frac{n+2^n}{(n+1)+2^{n+1}}=\lim_{n\to\infty}\frac{2^n\left(\frac{n}{2^n}+1\right)}{2^n\left(\frac{n}{2^n}+\frac{1}{2^n}+2\right)}=\frac{1}{2}$ $\quad -\frac{1}{2}<x+1<\frac{1}{2}$

$-\frac{3}{2}<x<-\frac{1}{2}$

крајеви: $x=-\frac{1}{2}$: $\sum_{n=1}^{\infty}(n+2^n)\frac{1}{2^n}=\sum_{n=1}^{\infty}\left(\frac{n}{2^n}+1\right)=\infty$ (дв) $\quad x\in\left(-\frac{3}{2},-\frac{1}{2}\right)$

$x=-\frac{3}{2}$: $\sum_{n=1}^{\infty}(n+2^n)\frac{(-1)^n}{2^n}=\infty$ (дв) према претходном

нцв ?
дв ?

$\boxed{\text{О.к. } x\in\left(-\frac{3}{2},-\frac{1}{2}\right)}$

1° $\lim_{n\to\infty}a_n=\lim_{n\to\infty}\frac{n+2^n}{2^n}=\lim_{n\to\infty}\frac{2^n\left(\frac{n}{2^n}+1\right)}{2^n}=1$ (дв)

$\sum_{n=1}^{\infty}nx^n=\frac{x}{(1-x)^2}$

$\sum_{n=0}^{\infty}x^n=\frac{1}{1-x}$

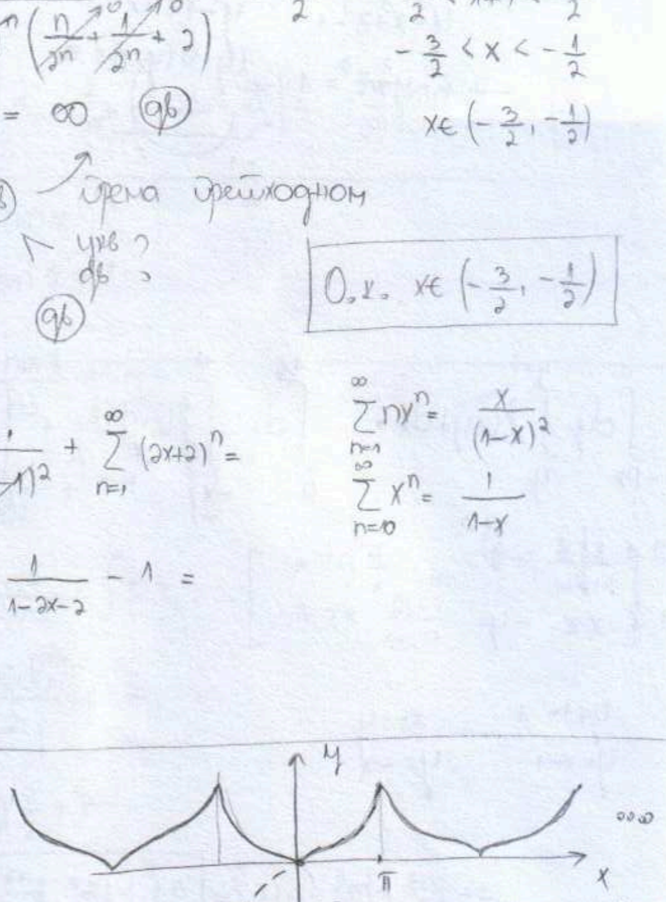
$\sum_{n=1}^{\infty}(n+2^n)(x+1)^n=\sum_{n=1}^{\infty}n(x+1)^n+\sum_{n=1}^{\infty}2^n(x+1)^n=\frac{x+1}{(1-x-1)^2}+\sum_{n=1}^{\infty}(2x+2)^n=$

$=\frac{x+1}{x^2}+\sum_{n=0}^{\infty}(2x+2)^n-1=\frac{x+1}{x^2}+\frac{1}{1-2x-2}-1=$

$=\boxed{\frac{x+1}{x^2}-\frac{1}{+2x+1}-1}$?

④ $f(x)=\operatorname{sh}x$, $x\in[0,\pi]$ $\quad$ – график
– a_0

– косинусни



парно продужење
$f(x)\cap\phi(x)$

$a_0=\frac{2}{l}\int_0^{l}f(x)dx=\frac{2}{\pi}\int_0^{\pi}\operatorname{sh}x\,dx=\frac{2}{\pi}\left(\int_0^{\pi}\frac{e^x}{2}-\int_0^{\pi}\frac{e^{-x}}{2}\right)=(*)$ $\quad \operatorname{sh}x=\frac{e^x-e^{-x}}{2}$

$④ = \frac{2}{\pi}\left[\frac{1}{2}e^x\Big|_0^\pi - \frac{1}{2}\begin{bmatrix} e^{-x}=t \\ -e^{-x}dx=dt \\ e^{-x}dx=-dt\end{bmatrix}\right] = \frac{2}{\pi}\left[\frac{1}{2}(e^\pi-1)+\frac{1}{2}e^{-x}\Big|_0^\pi\right] = \frac{2}{\pi}\left[\frac{1}{2}e^\pi-\frac{1}{2}+\frac{1}{2}(e^{-\pi}-1)\right] =$

$= \frac{2}{\pi}\left[\frac{e^\pi}{2}-\frac{1}{2}+\frac{e^{-\pi}}{2}-\frac{1}{2}\right] = \frac{\cancel{2}}{\pi\cancel{2}}\left(e^\pi-2+\frac{1}{e^\pi}\right) = \frac{1}{\pi}\,\frac{e^{2\pi}-2e^\pi+1}{e^\pi} = \boxed{\frac{1\,(e^\pi-1)^2}{\pi e^\pi}}$

---

5. $y'+1 = tg(x+y)$

$x+y=z \quad , z=z(x)$

$y = z-1$

$y' = z'-1$

$z'\cancel{-1}\cancel{+1} = tg\, z$

$z' = tg\, z$

$\frac{dz}{dx} = tg\, z$

$\frac{tg\, z}{dz} = \frac{1}{dx}$

$\frac{dz}{tg\, z} = dx \quad \Big|\int$

$\int\frac{dz}{tg z} = \int dx$

$\int\frac{\cos z}{\sin z}dz = \int dx \quad \begin{bmatrix}\sin z = t\\ \cos z dz = dt\end{bmatrix}$

$\int\frac{dt}{t} = \int dx$

$\ln\sin z = x+C$

$\boxed{\ln\sin(x+y) = x+C}$

- обично долази као 5. задатак:
  - д.ј. која разд. пр.
  - смена
  - Њутнова ф.

6. - нехомогена ~~(вишег реда)~~

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6. $y'''-y'' = 1-e^{-x}$

$y_N = y_H + y_P$

y_H: $r^3-r^2=0$

$r^2(r-1)=0$

$r_{1/2}=0 \quad r_3=1$

$e^{0\cdot x} \quad xe^{0\cdot x} \quad e^x$

$y_H = C_1+C_2x+C_3e^x$

y_P: $F(x)=1$, 0 је 0 реда 2 $\Big\}$ $y_P = Ax^2$

$y_P' = 2Ax$

$y_P'' = 2A$

$y_P''' = 0$

$-2A = 1$

$A = -\frac{1}{2}$

$y_P = -\frac{1}{2}x^2$

$F(x) = -e^{-x}$, -1 није 0 $\Big\}$ $y_P = Ae^{-x}$

$y_P' = -Ae^{-x}$

$y_P'' = Ae^{-x}$

$y_P''' = -Ae^{-x}$

$-Ae^{-x}-Ae^{-x} = -e^{-x}$

$-2Ae^{-x} = -e^{-x}$

$A = \frac{1}{2} \qquad y_P = \frac{1}{2}e^{-x}$

$\boxed{y_N = C_1+C_2x+C_3e^x - \frac{1}{2}x^2+\frac{1}{2}e^{-x}}$

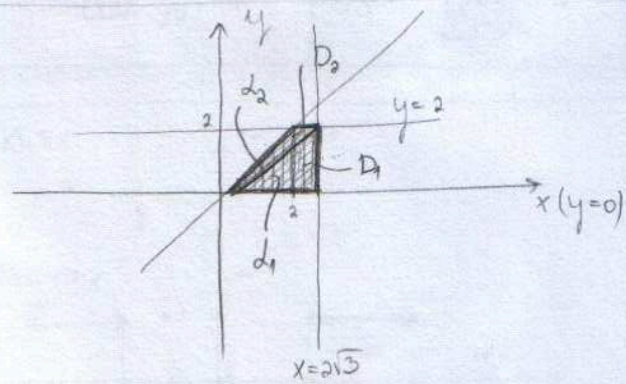
---

7. $y=0 \vee \quad x=\rho\cos\varphi$

$x=2\sqrt{3} \vee \quad y=\rho\sin\varphi$

$y=2 \vee$

$y=x \vee$



D_1: $x=2\sqrt{3}$

$\rho\cos\varphi = 2\sqrt{3}$

$\rho = \frac{2\sqrt{3}}{\cos\varphi}$

$\boxed{\begin{matrix} 0 \le \varphi \le \frac{\pi}{6} \\ 0 \le \rho \le \frac{2\sqrt{3}}{\cos\varphi}\end{matrix}}$

D_2: $y=2$

$\rho\sin\varphi = 2$

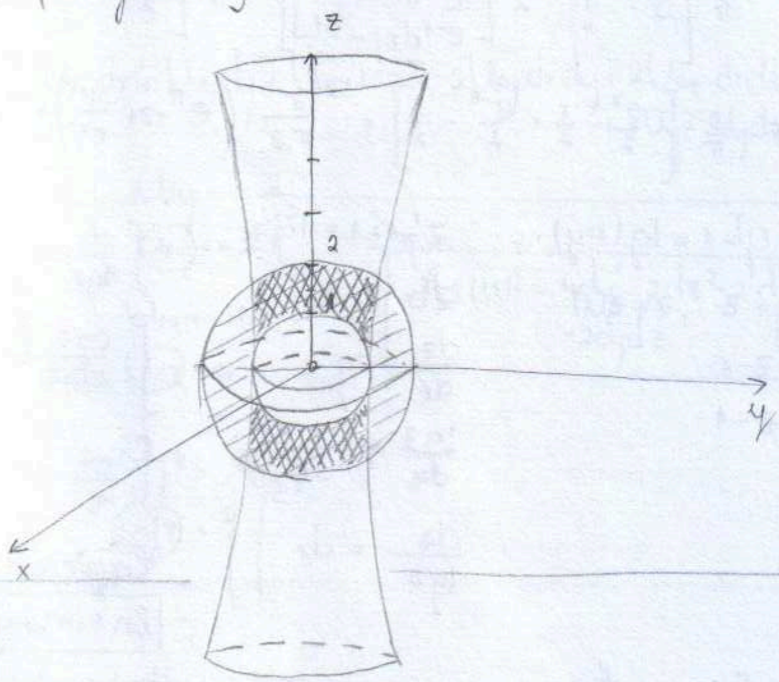
$\rho = \frac{2}{\sin\varphi}$

$\boxed{\begin{matrix} \frac{\pi}{6} \le \varphi \le \frac{\pi}{4} \\ 0 \le \rho \le \frac{2}{\sin\varphi}\end{matrix}}$

$tg\, d_1 = \frac{2}{2\sqrt{3}} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3} \Rightarrow d_1 = \frac{\pi}{6}$

$tg\, d_2 = ? \quad \frac{\pi}{4}-\frac{\pi}{6} = \frac{3\pi-2\pi}{12} = \frac{\pi}{12} = d_2$

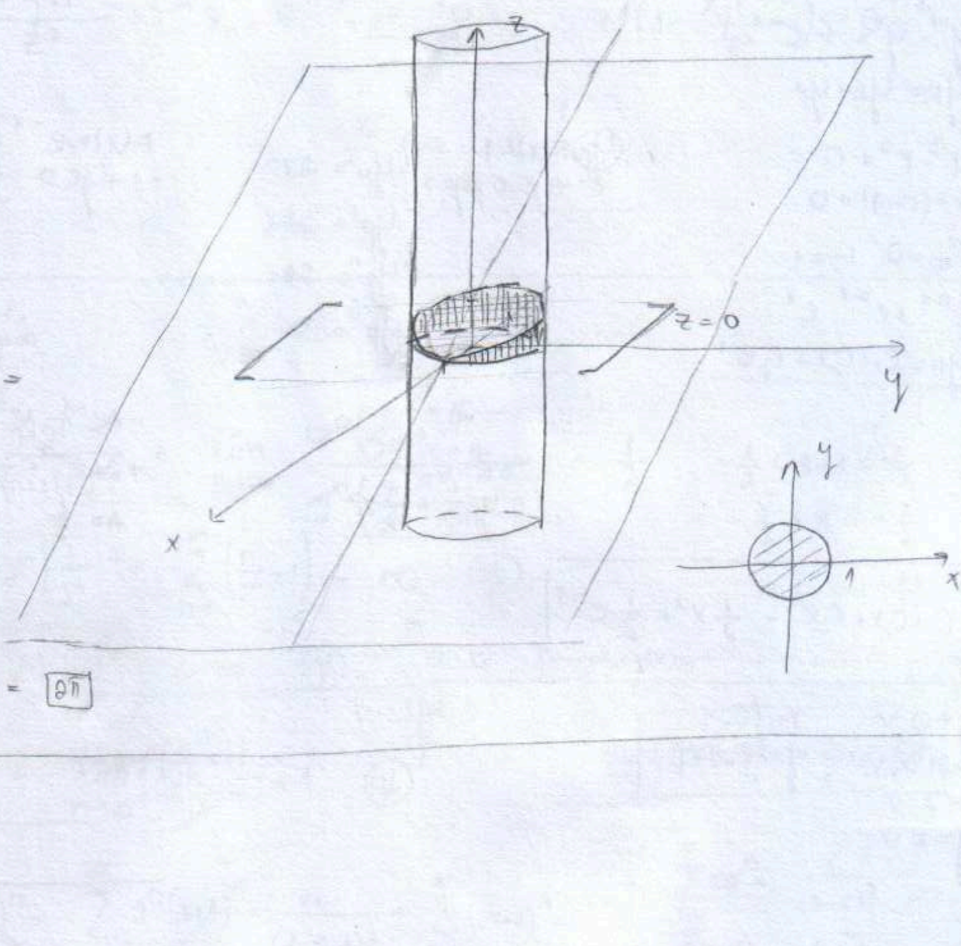
8. $T = \left\{ (x,y,z) \in \mathbb{R}^3 \mid 1 \le x^2+y^2+z^2 \le 4 \,,\; x^2+y^2-z^2 \le 1 \right\}.$



9. $x^2+y^2=1 \qquad P = ?$

$z=0$

$z = 1-x = -x+1$



$$P = \int_c (z_1 - z_2)\, ds = \int_c (-x+1)\, ds =$$

$$\left[\begin{array}{ll} ds = \sqrt{x_t'^2 + y_t'^2} & \\ x = \cos t & x_t' = -\sin t \\ y = \sin t & y_t' = \cos t \end{array}\right]$$

$$= \int_{-\pi}^{\pi} (-\cos t + 1)\sqrt{\sin^2 t + \cos^2 t}\, dt =$$

$$= -\int_{-\pi}^{\pi} \cos t\, dt + \int_{-\pi}^{\pi} dt =$$

$$= \cancel{-\sin t \Big|_{-\pi}^{\pi}}^{\,0} + t \Big|_{-\pi}^{\pi} = \pi + \pi = \boxed{2\pi}$$

(XV) 10.10.2007.

1. $z = z(x,y)$ $\quad z\ln(xyz) = \sqrt{\frac{x}{z}} \Big|'_x \Big|'_y$, $dz = ? = z_x dx + z_y dy$

$z_x \ln(xyz) + z\frac{\cancel{yz}}{xy\cancel{z}} = \frac{1}{2\sqrt{\frac{x}{z}}}\frac{z - xz_x}{z^2}$

$z_x \ln(xyz) + \frac{z}{x} = \frac{1}{2z}\sqrt{\frac{z}{x}} - \frac{1}{2}\sqrt{\frac{z}{x}}\frac{xz_x}{z^2}$

$z_x \ln(xyz) + \frac{1}{2}\sqrt{\frac{z}{x}}\frac{xz_x}{z^2} = \frac{1}{2z}\sqrt{\frac{z}{x}} - \frac{z}{x}$

$z_x\left(\ln(xyz) + \frac{1}{2}\sqrt{\frac{z}{x}}\frac{x}{z^2}\right) = \frac{1}{2z}\sqrt{\frac{z}{x}} - \frac{z}{x}$

$z_x = \dots$

$z_y \ln(xyz) + z\frac{\cancel{xz}}{xy\cancel{z}} = \frac{1}{2\sqrt{\frac{x}{z}}} \cdot \frac{-xz_y}{z^2}$

$z_y \ln(xyz) + \frac{xz_y}{2\sqrt{\frac{x}{z}}\cdot z^2} = -\frac{z}{y}$

$z_y\left(\ln(xyz + \frac{x}{2\sqrt{\frac{x}{z}}\cdot z^2}\right) = -\frac{z}{y}$

$z_y = \dots$

2. / 3. /

4. $\sum_{n=2}^{\infty}\frac{\ln n}{n}x^n$ $\quad R = \lim_{n\to\infty}\left|\frac{a_n}{a_{n+1}}\right| = \lim_{n\to\infty}\frac{\frac{\ln n}{n}}{\frac{\ln(n+1)}{n+1}} = \lim_{n\to\infty}\frac{(n+1)\ln n}{n\cdot\ln(n+1)} = \lim_{n\to\infty}\frac{\cancel{n}(1+\frac{1}{n})^{\to 0}\ln n}{\cancel{n}\ \ln n(1+\frac{1}{n})} =$

$= \lim_{n\to\infty}\frac{\ln n}{\ln n + \ln(1+\frac{1}{n})} = \lim_{n\to\infty}\frac{1}{1+\frac{\ln(1+\frac{1}{n})^{\to 0}}{\ln n_{\to 0}}} = 1$

$-1 < x < 1$

krajevi: $x=1$: $\sum_{n=2}^{\infty}\frac{\ln n}{n} \overset{\text{u.k.}}{\sim} \sum_{n=2}^{\infty}\frac{1}{n},\ p=1$ (dv)

$x=-1$: $\sum_{n=2}^{\infty}\frac{\ln n}{n}(-1)^n$ (dv) uvk? dv?

$1^\circ\ \lim a_n = \lim_{n\to\infty}\frac{\ln n}{n} = \lim_{n\to\infty}\ln \sqrt[n]{n}^{\to 1} = 0$

$2^\circ\ f(x)\downarrow\ ?\quad f(x) = \frac{\ln x}{x} \Rightarrow$

$f'(x) = \frac{\frac{1}{x}\cdot x - \ln x}{x^2} = \frac{1-\ln x}{x^2} < 0 \downarrow$ (uvk)

$1 - \ln x < 0$

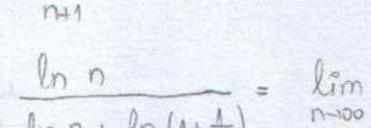
$\ln x > 1$

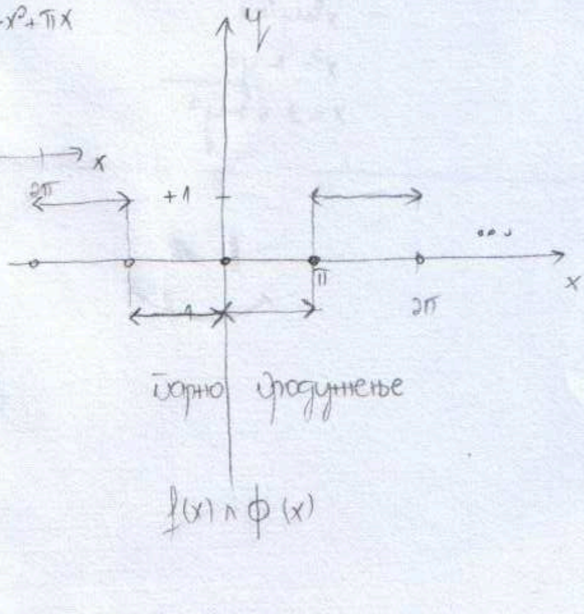
$x > e\ (2.7)^2\ \omega$

$\boxed{\text{O.k. } x\in[-1,1)}$

5. $f(x) = \operatorname{sgn} x(\pi - x),\ x\in[0,2\pi]$ $\quad$ – grafik $\quad y = -x^2 + \pi x$
– kosinusni $\quad$ – a_0

$\operatorname{sgn} x(\pi-x) = \begin{cases} -1 & x(\pi-x) < 0 \quad (\pi, 2\pi] \\ 0 & x(\pi-x) = 0 \quad 0,\pi \\ 1 & x(\pi-x) > 0 \quad (0,\pi) \end{cases}$





parno produženje

$f(x) \cap \phi(x)$

$a_0 = \frac{2}{l}\int_0^l f(x)dx = \frac{2}{2\pi}\left[+\int_0^\pi dx - \int_\pi^{2\pi}dx\right] = \frac{1}{\pi}\left(+x\Big|_0^\pi - x\Big|_\pi^{2\pi}\right) =$

$= \frac{1}{\pi}\left(\pi - (2\pi - \pi)\right) = \frac{1}{\pi}(\pi - \pi) = \boxed{0}$

(6.) $y^{V} + y''' = 2$

$y_N = y_H + y_p$

(y_H) $r^5 + r^3 = 0$

$r^3(r^2+1) = 0$

$r_{1/2/3} = 0 \quad r_{4/5} = \pm i \quad \alpha = 0 \quad \beta = 1$

$e^0 \quad xe^0 \quad x^2e^0 \quad \cos x \quad \sin x$

$y_H = C_1 + C_2x + C_3x^2 + C_4\cos x + C_5 \sin x$

(y_p) $F(x) = 2$
0 је 0 реда
3

$y_p = Ax^3$

$y_p' = 3Ax^2$

$y_p'' = 6Ax$

$y_p''' = 6A$

$y_p^{IV} = 6$

$y_p^{V} = 0$

$6A = 2$

$A = 3$

$\boxed{y_p = 3x^3}$

$\boxed{y_N = C_1 + C_2x + C_3x^2 + C_4\cos x + C_5 \sin x + 3x^3}$

(7.) $D = \{(x,y) \in \mathbb{R}^2 \mid x^2+y^2 \leq 1 \ \land \ y + |x| \geq 0\}$

$x = \rho\cos\varphi$

$y = \rho\sin\varphi$

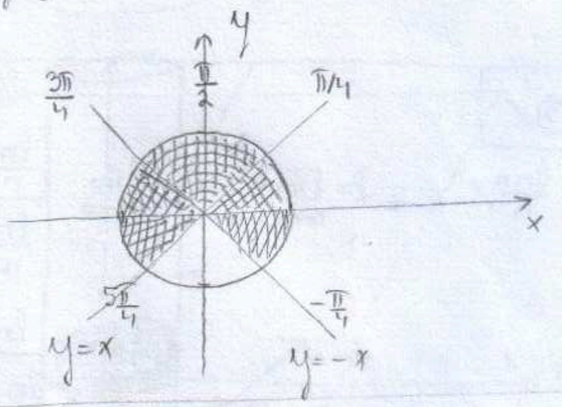
$y \geq -|x|$

$y \geq -x, \ x \geq 0$

$\lor$

$y \geq x, \ x < 0$

$\boxed{\begin{array}{c} -\frac{\pi}{4} \leq \varphi \leq \frac{5\pi}{4} \\ 0 \leq \rho \leq 1 \end{array}}$



(8.) исти као XIV (9.)

(9.) $\int_{-1}^{1} dx \int_{0}^{\sqrt{1-x^2}} f(x,y)\,dy = \int_{0}^{1} dy \int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} f(x,y)\,dx$

$x = 1$

$x = -1$

$y = 0$

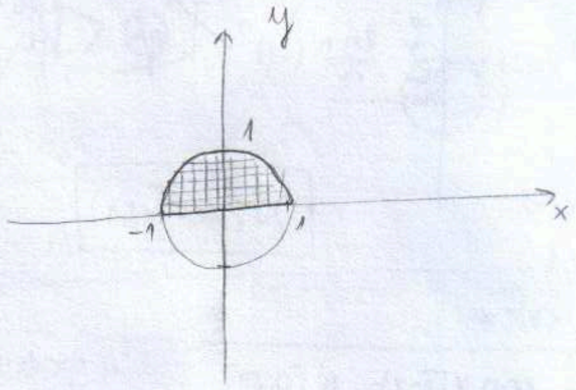
$y = \sqrt{1-x^2} \ )^2$

$y^2 = 1 - x^2$

$x^2 + y^2 = 1$

$x^2 = 1 - y^2$

$x = \pm\sqrt{1-y^2}$



Урош Мирковић

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